“Quantum-Psychological Model Of The Stock Market”

AUTHORS	Andrei Khrennikov
ARTICLE INFO	Andrei Khrennikov (2003). Quantum-Psychological Model Of The Stock Market. *Problems and Perspectives in Management*, *1*(1)
RELEASED ON	Wednesday, 26 November 2003
JOURNAL	"Problems and Perspectives in Management"
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

NUMBER OF REFERENCES	NUMBER OF FIGURES	NUMBER OF TABLES
0	0	0

© The author(s) 2025. This publication is an open access article.

BUSINESS PERSPECTIVES

businessperspectives.org

CHAPTER 4
GENERAL ISSUES IN MANAGEMENT

Quantum-Psychological Model of the Stock Market

Andrei Khrennikov*

Abstract. We use methods of classical and quantum mechanics for mathematical modelling of price dynamics at the financial market. The Hamiltonian formalism on the price/price-change phase space is used to describe the classical-like evolution of prices. This classical dynamics of prices is determined by ”hard” conditions (natural resources, industrial production, services and so on). These conditions as well as ”hard” relations between traders at the financial market are mathematically described by the classical financial potential. At the real financial market ”hard” conditions are not the only source of price changes. The information exchange and market psychology play important (and sometimes determining) role in price dynamics. We propose to describe this ”soft” financial factors by using the pilot wave (Bohmian) model of quantum mechanics. The theory of financial mental (or psychological) waves is used to take into account market psychology. The real trajectories of prices are determined (by the financial analogue of the second Newton law) by two financial potentials: classical-like (”hard” market conditions) and quantum-like (”soft” market conditions).

1. Introduction

Since the 1970s, the intensive exchange of information in the world of finances has become one of the main sources determining dynamics of prices. Electronic trading (that became the most important part of the environment of the major stock exchanges) induces huge information flows between traders (including foreign exchange market). Financial contracts are performed at a new time scale that differs essentially from the old ”hard” time scale that was determined by the development of the economic basis of the financial market. Prices at which traders are willing to buy (bid quotes) or sell (ask quotes) a financial asset are not more determined by the continuous development of industry, trade, services, situation at the market of natural resources and so on. Information (mental, market-psychological) factors play very important (and in some situations crucial) role in price dynamics. Traders performing financial operations work as a huge collective cognitive system. Roughly speaking classical-like dynamics of prices (determined) by ”hard” economic factors is permanently perturbed by additional financial forces, mental (or market-psychological) forces, see the book of J. Soros [1].

Unfortunately, at the present time we do not have mathematical models of cognitive phenomena that could provide an adequate description of the high level cognitive (in particular, conscious) processes, see [2] for the details. Of course, some primary cognitive features can be simulated by using neural networks, see e.g. [3]. However, it seems that high levels of cognitive organization could be never simulated on the neural network level, see [2]. It seems to be useful to consider the possibility that there is (at least) some analogy between conscious processes and quantum processes, see e.g. [2], [4]-[7]. In particular, there were attempts to use the pilot wave model of quantum mechanics (Bohmian mechanics, [8], [9]) to simulate cognitive phenomena, see [4], [5], [7].

In this paper we use methods of Bohmian mechanics to simulate dynamics of prices at the financial market. We start with the development of the classical Hamiltonian formalism on the

* International Center for Mathematical Modeling in Physics and Cognitive Sciences, MSI, University of Vaxjo, S-35195, Sweden, E-mail: Andrei.Khrennikov@msi.vxu.se

price/price-change phase space to describe the classical-like evolution of prices. This classical dynamics of prices is determined by "hard" financial conditions (natural resources, industrial production, services and so on). These conditions as well as "hard" relations between traders at the financial market are mathematically described by the classical financial potential. As we have already remarked, at the real financial market "hard" conditions are not the only source of price changes. The information and market psychology play important (and sometimes determining) role in price dynamics. We propose to describe this "soft" financial factors by using the pilot wave (Bohmian) model of quantum mechanics. The theory of financial mental (or psychological) waves is used to take into account market psychology. The real trajectories of prices are determined (by the financial analogue of the second Newton law) by two financial potentials: classical-like ("hard" market conditions) and quantum-like ("soft" market conditions).

Our quantum-like model of financial processes was strongly motivated by consideration by J. Soros [1] of the financial market as a complex cognitive system. Such an approach he called the theory of reflexivity. In this theory there is a large difference between market that is "ruled" by only "hard" economical factors and market at that mental factors play the crucial role (even changing the evolution of the "hard" basis, see [1]). J. Soros rightly remarked that "non mental" market evolves due to classical random fluctuations. However, such fluctuations do not provide an adequate description of mental market. He proposed to use analogy with quantum theory. However, it was noticed that directly quantum formalism could not be applied to the financial market, [1]. Traders differ essentially from elementary particles. Elementary particles behave stochastically due to perturbation effects provided by measurement devices, [10]. According to J. Soros, traders at the financial market behave stochastically due to free will of individuals. Combinations of a huge number of free wills of traders produce additional stochasticity at the financial market that could not be reduced to classical random fluctuations (determined by non mental factors). Here J. Soros followed to the conventional (Heisenberg, Bohr, Dirac, see, e.g. [10], [11]) viewpoint to the origin of quantum stochasticity. However, in the Bohmian approach (that is nonconventional one) quantum statistics is induced by the action of an additional potential, quantum potential, that changes classical trajectories of elementary particles. Such an approach gives the possibility to apply quantum formalism to the financial market. We also remark that recently it was demonstrated that, in fact, quantum-like (Hilbert space probabilistic) formalism could be applied to various statistical phenomena outside of the microworld, see [12]-[15].

It seems that quantum financial approach gives a new possibility to describe mathematically stochasticity at the financial market, compare to the classical stochastic approach starting with Bachelier doctoral thesis [16], see also e.g. [17]-[19]. We also refer to the book [20] containing an extended bibliography on the use of different physical models in social sciences and economy.

We hope that our paper might be readable by physicists and mathematicians as well as (mathematically thinking) economists. Therefore we try to use mathematics at a rather "primitive" level. We plan to present more advanced mathematical model in the next paper.

2. Classical phase space model

Let us consider a mathematical model in that n traders, a1,... ,an interact with one another and react to external economic (as well as political) information in order to determine the best price to buy or sell financial assets. We consider a price system of coordinates: There is the n-dimensional configuration space $Q = R_n$ of prices, $q = (q_1,\ldots ,q_n)$, where q_j is the price proposed by j^{th} trader. Here R is the real line. Dynamics of prices is described by the trajectory $q(t) = (q_1(t),\ldots ,q_n(t))$ in the configuration price space Q.

The reader may be surprised that we plan to use the whole real line R (and not only positive half-line) to describe price dynamics for trader. Well, it is clear what is the meaning of the price q = 1 dollar. But: What is the meaning of the price q = -1 dollar? We shall use the following interpretation. If trader a_j is selling a product (ask quotes), then $q_j \geq 0$. If aj is buying a product (bid quotes), then $q_j \leq 0$.

Other variable under the consideration is the price change variable:

$$v_j(t) = \dot{q}_j(t) = \lim_{\Delta t \to 0} \frac{q_j(t + \Delta t) - q_j(t)}{\Delta t}$$

see, for example, [20] on the role of the price change description. In real models we consider the discrete time scale ... Δt, 2Δt,. Here we should use discrete price change variable $z_j(t) = q_j(t + \Delta t) - q_j(t)$, see [20].

We denote the space of price changes by the symbol $V(\equiv R^n),\ \ v = (v_1,..., v_n)$. As in classical physics, it is useful to introduce the phase space $Q \times V = R^{2n}$, namely the price phase space. A pair (q,v) = (price, price change) is called a state of the financial market.[1]

We now introduce an analogue m of mass as the number of items (i.e., options) that trader presents to the market.[2] We call m the financial mass. Thus each trader has its own financial mass mj . The total price of her/his offer to the market is equal to $T_j = m_j q_j$.

We also introduce financial energy of trade as a function $H : Q \times V \to R$.

If we use the analogue with classical mechanicsc[3], then we could consider (at least for mathematical modeling) the financial energy of the form:

$$H(q, p) = \frac{1}{2} \sum_{j=1}^{n} m_j v_j^2 + V(q_1,..., q_n).$$

Here is $K = \frac{1}{2} \sum_{j=1}^{n} m_j v_j^2$ the kinetic financial energy and $V(q_1,... ,q_n)$ is the potential financial energy, m_j is the financial mass of j^{th} trader

The kinetic financial energy represents efforts of traders to change prices: higher price changes vj induce higher kinetic financial energies. The financial mass also plays the important role: if one trader, a_1, sells 1 item and other trader, a_2 sells 2 items and they both change the price in the same way, then a_2 has two times larger kinetic financial energy than a_1. We remark that the kinetic financial energy does not depend on the absolute magnitudes of prices (only on price changes). We also remark that high kinetic financial energy induces rapid changes of the financial situation at market. However, the kinetic financial energy does not give the attitude of these changes. It could be rapid economic growth as well as recession.

The potential financial energy V describes the interactions between traders $a_1,... ,a_n$ as well as external economic conditions. For example, we can consider the simplest interaction potential:

[1] Later we shall consider quantum-like states of the financial market. A state (q, v) is a classical state.

[2] 'Number' is typically a natural number m = 0, 1, . . . , because even in trade for "continuous products" (such as oil or gas) we use discrete units, i.e. ton or barrel.

[3] Why not? In principle, there is not so much difference between motions in "physical space" and "price space".

$$V(q_1,...,q_n)=\sum_{j=1}^{n}(q_i-q_j)^2. \tag{1}$$

The difference $|q_i - q_j|$ between prices of a_i and a_j is the most important condition for arbitrage. So the description of interactions between different traders by using the potential financial energy is straightforward. What is about the role of external conditions? It is the very complicated problem. In some sense V describes (if we forget about interactions between traders $a_1,...,a_n$) reactions of our traders to external conditions. There is the large variety of such conditions, economic as well as political. For example, suppose that we study the car-market. Here $a_1,...,a_n$ are car-traders (selling as well as buying cars). Then the potential financial energy depends, in particular, on oil price (that influences to car-prices)

We could never take into account all economic and other conditions that have influences to the market. Therefore by using some concrete potential V(q) we consider the very idealized model of financial processes. However, such an approach is standard for physical modeling where we also consider idealized mathematical models of real physical processes.

To describe dynamics of prices, it is natural to use the Hamiltonian dynamics on the price phase space. As in classical mechanics for material objects, it is useful to introduce a new variable $p = mv$, the price momentum variable. So, instead of the price change vector $v = (v_1,...,v_n)$, we shall consider the price momentum vector $p = (p_1,...,p_n)$, $p_j = m_j v_j$. The space of price momentums is denoted by the symbol P. The space $Q \times P$ will be also called the price phase space. Hamiltonian equations of motion on the price phase space have the form:

$$\dot{q}_j=\frac{\partial H}{\partial p_j}, \quad \dot{p}_j=-\frac{\partial H}{\partial q_j}, \quad j=1,...,n. \tag{2}$$

We also need the initial conditions:
$q_j(0) = q_j0$, $p_j(0) = p_j0$.

If the financial energy has form (1), i.e.,

$$H(q,p)=\sum_{j=1}^{n}\frac{p_j^2}{2m_j}+V(q_1,...,q_n),$$

then the Hamiltonian equations have the form

$$\dot{q}_j=\frac{p_j}{m_j}=v_j, \quad \dot{p}_j=-\frac{\partial V}{\partial q_j}.$$

The latter equation can be written in the form:

$$m_j\dot{v}_j=-\frac{\partial V}{\partial q_j}.$$

The quantity $\dot{v}_j(t)=\lim_{\Delta t\to 0}\frac{v_j(t+\Delta t)-v_j(t)}{\Delta t}$ is natural to call the price acceleration (change of price change). The quantity $f_j(q)=-\frac{\partial V}{\partial q_j}$ is called the (potential) financial force. We get the financial variant of the second Newton law:

$$m\dot{v} = f. \tag{3}$$

"The product of the financial mass and the price acceleration is equal to the financial force."

In fact, the Hamiltonian evolution is determined by the following fundamental property of the financial energy: The financial energy is not changed in the process of Hamiltonian evolution:

$$H(q_1(t), \ldots, q_n(t), p_1(t), \ldots, p_n(t)) = H(q_1(0), \ldots, q_n(0), p_1(0), \ldots, p_n(0)) = const.$$

We need not restrict our considerations to financial energies of form (1). First of all external (e.g. economic) conditions as well as the character of interactions between traders at the market depend strongly on time. This must be taken into account by considering time dependent potentials: V = V(t,q). Moreover, the assumption that the financial potential depends only on prices, V = V(t,q), in not so natural for the modern financial market. Traders have the complete information on price changes. This information is taken into account by traders for acts of arbitrage, see [20] for the details. Therefore, it can be useful to consider potentials that depend not only on prices, but also on price changes: V = V(t,q,v) or in the Hamiltonian framework: V = V(t,q,p). In such a case the financial force is not potential. Therefore, it is also useful to consider the financial second Newton law for general financial forces: $m\dot{v} = f(t, q, p)$.

Remark (On the form of the kinetic financial energy). We copied the form of kinetic energy from classical mechanics for material objects. It may be that such a form of kinetic financial energy is not justified by real financial market. It might be better to consider our choice of the kinetic financial energy as just the basis for mathematical modeling (and looking for other possibilities).

Example 1. ("Free trader") Let $n = 1$ and $V \equiv 0$. We get $\dot{p} = 0$ or $p(t) \equiv p_0$ and $\dot{q} = \frac{p_0}{m} = v_0$

Thus $q(t) = q_0 + v_0 t$. So, in the absence of concurrence and by neglecting external (economic and political conditions), trader will increase the price linearly. It is very natural behaviour.[1]

We now consider the situation in that $v_0 > 0$. Here trader will linearly decrease price. It at $t = t_0$ trader (on the basis of some information from market) decided to decrease the price and after this he could not get any information on economic and financial situation. So he/she continues to decrease the price. Of course, this is very idealized example. We could never find isolated trader. Any trader need a partner for arbitrage and, consequently, could use information on partner's behaviour.

3. 'Classical' Hamiltonian model of price dynamics and stock's market

The model of Hamiltonian price dynamics on the price phase space can be useful to describe a market that essentially depends on "hard" economic conditions: natural resources, volumes of production, human resources, ... In principle, such a model could be even used in plan economy: by introducing different potentials V(q1,... ,qn) we can regulate plan-"market", see, e.g. [21]. However, it seems that classical price dynamics could not be applied (at least directly) to financial markets, [1]. Well, "hard" economic conditions play the important role in forming of stock prices. However, it is clear that stock market is not based only on these "hard" factors. There

[1] If market has very large capacity. However, infinite capacity of market is a consequence of V≡0 : there is no external contraints.

are other factors, soft ones, that play the important and (sometimes even determining) role in forming of prices at the financial market. We could not define precisely these soft factors. We can say about market's psychology. These psychological factors became very important due to the rapid information exchanges performed by modern financial computer systems.

Negligibly small amounts of information (due to the rapid exchange of information) could imply large changes of prices at the financial market. We may speak about financial (psychological) waves that are permanently present at financial market. Sometimes these waves produce uncontrollable changes of prices disturbing the whole financial market (financial crises). Of course, financial waves depend on hard economic factors. However, these hard factors do not play the crucial role in forming of financial waves. Financial waves are merely waves of information.[1] We could compare behaviour of financial market with behaviour of a gigantic ship that is ruled by a radio signal. A radio signal with negligibly small physical energy can essentially change (due to information contained in this signal) the motion of the gigantic ship. If we do not pay attention on (do not know about the presence of) the radio signal, then we will be continuously disappointed by ship's behaviour. It can change the direction of motion without any "hard" reason (weather, destination, technical state of ship's equipment). However, if we know about the existence of radio monitoring, then we could find information that is sent by radio. This would give us the powerful tool to predict ship's trajectory. We now inform the reader that this example on ship's monitoring was taken from the book of D. Bohm and B. Hiley [4] on so called pilot wave quantum theory (or Bohmian quantum mechanics).

4. Financial pilot waves

According to Bohmian interpretation of quantum mechanics, a quantum system consists of a material body (e.g. an elementary particle) and a pilot wave. The latter "rules" the first. This pilot wave does not carry any physical energy. At least an exchange of physical energy between the pilot wave and quantum system has never been observed (however, see [4]).

If we interpret the pilot wave as a kind of physical field, then we should recognize that this is a rather strange field. It differs crucially from other physical fields, i.e., electromagnetic field. The latter carries physical energy. There are some other pathological features of the pilot wave field, see [9] for the detailed analysis. In particular, the force induced by this pilot wave field does not depend on the amplitude of wave. Thus small waves and large waves change equally the trajectory of an elementary particle. Such features of the pilot wave give the possibility to speculate, see [4], [5] that this is just a wave of information (active information). Hence, the pilot wave field does not describe the propagation of energy in the physical space-time, but the propagation of information. The pilot wave is more similar to a radio signal that guides a ship. Of course, this is just an analogy (because a radio signal is related to ordinary physical field, namely electromagnetic field). The more precise analogy is to compare the pilot wave with information contained in a radio signal.

We remark that the pilot wave (Bohmian) interpretation of quantum mechanics is not the conventional one. As we have already noted, there are a few critical arguments against Bohmian quantum formalism:

Bohmian theory gives the possibility to provide the mathematical description of the trajectory $q(t)$ of an elementary particle. However, such a trajectory does not exist according to the conventional quantum formalism.

Bohmian theory is not local, namely via the pilot wave field one particle "feels" another on large distances (without any exchange of physical energy).

As the pilot wave does not carry physical energy, this is not a physical field. Thus it is not an element of physical reality at all. It seems meaningless to study such structures that do not have any relation to physical reality.

[1] We may speak about psychological waves or even collective consciousness of traders, compare to Bohm-Hiley-Pilkkanen [4], [5] theory of active information.

Regarding to 1 and 2 we say that these disadvantages of theory will become advantages in our applications of Bohmian theory to financial market. Regarding to 3 we recall that already Bohm and Hiley [4] and Hiley and Pilkkanen [5] discussed the possibility to interpret the pilot wave field as a kind of information field. This information interpretation was essentially developed in works of A. Khrennikov [7] that were devoted to pilot wave cognitive models.

Our fundamental assumption is that traders at the modern financial market are not just classical-like traders. Their actions are ruled not only by classical-like financial potentials $V(t, q_1,..., q_n, p_1,..., p_n)$, but also (in the same way as in the pilot wave theory for quantum systems) by an additional information (or psychological, compare to [1], [7]) potential induced by a financial pilot wave.

Therefore we could not use the classical financial dynamics (Hamiltonian formalism) on the financial phase space to describe the real price trajectories. Information (psychological) perturbation of Hamiltonian equations for price and price change must be taken into account. To describe such a model mathematically, it is convenient to use such an object as a financial pilot wave that rules activity of each trader at the financial market.

The reader may ask: "Where is such a financial field defined?" In principle, it is possible to develop a model in that this field is distributed over the physical space (the surface of Earth with singularities at New-York, Tokyo, London, Paris, Frankfurt,...). However, we prefer to use information space, namely the price space Q, in our modeling. Thus the financial pilot wave is mathematically described as a function $\varphi : Q \to C$, where C is the set of complex numbers. It maps the price configuration $q = (q_1,..., q_n)$ for n traders, $a_1,... ,a_n$ into a complex number $\varphi(q)$, the amplitude of the price configuration. In some sense (see [7] for the details) $\varphi(q)$ describes the psychological influence of the price configuration q to traders. The reader may be surprised that there appeared complex numbers C. However, the use of these numbers is just a mathematical trick that provides the simple mathematical description of dynamics of the financial pilot wave. Before to go into mathematical details, we underline two important features of the financial pilot wave model:

1. All traders are coupled on the information level. The general formalism [8], [9] of the pilot wave theory says that if the function $\varphi(q_1,..., q_n)$ is not factorized, i.e.,

$\varphi(q_1, \dots, q_n) \neq \varphi(q_1), \dots, \varphi_n(q_n)$,

then by changing the price qi the trader ai will automatically change behaviour of all other traders aj, j $\neq$ i. . At the same time the "hard" economic potential $V(q_1,..., q_n)$ can be totally local: economic conditions for distinct traders can be independent. For example, $V(q_1,..., q_n) = q_1^2 + ... + q_n^2$ and $a_1,..., a_n$ are totally independent economically. Hamiltonian equation in the absence of the financial pilot wave have the form: $\dot{q}_j = p_j, \dot{p}_j = -2q_j, j = 1,2,..., n.$ Thus the classical-like price trajectory of a_j, $q_j(t)$, does not depend on dynamics of prices for other traders a_i, $j \neq i$.

However, if e.g. $\varphi(q_1,..., q_n) = ce^{i(q_1 q_2 + ... + q_{n-1} q_n)} e^{-(q_1^2 + ... + q_n^2)}$,

where $c \in C$ is some normalization constant, then financial behaviour of traders at the financial market is nonlocal (see further considerations). Everybody would immediately react to changes of other prices, despite stable domestic economic situation.

2. Reactions of traders a_j do not depend on the amplitude of the financial pilot wave: financial waves φ, 2φ, 100000φ will produce the same reactions of a_j. Such a behaviour of traders at the financial market is quite natural (if the financial pilot wave is interpreted as an information wave, the wave of financial information). The amplitude of an information signal does not play so

large role in the information exchange. The most important is the context of such a signal. The context is given by the shape of the signal, the form of the financial pilot wave function.

5 Dynamics of prices guided by the financial pilot wave

In fact, we need not develop a new mathematical formalism. We will just apply the standard pilot wave formalism (that was developed by D. Bohm [8], see also [4], [9] for elementary particles) to traders at the financial market. The fundamental postulate of the pilot wave theory is that the pilot wave (field) $\varphi(q_1,...,q_n)$ induces a new (quantum) potential $U(q_1,...,q_n)$ that perturbs the classical equations of motion. A modified Newton equation has the form:

$$\dot{p} = f + g, \ \text{где} \ f = -\frac{\partial V}{\partial q}, \quad g = -\frac{\partial U}{\partial q}.$$

We call the additional financial force g a financial mental force.

This force, $g(q_1,...,q_n)$, determines a kind of collective consciousness of the financial market. Of course, the g depends on economic and other "hard" conditions given by the financial potential $V(q_1,...,q_n)$ However, this is not the direct dependence. In principle, nonzero financial mental force can be induced by the financial pilot wave φ in the case of zero financial potential, V ≡ 0. So V ≡ 0 does not imly that U ≡ 0. Market psychology is not totally determined by economic factors. Financial (psychological) waves of information need not be generated by some changes in the real economic situation. They are mixtures of mental and economic waves. Even in the absence of economic waves, mental financial waves can have large influence to the financial market.

By using the standard pilot wave formalism we get the following rule for computing the financial mental force.

We represent the financial pilot wave $\varphi(q)$ in the form:

$$\varphi(q) = R(q)e^{iS(q)},$$

where $R(q) = |\varphi(q)|$ is the amplitude of $\varphi(q)$ (the absolute value of the complex $c = \varphi(q)$ number $c = \varphi(q)$ and $S(q)$ is the phase of $\varphi(q)$ (the argument of the complex number $c = \varphi(q)$. Then the financial mental potential is computed as

$$U(q_1,...,q_n) = -\frac{1}{R}\sum_{i=1}^{n}\frac{\partial^2 R}{\partial q_i^2}$$

and the financial mental force

$$g_j(q_1,...,q_n) = -\frac{\partial U}{\partial q_j}(q_1,...,q_n).$$

These formulas imply that strong financial effects are produced by financial waves having essential variations of amplitudes.

Example 2. (Financial waves with small variation have no effect) Let $R \equiv const$. Then the financial mental force $g \equiv 0$. As $R \equiv const$, j^{th} trader could not perturb market's conscious by varying his price q_j. The constant information field does not induce psychological financial effects at all. As we have already remarked the absolute value of this constant does not play any role. Waves of the constant amplitude R = 1 as well as R = 10100 produce no financial effect.

Let $R(q) = cq, \ c > 0$. This is the linear function; variation is not so large. As the result, here also $g \equiv 0$. No financial mental effects.

Example 3. (Successive speculations) Let $R(q)=c(q^2+d),\ c,d>0$. Here $U(q)=-\frac{2}{q^2+d}$ (it does not depend on the amplitude c!) and $g(q)=-\frac{4q}{(q^2+d)^2}$. The quadratic function varies essentially stronger than the linear function, and, as the result, such a financial pilot wave induces nontrivial financial mental force.

We analyse financial drives induced by such a force for traders selling options. Here q > 0 and g < 0. The financial mental force g stimulates trader a to decrease the price. For small prices, $g(q)\approx -4q/d^2$. If trader a increases his/her price q for options, then the negative reaction of the financial mental force becomes stronger and stronger. He/she is pressed by the financial market to stop increasing of the price q. However, for large prices, $g(q)\approx -4/q^3$. If the a could approach this range of prices (despite of the negative pressure of the financial market for relatively small q), then the a will feel decreasing of the negative pressure of the financial market. This model explains well successive speculative behaviour at the financial market.

Let R(q) = c(q4 + b), c, b > 0. Thus $g(q)=-\frac{bq-q^5}{(q^4+b)^2}$. Here behaviour of trader a is more complicated. We consider the case $q\geq 0$ (asking quotes). Let $d=\sqrt[4]{b}$. If the price q is changing from $q=0$ to $q=d$, then the a is motivated (by financial mental force *g(q))* to increase the price. The price q = d is critical for his/her financial activity. By psychological reasons (of course, indirectly based on the whole information on the financial market) he/she understands that it would be dangerous to continue to increase the price. Since $q = d$, he/she has the psychological stimuli to decrease the price.

Financial pilot waves $\varphi(q)$ with $R(q)$ that are polynomials of higher order could induce very complex behaviour. The interval $[0,\infty)$ is split in the collection of subintervals $0<d_1<d_2<...<d_n<\infty$ such that at each price level $q = d_j$ the trader changes his/her attitude to increase or to decrease the price.

In fact, we have considered just one dimensional model. In the real case we have to consider multidimensional models of huge dimensions. A financial pilot wave $\varphi(q_1,...,q_n)$ on such a price space Q induces splitting of Q into a large number of domains $Q=O_1\cup...\cup O_N$. Each domain Oj is characterized by the fixed collection of financial attitudes of traders at the financial market. For example, in O_1: a_1 has the drive to increase the price (and he/she is selling options, $q_1 \geq 0$), a_2 has the drive to decrease the price (and he/she is buying options $q_2 \leq 0$), ..., an has the drive to decrease the price (and he/she is selling options $q_n \geq 0$).

The only problem that we have still to solve is the description of time-dynamics of the financial pilot wave, φ(t,q). We follow to the standard pilot wave theory. Here $\varphi(t,q)$ is found as the solution of Schrodinger's equation.

Schrodinger's equation for the energy

$$H(q,p)=\frac{1}{2}\sum_{j=1}^{n}\frac{p_j^2}{m_j}+V(q_1,...,q_n)$$

has the form:

$$ih\frac{\partial\varphi}{\partial t}(t,q_1,...,q_n)=-\sum_{j=1}^{n}\frac{h^2}{2m_j}\frac{\partial^2\varphi(t,q_1,...,q_n)}{\partial q_j^2}+V(q_1,...,q_n)\ \varphi(t,q_1,...,q_n).$$

with the initial condition $\varphi(0, q_1, \dots q_n) = \psi(q_1, \dots q_n)$.

Thus, if we know $\varphi(0,q)$, then by using Schrodinger's equation we can find the pilot wave at any instant of time t, $\varphi(t,q)$. Then we compute the corresponding mental potential $U(t,q)$ and mental force $g(t,q)$ and solve Newton's equation.

We shall use the same equation to find the evolution of the financial pilot wave. We have only to make one remark, namely on the role of the constant h in Shrodinger's equation. In quantum mechanics (that deals with microscopic objects) h is the Planck constant. This constant supposed to play the fundamental role in all quantum considerations. However, originally h appeared as just a scaling numerical parameter for processes of energy exchange, see, especially, the paper of A. Einstein [22], see [12]-[15] on general investigations of the possibility to use quantum formalism in domains different from microworld. Therefore in our financial model we can consider h as a price scaling parameter, namely the unit in that we would like to measure price change. We do not present any special value for h. There are numerous investigations on price scaling, see, e.g. [20]. It may be that there could be recommended some special value for h related to the modern financial market, fundamental financial constant. However, it seems that $h = h(t)$ evolves depending on economic development.

We suppose that the financial pilot wave evolves via financial Schrodinger's equation (an analogue of Schrodinger's equation) on the price space. In general case this equation has the form:

$$ih\frac{\partial\varphi}{\partial t}(t,q) = \hat{H}\varphi(t,q), \qquad \varphi(0,q) = \psi(q),$$

^where $\hat{H}$ is self-adjoint operator corresponding to the financial energy given by a function H(q,p) on the financial phase space. Here we have to proceed in the same way as in ordinary quantum theory for elementary particles.

As the mathematical basis of the model, we use the space $L_2(Q)$ of square integrable functions $\varphi : Q \to C$, where Q is the configuration price space, $Q = R^n$ or some domain $O \subset R^n$: $\|\varphi\|^2 = \int_Q |\varphi(x)|^2 dx < \infty$

Here dx is the Lebesque measure, uniform probability distribution, on the configuration price space.

Of course, the uniform distribution dx is not the unique choice of the normalization measure on the configuration price space. By choosing dx we assume that in the absence of the pilot wave influence, e.g., for $\varphi(x) = const$, all prices 'have equal rights'. In general, this is not true. If there is no financial (psychological) waves the financial market still strongly depends on 'hard' economic conditions. In general, the choice of the normalization measure M must be justified by real relation between prices. So, in general the financial pilot wave φ belongs to the space $L_2(Q, dM)$ of square integrable functions with respect to some measure M on the configuration price space:

$$\|\varphi\|^2 = \int_Q |\varphi(x)|^2 dM(x) < \infty$$

In particular, M can be a Gaussian measure: $dM(x) = \frac{1}{(2\pi \det B)^{n/2}} e^{\frac{-(B^{-1}(x-\alpha), x-\alpha)}{2}} dx$,

where $B = (b_{ij})_{i,j=1}^n$ is the covariance matrice and $\alpha = (\alpha_1, \dots, \alpha_n)$ is the average vector. Parameters (b_{ij}) and α are determined by 'hard'-economic conditions. In particular, b_{ij} gives the covariation between prices of traders a_i and a_j . We recall that the matrice B is symmetric. Thus $b_{ij} = b_{ji}$. The α_j is the average of prices a_j , $\alpha_j = \overline{q}_j$, proposed by the trader a_j. The measure M describes classical random fluctuations at the financial market that are not related to 'quantum' effects. The latter effects are described in our model by the financial pilot wave. If the influence of this wave is very small we can use classical probabilistic models; in particular, based on the Gaussian distribution. The Gaussian model for price fluctuations was the first financial probabilistic model, see Bachelier [16]. In fact, Bachelier described price dynamics by Brownian motion. Therefore it would be even more natural to consider the Gaussian distribution of price changes. So, it is useful to study momentum representation for the pilot wave theory [23]. Instead of financial waves on the configuration financial space Q, we can consider financial waves $\varphi : P \to C$ on the momentum space. We recall that the momentum $p = mv$, where v, the velocity, describes price changes. Therefore, by following to Bachelier we have to consider the Gaussian representation, $\varphi \in L_2(P, dm)$, where P is the price change space.[1]

However, further investigations demonstrated that it seems that the Gaussian model for price changes is not the best one to describe price fluctuations. One of alternative models is based on Levy process, [17], [20]. Therefore it can be useful to investigate the 'quantum' financial model that is based on the Cauchy (Lorentzian) measure:

$$dM(p) = \frac{\gamma}{\pi} \frac{dp}{\gamma^2 + p^2}$$

on the momentum financial space. It would be interesting to provide comparative analysis of financial pilot wave models for dM (p) Gaussian and dM (p) Cauchy.

It seems that the Cauchy measure has some advantages. By using this measure we exclude from the consideration financial waves that increase at infinity as the linear function. So, $\varphi(p) \approx p^k, \;\; k \geq 1, \;\; p \to \infty$, are excluded from this model. It looks very natural, since prices (at least real financial market) could not change arbitrary quickly.

We now turn back to the general scheme, concentrating on the configuration representation, $\varphi : Q \to C, \;\; \varphi \in L_2(Q) \equiv L_2(Q, dx)$. This is the general quantum-like statistical formalism on the price space (compare to [12]-[15]).

As in the ordinary quantum mechanics, we consider a representation of financial quantities, observables, by symmetric operators in L2(Q). By using Schrodinger's representation we define price and price change operators by setting:

$\widehat{q}\varphi(q) = q_j \varphi(q)$ (the operator of multiplication by the qj-price)

$\widehat{p}_j = \frac{h}{i} \frac{\partial}{\partial q_j}$ (the operator of differentiation with respect to the qj-price, normalized by the scaling constant h (and that $-i = \frac{1}{i}$ provides symmetry of

Operators of price and price change satisfy to so called canonical commutation relations:

$$[\widehat{q}, \widehat{p}] = \widehat{q}\widehat{p} - \widehat{p}\widehat{q} = ih$$

By using this operator-representation of price and price changes we can represent every function H(q, p) on the financial phase-space as an operator $H(\widehat{q}, \widehat{p})$ in $L_2(Q)$. In particular, the financial energy operator is represented by the operator:

---

[1] However, it must be noticed that in the standard pilot wave theory (in the contrary to the ordinary quantum framework) there is no symmetry between coordinate and momentum representations. So, we have to be very careful by using the momentum representation; but, see recent works of B. Hiley [23].

$$\widehat{H}=\sum_{j=1}^{n}\frac{h^{2}}{2m_{j}}\frac{\partial^{2}}{\partial q_{j}^{2}}+V(q_{1},...,q_{n})$$

Here V(q) is the multiplication operator by the function V(q).

In this general quantum-like formalism for the financial market we do not consider individual evolution of prices. The theory is purely statistical. We could only determine the average of a financial observable A for some fixed state φ of the financial market:

$$\left\langle A\right\rangle_{\phi}=\int_{Q}A(\phi)(x)\overline{\phi}(x)dx$$

The use of Bohmian model gives the additional possibility to determine individual trajectories.

References

1. J. Soros, The alchemy of finance. Reading of mind of the market. J. Wiley and Sons, Inc., New-York, 1987.
2. R. Penrose, Shadows of the mind. Oxford Univ. Press, Oxford (1994).
3. D. J. Amit, Modeling of brain functions. Cambridge University Press, Cambridge (1989).
4. D. Bohm, and B. Hiley , The undivided universe: an ontological interpretation of quantum mechanics. Routledge and Kegan Paul, London (1993).
5. B. Hiley, P. Pylkka. In: Brain, mind and physics. Editors: Pylkkanen, Active information and cognitive science – A reply to Kiesepp¨anen, P., Pylkk¨aki, A.. IOS Press, Amsterdam (1997). o, P., Hautam¨
6. A. Yu. Khrennikov, Classical and quantum mechanics on information spaces with applications to cognitive, psychological, social and anomalous phenomena. Found. of Physics, 29, N. 7, 1065-1098 (1999).
7. A. Yu. Khrennikov, Classical and quantum mechanics on p-adic trees of ideas. BioSystems, 56, 95-120 (2000.).
8. D. Bohm, Quantum theory, Prentice-Hall. Englewood Cliffs, New-Jersey (1951).
9. P. Holland, The quantum theory of motion. Cambridge University press, Cambridge, (1993).
10. W. Heisenberg, Physical principles of quantum theory. Chicago Univ. Press, 1930.
11. P. A. M. Dirac, The Principles of Quantum Mechanics. Claredon Press, Oxford, 1995.
12. A. Yu. Khrennikov, Ensemble fluctuations and the origin of quantum probabilistic rule. Rep. MSI, Vaxjo Univ., 90, October (2000).
13. A. Yu. Khrennikov, Linear representations of probabilistic transformations induced by context transitions. Preprint quant-ph/0105059, 13 May 2001.
14. A. Yu. Khrennikov, 'Quantum probabilities' as contex depending probabilities. Preprint quant-ph/0106073, 13 June 2001.
15. A. Yu. Khrennikov, Interpretations of probability. VSP Int. Publ., Utrecht, 1999.
16. L. Bachelier, Theorie de la speculation. Ann. Sc. l'Ecole Normale Superiere, 111-17, 21-86 (1990).
17. B.B. Mandelbrot, The variation of certain speculative prices. J. Business, 36, 394-419 (1963).
18. P. Bak, K. Chen, J. Scheinkman and M. Woodford, Aggregate fluctuations from independent sectorial shocks: self-organized criticality in a model of production and inventory dynamics. Ricerche Economiche, 47, 3-30 (1993).
19. P. H. Cootner, ed., The random character of stock market prices. MIT Press, Cambridge, 1964.

20. R. N. Mantegna and H. E. Stanley, An introduction to econophysics. Cambridge Univ. Press, 2000.
21. O. Choustova, Price and the problem of the study of people's request to goods. Proc. of Conf. Role and importance of price in the evaluation of fucntioning of enterprices. 33-35, Ijevsk, 1987 (in Russian).
22. A. Einstein, Die Plancksche Theorie der Stralung und die Theorie der spezifischen Warme. Ann. der Physik, 22 (1907).
23. B. Hiley, Non-commutavive geometry, the Bohm interpretation and the mind-matter relationship. To appear in Proc. CASYS 2000, LIege, Belgium, 2000.

Core issues in German strategic management research

Kurt Matzler, Hans H. Hinterhuber, Stephan A. Friedrich, Heinz K. Stahl*

Abstract. This study attempts to identify core issues of German strategic management research. Departing from an analysis of core issues of the last decade, the authors identify future directions of research into strategic management and provide suggestions and recommendations for a possible research agenda. In order to accomplish these objectives, the following methods were used. First, a content analysis of seven leading German journals was conducted for the years 1990 to 2000; over 400 articles were categorized according to their central issues as well as to their methodology. Then, a Delphi-study among leading German researchers in strategic management was conducted to identify current and future research directions, challenges and developments in the field. Finally, the authors present the results of an empirical study among 149 researchers in Germany, Austria and Switzerland, which identifies the problems, weaknesses and challenges of German research in business administration in general. Suggestions are made related to how these problems can be overcome.

Key Words: scientific research, strategic management, empirical method, conceptual method, business administration.

Introduction

Hermann Simon, in an article published in 1993, accused German business research of suffering from a "black hole-syndrome" (Simon, 1993): It received substantial information from outside, but did not communicate to the outside. Simon believed that German research output was largely unknown to the international academic community and called for a dramatic change towards more international integration.

In their article, "Making and Measuring Reputations", Baden-Fuller, Ravazzolo and Schweizer (2000) presented a ranking of European business schools and university departments on the basis of their published work in top quality international management journals. As for the German-speaking countries, the results are rather disappointing: None of the business schools or university departments made it to the top thirty-eight.

The number of publications in leading international journals is not worth mentioning. Consequently, the international scientific community is practically unaware of what is done in the German speaking countries. Thus the objectives of this paper are: (a) to provide an insight into the German research priorities in Strategic Management, (b) to discuss the main issues and deficits within this area and (c) to recommend on how to overcome the lack of international integration.

* Department of General and Tourism Management, University of Innsbruck/Austria, Universitaetsstrasse 15 6020 Innsbruck, Austria Tel. +43-512-507-7185 Fax + 43-512-507-2968 e-mail: kurt.matzler@uibk.ac.at