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Excluding Sum Stable Distributions as an Explanation of Second Moment Condition Failure – The Australian Evidence

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Abstract

This paper examines the issue of stock return moments in the Australian stock market. The existence of at least second moments is a fundamental assumption of underlying finance theory. We determine, using characteristic exponent point estimates, that the population variance may be infinite but on the same data, we also find that Hill-estimates are above 2 for all stocks, indicating that second moments do exist. This conflicting result is resolved by setting up a simulation experiment in which we show that the empirical combination of the Hill-estimate and the characteristic exponent lies outside the simulated confidence intervals for sum stables. This enhances the evidence for the existence of second moments in Australian stock returns.

Key words: sum stable distributions, Hill-estimator, second moment.

JEL Classification: C14, C22, G12.

1. Introduction

Undertaking empirical financial analysis is the cornerstone of most financial and investment research. However, when applying econometric procedures a number of assumptions are made regarding the existence of certain moments. For example, covariance stationary models require at least the first two moments to exist. If they do not, then the basis of the capital asset pricing model which undermines tests of market efficiency and event studies, is ruled out as a correct form of analysis.

The appropriate descriptive distribution for the returns of various speculative asset prices and the existence of second and higher moments have been debated since the 1960s. Mandelbrot (1963) was the first to suggest that observed fat tails could be explained by relying on the generalised central limit theorem (Feller, 1971). This generalised central limit theorem states that the sums of independent and identical random variables with infinite variance converge to a (non-normal) sum stable distribution. Mandelbrot's suggestion of infinite second moments meant that volatility, as measured by standard deviation, was a precarious sample dependent risk measure that would explode if longer time series were observed. To date, the standard procedure in the finance profession has been to ignore Mandelbrot's hypothesis and assume that his exploding sequential standard deviation plots could be generated by other fundamentals, such as time varying volatility. However, concern about the possibility of an infinite variance was reawakened when GARCH(1,1) models were shown to be borderline to integrated GARCH models (Engle and Bollerslev, 1986).

More recently, Jansen and de Vries (1991), Loretan and Phillips (1994), Hiemstra and Jones (1995), and Pagan (1996) have produced empirical evidence consistent with the existence of second moments. This research directly focused on the tail of the empirical distribution, instead of fitting a pre-specified density function to the full data set. The underlying logic is that the latter method will be unduly influenced by the vast majority of observations lying in the centre of the distribution and, therefore, will not carry appropriate information about the tails. In order to alleviate this problem, a distinction was made between distributions with exponentially declining tails (like the tails of the normal distribution) and distributions having a tail that declines with a power law (fat tailed distributions).

Given that fat tail distributions exist in most financial return series (Bollerslev, 1987) researchers assume the tails of the distribution to behave according to the Pareto (power) law, that is $\Pr(X>x) = k \cdot x^{-\alpha}$, with k constant and tail index α . Estimating this tail index (α) is of interest and is useful, in at least two respects. First, it provides a statistically rigorous way of discriminating between various suggested stochastic process candidates for modelling the unconditional distribution of financial returns. Although the sum stable distributions and other fat tailed processes, such as

the Student- t distribution and non-integrated ARCH models are not nested, they are *nested in the tail of the distribution*. Moreover, Jansen and de Vries (1991) and Hols and de Vries (1991) exploit the property that $\alpha < 2$ for stable processes, whereas otherwise α is restricted only to be positive. Finding α 's that are statistically significantly higher than 2 hence rules out the sum stable distributions as suitable approximations for unconditional return distributions. At the same time this finding would establish the appropriateness of the mean-variance paradigm. Second, under the assumption that the tails are truly from the Pareto type, α can be used to determine the maximum number of moments that exist since only moments up to the value of α exist (Loretan and Phillips, 1994).

The estimation of the tail index and the statistical inference, however, lead to various statistical problems.

1. If the tail index is estimated assuming that the true distribution is sum stable, α is *a priori* restricted to be below 2, which makes it impossible to discriminate between alternative stochastic processes based on α .
2. Alternatively, one can assume that the tail of the distribution is a Pareto type. Then α can be estimated in an unrestricted way by means of a Hill-estimator (Hill, 1975). Of note is that theoretical standard deviations on the Hill-estimator have usually been derived under the assumptions of independent realisations and Pareto type tails. This has induced further research.
 - a. Kearns and Pagan (1997) have shown that GARCH-type dependence causes the theoretical standard errors to be so large that the tests on α being smaller or larger than 2 are virtually always inconclusive. The results of Kearns and Pagan again question the conclusions of Jansen and de Vries (1991) and Hols and de Vries (1991) that second moments do exist.
 - b. Further, Groenendijk, Lucas and de Vries (1995) in their Tables 1 and 3, implicitly show that the assumption of Pareto type tails is crucial as departures from this assumption lead to severe biases in the estimation of α . Groenendijk, Lucas and de Vries report that for the Student- t distribution with 30 degrees of freedom the number of existing moments is estimated to be as low as 1.767 and even after applying a bias correction only 5.403 moments were found to exist.

Taken together, this research takes us back to basics. Even the existence of the second moment becomes questionable and this, for the field of finance, becomes a fundamental issue. In this paper, we estimate the tail index α for a number of Australian stocks based on the Hill-estimator. Our empirical results are consistent with the US evidence of Loretan and Phillips (1994) and the UK evidence of Omran (1997). Given the findings of Kearns and Pagan (1997), however, we are aware that the theoretical standard deviations used by Loretan and Phillips (1994) and Omran (1997) are severely downward biased in the presence of dependency in asset returns. Based on simulated combinations of Hill-estimates and characteristic exponents for truly sum stables, we are able to strongly reject the sum stable law as a possible candidate for the unconditional distribution of stock returns.

Our research does *not* prove that the second moment is finite beyond any reasonable doubt, but it does show that the moment condition failure due to the sum stable hypothesis does not apply. Hence, the integrated GARCH process studied by Kearns and Pagan (1997) can still be regarded as a serious alternative. But, older research (e.g. Lastrapes, 1989) does cast some doubt on this hypothesis since structural breaks often cause integrated GARCH like behaviour. We conclude that it is reasonable to assume the existence of second moments in Australian stock returns.

The remainder of the paper is organised as follows. In section 2 we explain how the tail index can be estimated using a Hill-estimator. In section 3 we describe the data and section 4 reports the results of the Hill estimates. In section 5 we conduct a simulation experiment in order to judge the reasonableness of the sum stable hypothesis as an approximation of the unconditional distribution of stock returns and strongly reject the hypothesis. Section 6 concludes the paper.

2. Estimating the tail index

Let $\{x_t\}$ be an identically and independently distributed series following a distribution with asymptotic Pareto-type tails. Pareto-type tails imply that

$$\log \Pr(X > x) = k - \alpha \log x$$

or

$$\log x = k' - \gamma \log \Pr(X > x), \quad (1)$$

where $\gamma = 1/\alpha$. If we conveniently assume that the N data points have been sorted to produce the order statistics $x_{(1)} > x_{(2)} > \dots > x_{(N)}$, then $\Pr(X > x_{(j)})$ can be estimated by the empirical survival function (i.e. the number of observations larger than $x_{(j)}$ divided by N).

Equation (1) reveals that one can estimate γ by fitting a straight line into the $\log x - \log \Pr(X > x)$ plane. Any two points $x_{(j)}$ and $x_{(l)}$ can produce an estimate of γ . Obviously, one of them should be, depending on the tail under consideration, the most extreme observation, $x_{(1)}$ or $x_{(N)}$. The other point, $x_{(m)}$, still has to be a point in the tail of the distribution but, to date, no strong unambiguous arguments exist to determine the starting point of the tail region. However, Dumouchel (1983) suggests that $m < 0.1 \cdot N$ is a practical rule of thumb and common practice varies m between 1% and 10% of the sample size in order to judge the robustness of the estimate.

Several estimators, all based on the intuition behind equation (1), have been suggested. Kearns and Pagan (1997) performed simulation experiments in order to calculate the bias and the efficiency of the Picands (1975), the Hill (1975) and the de Haan and Resnick (1980) estimator. Bias and efficiency were evaluated under the null hypothesis of sum stable distributed returns on the one hand and under the null hypothesis of integrated GARCH processes on the other. Although a small bias remained, under both null hypotheses, the Hill-estimator proved to be the most efficient estimator, viz:

$$\hat{\gamma}_{hill} = \left[\frac{1}{m-1} \sum_{i=1}^{m-1} \log x_{(i)} \right] - \log x_{(m)}, \quad (2)$$

$$\text{and } \hat{\alpha}_{hill} = 1/\hat{\gamma}_{hill}.$$

Kearns and Pagan (1997) signalled a cautionary note by showing that the theoretical standard deviations for the Hill-estimator derived by Hall (1982) and Goldie and Smith (1987) may underestimate the true standard errors whenever there is dependence of the integrated GARCH type. Mittnik, Paolella and Rachev (1998) also showed that the small sample performance of the Hill-estimator does not resemble its asymptotic behaviour, even for large samples of 10,000 observations. Taking these findings into account, we do not compute the theoretical standard deviations but instead build an alternative simulation based approach in order to assess moment condition failure caused by the infinite variance problem.

3. Data and descriptive statistics

In order to undertake the empirical application using the Hill-estimator we selected 23 actively traded stocks on the Australian Stock Exchange (ASE). The data were sourced from Datastream. Given the importance of sample size in the estimation procedures we required the stocks to have a full history from January 1985 until July 2000 and this provided a sample of 3,891 daily observations. Returns were calculated as continuously compounded, that is the logarithm of the ratio of two successive prices after adjusting for stock splits, bonus shares and dividends.

Table 1 summarises some basic descriptive statistics. Average annualised returns vary between -1.4% and 20.4%, with historical annualised volatilities ranging between 21.4% and 39.9%. Clearly, the daily returns are not normally distributed as can be observed from the high skewness and kurtosis figures. First order autocorrelations based on the returns, r , are small, autocorrelations on the absolute returns and on the squared returns are more prominent, indicating GARCH type dynamics are present (see Ding, Granger and Engle (1993) for a discussion of the autocorrelations of absolute returns). Hence, the strict assumptions of the Hill-estimator are not fulfilled.

Table 1

Descriptive statistics for 23 Australian stocks

	Average return	Standard Deviation	Skewness	Kurtosis	AC(r)	AC(r)	AC(r ²)
National Australian Bank	18.9%	22.5%	-1.24	20.63	0.08	0.21	0.14
Lend Lease Corporation	16.6%	25.1%	-4.52	21.17	0.07	0.21	0.05
Amcor	11.1%	24.6%	-4.62	123.88	0.03	0.25	0.08
CSR	7.6%	26.4%	-1.12	27.04	-0.00	0.28	0.21
Brambles Industry	20.4%	25.8%	-3.95	110.08	0.03	0.25	0.08
Coles Myer	12.9%	22.8%	-1.71	30.49	0.02	0.26	0.24
Pioneer International	12.0%	28.8%	0.06	30.64	0.07	0.19	0.08
Fosters Brewing Group	11.3%	30.4%	-1.02	24.92	0.05	0.24	0.11
Rio Tinto	13.4%	29.2%	-2.17	54.13	0.13	0.25	0.07
Broken Hill Proprietary	13.6%	23.3%	-0.37	14.69	0.07	0.17	0.05
Coca-Cola Amatil	11.8%	30.2%	-1.59	37.66	0.07	0.32	0.19
Southcorp	14.4%	27.6%	-0.14	12.20	0.00	0.22	0.25
Comalco	12.4%	33.5%	-1.27	28.31	0.08	0.19	0.15
General Pr. Tst.	9.5%	21.4%	-2.99	72.99	-0.07	0.18	0.05
Santos	4.4%	26.9%	-1.02	18.84	0.04	0.19	0.06
Australian Gas and Light	14.4%	28.9%	-0.60	15.19	0.05	0.22	0.24
QBE Insurance Group	17.9%	27.1%	-1.12	27.41	0.03	0.12	0.04
MIM	-1.4%	39.9%	-1.79	43.15	0.06	0.21	0.03
North	8.9%	39.6%	-2.82	84.86	0.04	0.20	0.05
Westpac Banking	7.5%	25.1%	-1.45	27.13	0.09	0.19	0.10
Westfield Holdings	19.5%	33.9%	-4.61	110.68	0.01	0.19	0.03
Woodside Petroleum	14.8%	34.0%	-1.84	46.85	0.01	0.27	0.10
WMC	7.0%	32.4%	-1.74	42.65	0.11	0.20	0.04

Averages and standard deviations are annualised figures and are based on a daily data set starting in January 1985 and ending in July 2000 (3,891 returns). Annualisation was based on 250 trading days with the $\sqrt{250}$ rule applied to calculate standard deviations.

4. Results of the Hill-estimator

Before applying the Hill-estimator we first demean the series by regressing the data on five weekday dummies, and in a second step, on twelve monthly dummies to eliminate potential day of the week and seasonal effects (see Loretan and Phillips, 1994). Finally, we passed the residuals from these regressions through an AR filter in order to remove any linear serial correlation. Because choosing m remains a contentious issue, we varied m between 40 and 400 with steps of 40 similar in nature to those studied by Loretan and Phillips (1994) and Omran (1997).

The point estimates in Table 2 are almost always higher than 2 and usually less than 4, ranging in value from 1.91 to 4.28. Based on this evidence, it would be tempting to discard the infinite variance stable distribution as an appropriate description of the unconditional distribution of stock returns. Unfortunately, theoretical standard deviations are not very useful given the GARCH-type dependence present in the time series (see Table 1). On the other hand, the point estimates indicate that the existence of the fourth moment is doubtful since almost all the Hill estimates are below 4 and similar to Omran (1997) they become smaller than four as the value of m increases. If this is so, this result casts doubt on the validity of ARCH-tests and other tests on second moments that require the existence of four moments. However, Hill-estimates only provide an accurate estimate of the existing number of moments under the assumption of strict Pareto tails and therefore for other distributions the Hill-estimate may be severely downward biased. To illustrate this point we simulated 1000 runs from a Student- t distribution with 5 degrees of freedom and 2000 observations. From this experiment the average number of existing moments was estimated to be 4.2 and decreasing the degrees of freedom to 4 resulted in an average number of 3.65. We

also note that these averages are downward biased estimates of the existing number of moments, but regardless, they prevent us making conclusions with respect to higher moments.

Table 2

Hill-estimates of the tail index

Company $m=$	40	80	120	160	200	240	280	320	360	400
National Australian Bank	3.03	2.97	2.83	2.91	2.79	2.59	2.51	2.32	2.23	2.14
Lend Lease Corporation	2.84	3.20	2.86	2.81	2.74	2.65	2.66	2.55	2.48	2.37
Amcor	3.49	3.27	2.86	2.68	2.52	2.51	2.35	2.19	2.18	2.13
CSR	2.86	2.91	2.80	2.90	2.77	2.70	2.62	2.50	2.35	2.24
Brambles Industry	2.75	2.73	2.67	2.97	2.83	2.71	2.56	2.52	2.39	2.27
Coles Myer	2.60	2.88	2.80	2.79	2.88	2.59	2.55	2.32	2.31	2.19
Pioneer International	3.09	3.42	3.31	2.91	2.73	2.53	2.49	2.43	2.36	2.31
Fosters Brewing Group	2.76	2.62	2.78	2.69	2.61	2.68	2.56	2.57	2.47	2.23
Rio Tinto	2.64	2.93	2.89	2.94	2.73	2.73	2.56	2.49	2.37	2.38
Broken Hill Proprietary	4.28	3.98	3.46	3.42	3.25	3.07	2.77	2.66	2.48	2.43
Coco-Cola Amatil	3.25	3.25	2.79	2.58	2.40	2.30	2.23	2.20	2.11	2.01
Southcorp	3.23	2.93	2.68	2.65	2.58	2.52	2.58	2.53	2.57	2.40
Comalco	3.48	3.15	3.10	2.72	2.65	2.56	2.55	2.34	2.35	2.29
General Pr. Tst.	2.98	3.31	3.18	3.08	2.80	2.91	3.02	2.65	2.41	2.27
Santos	2.92	3.20	3.14	3.12	3.21	3.13	3.03	2.86	2.64	2.53
Australian Gas and Light	2.21	2.37	2.41	2.46	2.51	2.49	2.56	2.56	2.55	2.36
QBE Insurance Group	2.69	2.81	2.85	2.64	2.49	2.37	2.13	1.99	1.94	1.91
MIM	2.92	3.27	3.41	3.41	3.25	3.14	2.89	2.86	2.63	2.59
North	2.55	3.08	3.13	2.81	2.87	2.67	2.43	2.39	2.35	2.30
Westpac Banking	3.61	3.53	3.41	3.14	2.97	2.90	2.77	2.69	2.44	2.38
Westfield Holdings	2.24	2.67	2.58	2.34	2.15	2.15	2.17	2.07	2.06	1.97
Woodside Petroleum	2.30	2.73	2.57	2.45	2.40	2.39	2.32	2.29	2.39	2.28
WMC	2.78	3.20	3.47	3.37	3.33	3.22	3.08	2.81	2.72	2.57

Estimates are based on daily returns computed from January 1985 through July 2000 (3,891 observations).

Recognising the flaws in the applicability of the theoretical standard deviations places us, along with previous authors, in a weak position. We cannot draw strong conclusions on the upper bound of the existing moments due to the downward bias in the Hill-estimates. Of greater concern is that the sum stable hypothesis, which implies infinite variances, cannot be rejected in a statistically rigorous way using Hill-estimates and their standard errors. We extend our analysis in the next section.

5. Discarding the sum stable hypothesis

Sum stable distributions are usually parameterised in terms of their log-characteristic function since, for most sum stables, the density function has no analytical closed-form expression. The log-characteristic function is determined by four parameters (McCulloch, 1996). First, the location parameter δ has the potential to shift the distribution to the left (negative values) or right (positive values). Second, the positive scale parameter γ expands or contracts the distribution about δ . Third, a skewness parameter β , whose absolute value is constrained to be less or equal to one, indicates the symmetry of the distribution. Finally, the characteristic exponent α^* (positive and less than or equal to 2) governs the tail behaviour of the distribution and should be equal to the tail index α . Therefore, if stock returns were truly sum stable, we can estimate the tail index through the direct estimation of the characteristic exponent of the distribution.

In order to further estimate the parameters of the sum stable distribution, we applied the McCulloch (1986) procedure used by Ghose and Kroner (1995). Akgiray and Lamoureux (1989)

and Bates and McLaughlin (undated) both showed that McCulloch's (1986) technique provides robust all round estimators for the parameters of the sum stable distributions (with $\alpha > 0.6$). Table 3 reports the estimated α^* s and the other distribution parameters for our sample of Australian stocks. The characteristic exponents were estimated by using two alternative assumptions. In the first part of the table all parameters are freely estimated, whereas in the second part a symmetry restriction ($\beta=0$) is imposed. Without any formal statistical testing, one can observe that the differences between the two sets of estimates for the characteristic exponents are very marginal. All point estimates are in the range 1.1 to 1.7 with estimates around 1.5 the most representative and these results are in line with the evidence of Fama (1965), Blattberg and Gonedes (1974), and Fielitz and Rozelle (1983).

Table 3

Estimates of the parameters of the sum stable distribution

	α^*	β	γ	δ	α^*	γ	δ
National Australian Bank	1.537	0.052	0.007	0.001	1.537	0.007	0.000
Lend Lease Corporation	1.521	0.135	0.008	0.001	1.523	0.008	0.000
Amcor	1.485	0.041	0.007	0.000	1.486	0.007	0.000
CSR	1.577	-0.020	0.009	0.000	1.577	0.009	0.000
Brambles Industry	1.607	0.153	0.008	0.001	1.608	0.008	0.000
Coles Myer	1.493	0.057	0.007	0.000	1.494	0.007	0.000
Pioneer International	1.549	0.071	0.009	0.001	1.549	0.009	0.000
Fosters Brewing Group	1.531	0.082	0.009	0.001	1.532	0.009	0.000
Rio Tinto	1.579	0.162	0.009	0.001	1.580	0.009	0.000
Broken Hill Proprietary	1.579	0.082	0.008	0.000	1.579	0.008	0.000
Coca-Cola Amatil	1.438	0.047	0.009	0.000	1.439	0.009	0.000
Southcorp	1.586	0.160	0.009	0.001	1.587	0.009	0.000
Comalco	1.502	0.080	0.010	0.001	1.503	0.010	0.000
General Pr. Tst.	1.700	-0.056	0.007	0.000	1.700	0.007	0.000
Santos	1.541	0.028	0.009	0.000	1.542	0.009	0.000
Australian Gas and Light	1.597	0.139	0.009	0.001	1.598	0.009	0.000
QBE Insurance Group	1.308	0.041	0.007	0.000	1.310	0.007	0.000
MIM	1.581	0.025	0.013	0.000	1.581	0.013	0.000
North	1.576	0.015	0.012	0.000	1.576	0.012	0.000
Westpac Banking	1.553	0.040	0.008	0.000	1.553	0.008	0.000
Westfield Holdings	1.158	0.059	0.006	0.001	1.162	0.006	0.000
Woodside Petroleum	1.470	0.127	0.009	0.001	1.473	0.009	0.000
WMC	1.645	0.107	0.011	0.001	1.645	0.011	0.000
Mean	1.53	0.07	0.01	0.00	1.53	0.01	0.00
Standard deviation	0.02	0.01	0.00	0.00	0.02	0.00	0.00

Estimates are based on 3,891 daily returns (January 1985 to July 2000) using the McCulloch-technique. The second to fifth columns contain unrestricted estimates, the last three columns contain estimates with a symmetry restriction imposed. H. McCulloch kindly provided the GAUSS-code.

Whilst the results of Table 3 are also in line with previous US research, the implied conclusion of finding infinite variances contradicts the point estimates of the Hill-estimator in Table 2 (that are also in line with prior findings). Recall that the tail index nests various models in the tail of the distribution. The Hill-estimator and the McCulloch estimator are two estimators that should produce the same "alpha" under the assumption that the observations are independent and have true Pareto-type tails. We note the discrepancy we document is not unique to Australian stock returns. Ghose and Kroner (1995, Table 4) report similar discrepancies for currencies (GBP, DEM, CHF, JPY), stocks (S&P500) and commodities (soybeans, live cattle and live hogs).

In order to discard the sum stable hypothesis without relying on the theoretical standard deviations on the Hill-estimator, we conduct a simulation experiment. In accordance with McCulloch (1996) we set the estimates of β and δ to zero and from observation from Table 3 the scale parameter (γ) is set to 0.01. Given these inputs, we generated sum stable distributions using the MATLAB-algorithm provided by H. McCulloch on the Mathworks-website and obtained characteristic exponents varying between 1 and 2 with a step size of 0.1. For each characteristic exponent we generated 1,000 series of 3,891 observations and for each series the Hill-estimator was determined and the empirical distribution of the Hill-estimators computed.

Figures 1 and 2 plot for each simulated characteristic exponent the minimum and maximum Hill-estimates along with their simulated 95% and 99% confidence interval. Hence, the lowest (highest) line connects the minimum (maximum) Hill-estimates that were generated over 1000 runs for characteristic exponents varying from 1 to 2. The 95% and 99% bounds were also plotted to give an indication about the likelihood of Hill-estimate characteristic exponent combinations. In Figure 1, m was set to 40, whereas Figure 2 was based upon m equal to 400 determined because they represent the upper and lower cut off rates used to define the tails (see also Table 2). This graph has also been produced for all other m values reported in Table 2 but they provided similar qualitative results.

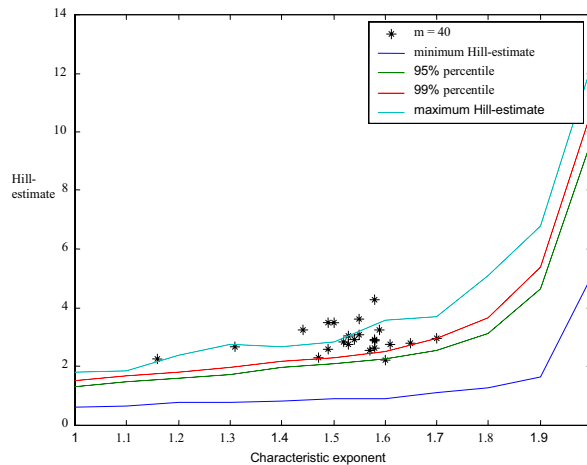


Fig.1. Positioning Australian stocks in the Hill-estimate characteristic exponent plane

Finally, the 23 Australian stocks were positioned in the Hill-estimate characteristic exponent plane. The Hill-estimates used are the same as those reported in Table 2 whereas the characteristic exponents are the ones estimated in Table 3. If stock returns follow the sum stable law we expect to find the stocks scattered (at the 95% level) between the minimum and 95% line. For $m=40$ we notice that only 1 stock falls within the (one-sided) 95% confidence interval. Most stocks are situated between the 95% and the 99% line and several stocks position above the maximum Hill-characteristic exponent line. For Figure 2 ($m=400$) the results are even more dramatic. All but one stock is positioned above the simulated maximum Hill-characteristic exponent line and both figures show that it is extremely unlikely that the real stock returns are being characterised by an unconditional sum stable law.

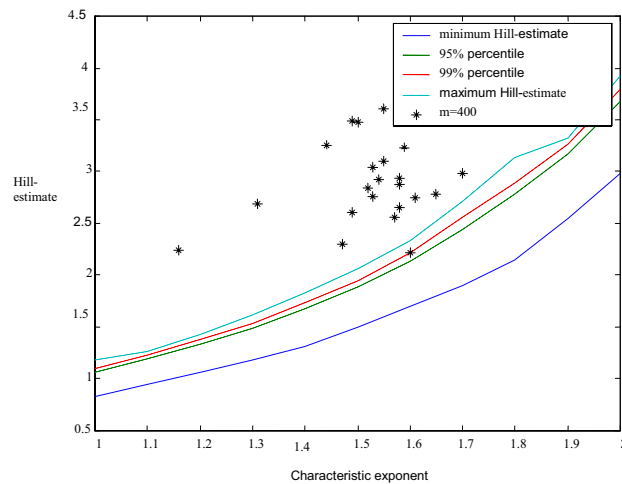


Fig. 2. Positioning Australian stocks in the Hill-estimate characteristic exponent plane

6. Conclusions

If one assumes that stock returns behave according to the sum stable law, point estimates for the tail index of approximately 1.5 will be observed, implying that second moments do not exist. This is the result we observe for the selected Australian stocks that have a full daily data set over 15 years and is in line with previous US research findings. On the other hand, Hill-estimates for the tail index estimated for all stocks provided point estimates above 2, hence providing contradictory support for the existence of second moments. However, the sum stable hypothesis can be firmly rejected since combinations of the Hill-estimate and the characteristic exponent produced by the real data, are extremely unlikely for sum stables.

In general the results of our research lead us to make the following conclusions. First, there is no real evidence that the variance does not exist and this provides support for a number of fundamental assumptions contained in the disciplines of finance and investment analysis. Second, assuming that all moments exist is still dangerous. The existing literature in this area is not helpful and econometricians should handle this problem with care.

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