“Capital Structure Model (CSM): correction, constraints, and applications”

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CAPITAL STRUCTURE MODEL (CSM): CORRECTION, CONSTRAINTS, AND APPLICATIONS

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Abstract

This paper extends the Capital Structure Model (CSM) research by performing the following tasks. *First*, a correction is offered on the corporate tax rate adjustment found in the break-through concept of the levered equity growth rate (g_L) given by Hull (2010). This correction is important because g_L links the plowback-payout and debt-equity choices and so its accuracy is paramount. *Second*, this paper introduces a retained earnings (*RE*) constraint missing from the CSM growth research when a firm finances with internal equity. The *RE* constraint governs the plowback-payout and debt-equity choices through the interdependent relation between *RE* and interest payments (*I*). *Third*, a by-product of the *RE* constraint is a second constraint that governs a no-growth situation so that *I* does not exceed the available cash flows. *Fourth*, with the g_L correction and two constraints in place, updated applications of prior research and new applications are provided. These applications reveal lower gain to leverage (G_L) values than previously reported with more symmetry around the optimal debt-to-equity ratio (*ODE*) while minimizing steep drop-offs in firm value. For larger plowback ratios, the optimal debt level choice can change. The new constraints serve to point out the need for further research to incorporate external financing within the CSM framework.

Keywords Capital Structure Model, gain to leverage, levered equity growth rate

JEL Classification G32, G35, C02

INTRODUCTION

Since the publication of Capital Structure Model (CSM) with growth by Hull (2010), five papers on the CSM have been published. They include two theoretical extensions, two instructional papers, and one applied paper. The two theoretical extensions cover wealth transfers (Hull, 2012) and changes in tax rates (Hull, 2014b). The two instructional papers provide pedagogical exercises on growth (Hull, 2011) and wealth transfers (Hull, 2014a). The applied paper (Hull & Price, 2015) concerns pass-through enterprises where corporate tax rates are nonexistent.

At the root of Hull (2010) CSM growth research is the concept of the levered equity growth rate (g_L). Prior to the development of the CSM, the finance world had no concept of g_L and thus no variable to directly link the plowback-payout and debt-equity choices as interdependent selections when applied to a perpetuity gain to leverage (G_L) equation resulting from a debt-equity exchange. The concept of g_L remains absent in the dividend valuation model (DVM) with growth. Unlike the Hull (2010) CSM, the DVM with growth offers a growth rate that does not distinguish between a firm having debt and not having debt.

In this paper, the primary goals are to correct the g_L equation given by Hull (2010) where $(1-T_C)$ was misplaced, provide two missing constraints not found in the extant CSM research, and offer updated and new applications using the new g_L and constraints. One constraint governs the use of retained earnings (RE) and tells us when RE cannot be maintained because of too much interest (I). The RE constraint also embodies a constraint for a no growth situation that was missing in Hull (2007)[1]. Both constraints involve monitoring large debt issues that can lead to increasing I values that cause the firm to exhaust cash flows needed to satisfy the plowback-payout decision. When a constraint is violated before reaching an optimal debt-equity ratio (ODE), then it signals the firm needs external financing to attain its ODE.

Our applications highlight the differences between the old g_L without constraints and our new g_L with constraints. With the g_L correction in place, we find lower G_L values than previously reported and more symmetry around the ODE while minimizing steep drop-offs in firm value. Except for lower plowback ratios $(PBRs)$, managerial decision-making in terms of choosing an optimal debt choice can be affected by the g_L correction. Finally, it appears that maximum G_L values are often achieved before a constraint sets in with an exception being when larger $PBRs$ are used with larger debt levels.

The remainder of our paper is as follows. Section 1 provides a literature review of capital structure research. Section 2 corrects three equations related to g_L and introduce the two new constraints. Section 3 presents an overview of major CSM equations used in our applications. Section 4 reports results from updated and new applications using the new g_L and the two constraints. Section 5 provides a discussion of results. Final section gives conclusions and future research possibilities.

1. LITERATURE REVIEW

In this section, we overview the MM and Miller tax models, trade-off theory (TOT) and pecking order theory (POT).

1.1. MM and Miller tax models

The perpetuity gain to leverage (G_L) research originates with Modigliani and Miller (1963), referred to as MM. Given their simplifying assumptions of an unlevered situation, no growth, no personal taxes and riskless debt, MM contend that:

$$G_L = T_C \cdot D, \tag{1}$$

where T_C is the effective corporate tax rate and D is perpetual riskless debt. With no personal taxes, D is:

$$D = \frac{I}{r_F}, \tag{2}$$

where I is the perpetual interest payment and r_F is the riskless cost of debt. Miller (1977) extends (1) by including personal taxes so that:

$$G_L = (1-\alpha)\cdot D, \tag{3}$$

where $\alpha = (1-T_E)\cdot(1-T_C)/1-T_D$, T_E, and T_D are the effective personal tax rates on equity and debt, respectively, and D now includes personal taxes and risky debt (r_D) such that:

$$D = \frac{(1-T_D)\cdot I}{r_D}. \tag{4}$$

1.2. TOT versus POT

The MM (1963) research stresses the benefits of debt. MM extensions focus on debt-related costs consisting of bankruptcy costs (Baxter, 1967; Kraus & Litzenberger, 1973) and agency costs (Jensen & Meckling, 1976; Jensen, 1986). This line of research, referred to as trade-off theory

1 While on the subject of corrections, we would note an error in a footnote, page 7, in Hull (2007) where it was stated that an ODE could, under simplifying assumptions, be approximated by ar_U/r_D. Even withstanding the advances introduced since that article, tests indicate this approximation overestimates ODE and so the derivational process behind this claim needs to be revisited.

(TOT), emphasizes both the benefits and costs of leverage and argues for the existence of an optimal debt-equity ratio (ODE). In contrast to TOT research, Miller (1977) and Warner (1977) argue that debt-related effects can be inconsequential such that no unique ODE occurs. However, subsequent researchers (Altman, 1984; Fischer et al., 1989; Kayhan & Titman, 2007) provide evidence that debt-related effects are important such that ODE exists.

The Capital Structure Model (CSM) research, overviewed in section 3, is consistent with TOT as its equations allow for both positive and negative effects from debt that lead to an ODE. As seen in Section 4, the CSM produces values consistent with TOT researchers (Graham, 2000; Korteweg, 2010; Van Binsbergen et al., 2010) who find maximum G_L values that increase firm value from 4% to 10%.

Pecking order theory (POT) provides the main challenge to TOT. Donaldson (1962), an early POT proponent, offers a pecking order where managers prefer internal equity financing for growth. If internal equity financing is lacking, he recommends asset conversion followed by debt issuance with external equity issuance the last resort. Myers and Majluf (1984) extend Donaldson emphasizing asymmetric information between managers and investors. Since investors lack information on the firm's prospects, they fear that managers will issue equity when overvalued and so will bid the price down if a new issue takes place. Consequently, prohibitive asymmetric information costs can result when using external equity. Besides asymmetric information, the financing resource ordering can stem from agency conflicts and taxes.

2. EXTENDING THE CSM WITH A g_L CORRECTION AND NEW CONSTRAINTS

In this section, we provide background for the CSM unlevered equity growth rate (g_U) and levered equity growth rate (g_L). All g_L related-formulations include our correction on how cash flows are adjusted for corporate taxes. We also introduce the no growth and RE constraints.

2.1. Double taxation on retained earnings

Hull (2010) refers to the before-tax cash flows from operating assets as CF_{BT}. The amount of CF_{BT} used for internal growth is the retained earnings (RE) and the amount earmarked for payment to equity is C. Hull defines the before-tax plowback ratio as $PBR = RE/CF_{BT}$ and the before-tax payout ratio as $POR = C/CF_{BT}$. If RE and C are lowered only by corporate taxes, the same values for PBR and POR occur. However, because a portion of CF_{BT} (namely, C) is taxed at the personal level, this causes PBR and POR to both change if we consider personal taxes.

Hull (2010) provides two definitions for the before-tax perpetual unlevered cash flow that results from growth (R_U). For the first definition, Hull has:

$$R_U(\Delta C) = g_U \cdot C. \tag{5}$$

Hull notes that the cost to produce R_U is the corporate taxes paid on RE before it can produce its own taxable income subject to corporate taxes. This double corporate tax when using internal financing is a fact researchers overlook when accounting for the cost of using internal equity. In response to this fact, Hull offers a second definition for R_U given as $R_U = r_E \cdot (1 - T_C) \cdot RE$, where r_E is the expected return on after-corporate tax retained earnings. Noting that r_E represents the long-run unlevered equity rate of r_U, Hull views R_U as:

$$R_U = r_U \cdot (1 - T_C) \cdot RE. \tag{5a}$$

The flotation expenses (T_F) of external financing (EF) are a cheaper form of financing for Hull. For this form, $R_U = r_U \cdot (1 - T_F) \cdot EF$. Since $T_F < T_C$, less external funds are needed to generate the same R_U. For a levered firm, the perpetuity before-tax cash flow from growth (R_L) is:

$$R_L = r_L \cdot (1 - T_C) \cdot RE, \tag{5b}$$

where r_L is the levered equity rate. For external financing, we have $R_L = r_L \cdot (1 - T_F) \cdot EF$.

With debt, there are added funds available to equity, namely, the tax shield from the interest payment. If retained earnings gets the same percent given by the firm's *PBR*, then an incremental retained earnings from the tax shield (ΔRE_{TS}) of $PBR(T_C)I$ could be added to (5b) to prevent underestimating R_L. However, it can be argued that the cash flow effect from this tax shield is already captured by the gain to leverage and, in that sense, it is already accounted for in firm value as a residual cash flow for equity. As seen later, this cash flow is in the variable *G* when used to formulate the levered equity growth rate (g_L) equations given in (7) and (7a) where we imply that *G* contains the tax shield being used for *RE*. Thus, at least for now, we will ignore this possibility that R_L is underestimated and reserve it for future exploration when a *PBR*, other than a before-tax *PBR*, is the focus.

2.2. Equity growth rates and critical points

Rearranging (5) gives the unlevered equity growth rate as:

$$g_U = \frac{\Delta C}{C} = \frac{R_U}{C}, \tag{6}$$

where C grows at g_U for far-reaching periods. Since C increases each period, it stands to reason that the financing to support that increase would also be increasing in a similar manner such that g_U could also be expressed as $\Delta RE/RE$ It should be noted that these increases are on a per share basis.

Hull (2010) discusses the *minimum* g_U that a no growth unlevered firm must attain so that its equity value will not fall if it chooses to grow through RE. Hull shows the minimum g_U must equal $r_U PBR$. Hull (2010) suggests a similar expression holds for a levered firm, namely, *minimum* $g_L = r_L PBR$. Using the equation of *minimum* $g_U = r_U PBR$ along with (5) and (5a), Hull demonstrates that the *minimum* g_U implies $PBR = T_C$ where T_C is both the cost of internal equity financing and also the *minimum* PBR needed to insure that growth does not decrease unlevered equity value. Hull labels the point where the PBR equals the cost of financing as the critical point (CP). Of extreme importance, for a corporation that uses internal financing, $PBR > T_C$ must hold if growth is to add value. CP gives the minimum starting value for setting the PBR because managers should not undertake growth unless $PBR \geq CP$. To the extent EF is used instead of RE, CP falls and a smaller PBR can add value to equity.

The general discussion found from the popular press to standard academia articles appears to refer to the *minimum* g_L as the sustainable growth rate, which can imply a maximum growth rate based on the company's RE. For example, Investopedia states that a company's sustainable growth rate is the product of its return on equity and the fraction of its profits that is plowed back into the firm (i.e., r_L PBR)[2]. It adds that this means a firm can safely grow at this rate using its own revenues to remain self-sustaining and can seek outside funding if it wants to accelerate its growth at higher rate. However, nothing from standard sources appears to mention that this sustainable growth rate must be greater than the cost of financing if firm value is to increase. The standard discussions also do not distinguish between unlevered and levered growth rates. This is akin to the Dividend Valuation Model with growth that also does not differentiate between unlevered and levered growth rates.

2.3. Correction on the break-through concept of the levered equity growth rate

We will now correct the levered equity growth rate (g_L) given by Hull (2010). Equations (5c), (7) and (7a) are the equations affected by the correction[3].

The simplest way to create g_L is to adjust (6) where $g_U = R_U/C$ for a levered situation. In doing this, we begin by replacing R_U with R_L where the latter was given in (5b). We next replace C with all levered cash flows not earmarked for RE. Besides C, we have the perpetual cash flow from the gain to leverage (G) that is defined on a before-tax basis as given later in (14). We not only know that C will be reduced by I but we also know that I has a corporate tax benefit that occurs before C is taxed. Thus, we multiply I by $(1-T_C)$.

2 Retrieved October 10, 2017 from http://www.investopedia.com/terms/s/sustainablegrowthrate.asp

3 All three equations in Hull (2010) had expressions where $(1-T_C)$ was effectively a divisor of interest (I) when it should have been the multiplicand.

Given the above adjustments, we define the levered equity growth rate as:

$$g_L = \frac{R_L}{C + G - (1 - T_C) \cdot I}, \qquad (7)$$

where the amount of debt issued must not be too high so that I in (7) causes g_L to become large and unsustainable. In fact, if $(1 - T_C) \cdot I > C + G$ holds, then RE would have to fall, since debt owners have to be paid first. Thus, the break-through concept of g_L indicates that a growth firm is limited in its debt-equity choices and managers must exercise prudence and caution in choosing reasonable plowback and leverage choices if they are to avoid financial loss.

While perhaps not the last word, we think for now the new g_L in (7) is a better representative than given by Hull (2010) where $(1 - T_C)$ was misplaced in the denominator. Two points favor the new g_L as given in (7). *First*, the tax deduction given by I occurs prior to the taxing of C as part of net income (similar for G, which is computed on a before-tax basis). Thus, it should arguably be accounted for in the manner now found in (7). *Second*, as will be shown later in our applications when using the new g_L, we find results that are more symmetrical about ODE while minimizing steep drop-offs in value. We believe these results better represent what actually occurs.

2.4. Equilibrating unlevered and levered growth rates

Hull (2010) uses equations (5) and (5a) for R_U to get what he calls an equilibrating unlevered equity growth rate $(equilibrating\ g_U)$, which is the rate that balances the two formulations for R_U. Equating these two equations and solving for g_U gives:

$$equilibrating\ g_U = \frac{r_U \cdot (1 - T_C) \cdot RE}{C}, \qquad (6a)$$

where (6a) provides a g_U value such that (5) and (5a) will have the same R_U value. Similarly, Hull (2010) notes there are two equations involving R_L that can be used to get what he calls an equilibrating levered equity growth rate $(equilibrating\ g_L)$. *First*, we have equation (5b) where $R_L = r_L \cdot (1 - T_C) \cdot RE$. *Second*, we have the below equation that results from rearranging (7):

$$R_L = g_L \left[C + G - (1 - T_C) \cdot I \right]. \qquad (5c)$$

Equating these equations and solving for our equilibrating g_L, we get:

$$equilibrating\ g_L = \frac{r_L (1 - T_C) \cdot RE}{C + G - (1 - T_C) \cdot I}, \qquad (7a)$$

where (7a) gives a g_L value such that (5b) and (5c) give the same R_L.

2.5. Constraints and maximum equilibrating g_L

Assume an unlevered firm with *PBR* set to achieve its equilibrating g_L at it *ODE* where G_L is maximized. A key question is as follows. With internal financing, is it possible that a firm cannot issue enough debt to achieve its ODE before the firm pays out so much I that RE cannot be maintained? We answer this question by investigating (7a). With *RE* fixed by the chosen PBR to achieve the firm's desired growth, the denominator of $C + G - (1 - T_C) \cdot I$ reveals that $C + G$ must be greater than $(1 - T_C) \cdot I$. If not, then RE would have to relinquish some of its funds to service debt. From (7a), we see that the following RE constraint must hold:

$$C + G - (1 - T_C) \cdot I \geq RE. \qquad (8)$$

If this constraint does not hold, we no longer have enough RE to maintain the chosen growth. To illustrate, assume an unlevered firm investing \$30 of every \$100 of its CF_{BT} in RE. This leaves \$70 for C. Suppose the firm is aggressive in its leverage choice and chooses to retire 60% of its unlevered firm value (E_U). Further assume $T_C = 0.25$, $I = \$58$, and $G = \$3$. Inserting the given values into (8) and solving, we have $\$29.5 \geq \$30$. Since $\$29.5$ is not greater than or equal to $\$30$, our constraint does not hold and the firm has issued too much debt and cannot cover its RE requirement. Such a firm would have to turn to external equity to achieve its desired growth.

If $RE = 0$ (no growth situation where $PBR = 0$), then (8) implies that the following no-growth constraint must hold:

$$C + G \geq (1 - T_C) \cdot I. \qquad (8a)$$

If this constraint does not hold, we no longer have enough cash flows to cover debt payments. The constraint given by (8a) would likely only be violated for very high debt levels where $G < 0$ occurs due to a negative G_L.

3. CSM EQUATIONS NEEDED FOR APPLICATIONS

Before we offer updated and new applications using the new g_L, it is necessary to overview the major CSM equations used in these applications. Thus, in this section, we briefly present those equations covering situations for no growth, growth, wealth transfers, and changes in tax rates.

3.1. No growth CSM

Keeping the MM and Miller unlevered and no-growth conditions, Hull (2007) derives a CSM equation incorporating discount rates dependent on the leverage change. This equation is:

$$G_L^{D\to E} = \left[1 - \frac{\alpha r_D}{r_L}\right] \cdot D - \left[1 - \frac{r_U}{r_L}\right] \cdot E_U, \qquad (9)$$

where $D \to E$ indicates debt-for-equity exchange, the first component captures a positive tax-agency effect, and the second component represents financial distress costs (captured by increasing r_L values as debt increases) such that this component's negativity can offset the positive first component as debt increases. The reverse of equation (9) can be derived if a levered firm becomes unlevered. Equity-for-debt equations can also be derived for the other CSM extensions involving growth, wealth transfers, and changes in tax rates.

3.2. Growth CSM

Hull (2010) extends (9) by incorporating growth. His growth CSM equation is:

$$G_L^{D\to E} = \left[1 - \frac{\alpha r_D}{r_{Lg}}\right] \cdot D - \left[1 - \frac{r_{Ug}}{r_{Lg}}\right] \cdot E_U, \qquad (10)$$

where r_{Ug} and r_{Lg} are the growth-adjusted discount rates on unlevered and levered equity, $r_{Ug} = r_U - g_U$ with r_U and g_U the borrowing and growth rates for unlevered equity, and $r_{Lg} = r_L - g_L$ with r_L and g_L the borrowing and growth rates for levered equity.

3.3. CSM for levered situation with a wealth transfer

Hull (2012) incorporates a levered situation within the CSM framework and derives G_L equations showing how a wealth transfer (linked to a shift in risk between debt and equity) impacts firm value for incremental leverage changes. For incremental changes over time, we have to distinguish between values before and after the increment. Hull has "1" denotes less levered values and "2" signifying more levered value and is also used to refer to the new debt for debt-for-equity increments and the retired debt for equity-for-debt increments.

Debt-equity CSM equations for a levered situation focus on how the less levered cost of debt (r_{D1}) might change. The three outcomes for r_{D1} are no change, an increase, and a decrease. *First*, for no change in r_{D1}, Hull (2012) shows:

$$G_{L_2}^{D\to E} = \left[1 - \frac{\alpha r_{D_2}}{r_{Lg_2}}\right] \cdot D_2 - \left[1 - \frac{r_{Lg_1}}{r_{Lg_2}}\right] \cdot E_{L_1}, \qquad (11)$$

where the "2" in $G_{L_2}^{D\to E}$ indicates at least one prior leverage change; D_2 is the new debt; r_{D_2} is the cost of D_2, r_{Lg_2} is the growth-adjusted levered equity discount rate after the debt-for-equity increment with $r_{Lg_2} = r_{L_2} - g_{L_2}$ where r_{L_2} and g_{L_2} are equity's discount and growth rates; r_{Lg_1} is the growth-adjusted levered equity discount rate before the increment with $r_{Lg_1} = r_{L_1} - g_{L_1}$ where r_{L_1} and g_{L_1} are equity's discount and growth rates; and, E_{L_1} is the less levered equity value that occurs before the increment. Equation (11) represents the situation with no wealth transfer from D_1 (older debt) to E_{L_2} (remaining equity). This is not the case for the next two derivations where D_1 is affected through the change in r_{D_1}.

Second, for an increase in r_{D_1} where the claims of old debt (D_1) are diluted by the new debt (D_2), Hull (2012) shows:

$$G_{L_2}^{D\to E} = \left[1 - \frac{\alpha r_{D_2}}{r_{Lg_2}}\right] \cdot D_2 - \left[1 - \frac{r_{Lg_1}}{r_{Lg_2}}\right] \cdot E_{L_1} - \left[1 - \frac{r_{D_1}}{r_{D_1\uparrow}}\right] \cdot \mathrm{D}_1 \qquad (11a)$$

where $r_{D_1\uparrow}$ is r_{D_1} after its risk shifts upward from issuing D_2. The last component is negative and identical to the fall in D_1 caused when its discount rate increases from r_{D_1} to $r_{D_1\uparrow}$. Third, for a decrease in r_{D_1} (the less likely outcome), Hull (2012) shows:

$$G_{L_2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]\cdot D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]\cdot E_{L_1} - \left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]\cdot \mathrm{D}_1, \tag{11b}$$

where the last component of $-\left[1-\left(r_{D_1}/r_{D_1\downarrow}\right)\right]\cdot \mathrm{D}_1$ is positive because $r_{D_1} > r_{D_{1\downarrow}}$.

3.4. CSM with change in tax rates

Hull (2014b) extends prior CSM equations by allowing tax rates to be dependent on leverage. In the process, he discovers a new α variable found in the second component of CSM equations that he labels α_2. This adds to the prior α variable in the first component that he now calls α_1. Hull shows that managers should not ignore α_2 due to its potential strong effects on the debt-for-equity choice. To derive his new CSM equation with tax rate changes, Hull labels the tax rates prior to the debt-equity increment as T_{C_1}, T_{E_1} and T_{D_1}. Afterwards, they are called T_{C_2}, T_{E_2} and T_{D2}. With wealth transfers included along with tax rate changes, Hull shows:

$$G_{L_2}^{D\to E} = \left[1-\frac{\alpha_1 r_{D_2}}{r_{Lg_2}}\right]\cdot D_2 - \left[1-\frac{\alpha_2 r_{Lg_1}}{r_{Lg_2}}\right]\cdot E_{L_1} - \left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]\cdot \mathrm{D}_1, \tag{12}$$

where $\alpha_1 = \left(1-T_{E_2}\right)\cdot\left(1-T_{C_2}\right)/1-T_{D_2}$ and increases with debt, and

$\alpha_2 = \left(1-T_{E_2}\right)\cdot\left(1-T_{C_2}\right)/\left(1-T_{E_1}\right)\cdot\left(1-T_{C_1}\right)$

decreases with debt.

3.5. The G variable in g_L equations

As residual owners, Hull (2010) argues that the perpetual before-tax cash flow from G_L falls within the domain of the equity owners and thus is discounted at the same rate as CF_{BT}. This perpetuity cash flow is called G.

In terms of (10), G_L can be expressed as:

$$G_L = \frac{\left(1-T_E\right)\cdot\left(1-T_C\right)\cdot G}{r_{Lg}}. \tag{13}$$

Solving for G in (13), we get:

$$G = \frac{r_{Lg}\cdot G_L}{\left(1-T_E\right)\cdot\left(1-T_C\right)}, \tag{14}$$

where G can be positive or negative depending on the value for G_L. G influences the cash flows available for payout as G belongs to the residual equity owners. As seen earlier, it influences g_L.

3.6. Coefficients in CSM equations

Hull (2010) represents the CSM equations for G_L in terms of positive and negative coefficients that multiply security factors. For example, Hull represents equation (10) as:

$$G_L = n_1 D - n_2 E_U, \tag{15}$$

where $n_1 = \left[1-\left(\alpha r_D/r_{Lg}\right)\right]$, $n_2 = \left[1-\left(r_{Ug}/r_{Lg}\right)\right]$, and $n_1 > n_2$ will hold until a large leverage ratio is reached. The initial large positive gap between $n_1 - n_2$ narrows as debt increases due to the fact that n_1 decreases with debt, while n_2 increases with debt. Hull finds values for n_1 and n_2 fall as a firm's plowback ratio increases with the gap of $n_1 - n_2$ narrowing as the firm nears its optimal PBR that maximized G_L.

4. RESULTS AND DISCUSSION FROM THE CSMGL APPLICATIONS

This section gives updated and new applications. Based on these applications, we report findings comparing the old g_L versus the new g_L and the RE constraint. We also report results using the no growth constraint.

4.1. CSM GL application with growth, no wealth transfer, and no tax change

Appendix A and Appendix B present applications that repeat those given by Hull (2010) using (10) except we use the new g_L given in (7a) and the two new constraints. The four gray-shaded cells (with bold print) in the rows above the chart in Appendix A correspond to the maximum (max) G_L as a fraction of unlevered equity (E_U) for four *PBR* choices. The values for G_L/E_U are consistent with empirical research cited earlier. The first two gray-shaded cells for *PBRs* of 0 and 0.15 have max G_L/E_U values that correspond to $DC = 0.3$, which means 30% of E_U is retired by debt to maximize firm value if these *PBRs* are chosen. These two *DCs* agree with Hull (2010) for his *PBRs* of 0 and 0.15 that use the old g_L. The third gray-shaded cell for the $PBR = 0.3$ row corresponds with the max G_L/E_U that occurs at $DC = 0.4$. This differs from Hull (2010) where a *PBR* of 0.3 corresponds with a max G_L at $DC = 0.6$. As seen in Appendix A, we had to increase *PBR* to 0.35 to achieve the max G_L at $DC = 0.6$. From this appendix, the four max G_L/E_U values of 0.065, 0.062, 0.072, and 0.129 for the new g_L compare to 0.065, 0.069, 0.156, and 0.217 for the old g_L.

As seen in the chart in Appendix A, when the *RE* constraint given in (8) is violated, no value is assigned to G_L/E_U and so the trajectory for that *PBR* terminates. For example, for $PBR = 0.35$ the trajectory terminates where $G_L/E_U = 0.129$. If there are no violations of a constraint, then a trajectory converges to zero once all debt is retired as the firm becomes unlevered at that point and reverts back to $PBR = 0$.

The gray-shaded columns (with bold print cells) above the chart in Appendix B give variable values where the largest max G_L/E_U value occurs, which is for optimal choices of $PBR = 0.39$ and $DC = 0.4$. This appendix is often consistent with Hull (2010) as follows. *First*, once we reach *PBR* of 0.42, positive G_L/E_U values no longer occur for any *DCs* for the old g_L and new g_L. *Second*, the optimal *DC* remains constant at 0.3 for low *PBR* values, but once we reach $DC = 0.25$, the optimal *DC* are typically lower when using the new g_L. Lower optimal *DCs* occur for either lower *PBRs* or higher *PBRs* and this is true for the old g_L and new g_L. *Third*, greater G_L/E_U values occur for higher *DCs* and this holds for the old g_L and new g_L. Regardless, using the new g_L equation yields lower G_L/E_U values. Fourth, for the old g_L and new g_L, greater G_L/E_U values occur for lower values of the coefficient differential of $n_1 - n_2$. Fifth, compared to their peak *ODEs*, the old g_L and new g_L show lower *ODEs* for either lower *PBRs* or higher *PBRs*.

Finally, the number of times the *RE* constraint is violated is given in the last row. As expected, for higher *PBRs*, there are more violations of the *RE* constraint. For a *PBR* of 0.42, there are six violations among the nine *DC* choices. This compares to zero violations for a *PBR* of 0.05.

Appendix C provides three charts that updates the instructional CSM growth paper of Hull (2011) using (10). As seen in the first chart, old g_L and new g_L values begin to noticeably diverge when we reach $DC = 0.5$ as the old g_L becomes negative. The new g_L continues to increase until the *RE* constraint sets in after $DC = 0.7$. From the second chart, we find that G_L has a dramatic fall after $DC = 0.5$ for the old g_L, as the old g_L becomes negative after this point. After $DC = 0.7$, the free fall would begin for the new g_L except for the fact that the *RE* constraint is violated stopping the trajectory. From the third chart, we see the debt-to-firm value ratio *DV* increases for the old g_L and new g_L. For the old g_L, there is a major rise of about 100% from 0.4 to 0.81 when the *DC* goes from 0.5 to 0.6. *ODEs* using the old g_L and new g_L are similar at 0.4 and 0.46, respectively, reflecting the fact both have the same optimal *DC* of 0.5.

4.2. CSM G_L application with growth, a wealth transfer, and no tax change

We now repeat the applications found in Hull (2012) using the CSM with wealth transfers and the new g_L equation.

4.2.1. Asset substitution problem

The application for the asset substitution problem involves the claim by Leland (1998) that a tax shield effect from debt is greater than an agency costs of debt related to asset substitution. To examine this claim, we compare the tax shield component of (11a), $\left[1-\left(\alpha r_{D_2}/r_{Lg_2}\right)\right]\cdot D_2$, with the asset substitution or wealth transfer component of (11a), $-\left[1-\left(r_{D_1}/r_{D_1\uparrow}\right)\right]\cdot D_1$. Following Hull (2012), we set the outstanding debt (D_1) equal to the new debt (D_2) so we can compare $-\left[1-\left(r_{D_1}/r_{D_1\uparrow}\right)\right]$ and $\left[1-\left(\alpha r_{D_2}/r_{Lg_2}\right)\right]$ with no advantage to D_1 or D_2 being greater. From an absolute value standpoint and substituting in $r_{Lg_2}=r_{L_2}-g_{L_2}$, the advantage to the tax shield occurs when $\left|r_{D_1}/r_{D_1\uparrow}\right|>\left|\alpha r_{D_2}/r_{L_2}-g_{L_2}\right|$.

Due to space constraints, we do not report all details with numbers, but only the most important outcomes. When going from the 20% to 40% debt levels like Hull, we find that the Leland claim holds. This is true even if we adjust for a wealth transfer due to risk shift from debt to equity. Using the Hull optimal debt level of 50% and extrapolating to get 25% debt level values, we find that the Leland claim still holds. This latter application using the new g_L differs from Hull (2012) where the Leland claim did not hold when using the old g_L. Disregarding the fact $D_2>D_1$ (which can heavily favor rejection of Leland), if we go from the 10% to 45% debt level and adjust for risk, we discover the Leland claim does not hold. Thus, even ignoring the fact $D_2>D_1$, we see the possibility of still rejecting the Leland notion as a firm attempt to reach its ODE. In conclusion, while the results using the new g_L is more likely to favor the Leland claim, we can see there are still scenarios where the Leland claim would not hold.

4.2.2. Underinvestment problem

In regards to the underinvestment notion, Myers (1977) suggests that equity would not want to plow RE into lower risk projects that favor debt. Similarly, equity would not want to approve an equity-for-debt transaction if the new equity favors remaining debt owners by making their cash flows safer at equity's expense. For both cases, the decision to increase equity would not be desired by equity owners if debt profited at their expense.

To examine the underinvestment problem, we use the equity-for-debt equation of Hull (2012) given as:

$$G_{L_2}^{E\to D}=\left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]\cdot E_{L_1}-\left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]\cdot D_2+\left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]\cdot \mathrm{D}_1, \tag{16}$$

where $\left[1-\left(r_{D_1}/r_{D_1\downarrow}\right)\right]\cdot \mathrm{D}_1<0$ because $r_{D_1}>r_{D_1\downarrow}$. Equity owners would pursue an equity-for-debt exchange if $G_{L_2}^{E\to D}>0$ such as when a positive first component dominates negative second and third components.

In revisiting the underinvestment problem of Hull (2012) using his numbers but with the new g_L, we find that an optimal debt-to-firm value (ODV) of 0.46 is achieved with an optimal DC of 0.5. For the old g_L, an ODV of 0.40 is attained with same optimal DC of 0.5. Assuming the fall in debt's discount rate is from a debt level of 60% to 50%, we get

$$\begin{aligned}G_{L_2}^{E\to D}&=\left[1-\left(r_{Lg_1}/r_{Lg_2}\right)\right]\cdot E_{L_1}-\\&-\left[1-\left(\alpha r_{D_2}/r_{Lg_2}\right)\right]\cdot D_2+\left[1-\left(r_{D_1}/r_{D_{1\downarrow}}\right)\right]\cdot D_1=\\&=-\$0.222B-\$0.186B+\left(-\$0.251B\right)=\\&=-\$0.659B.\end{aligned}$$

The negative value of $-\$0.251B$ in the third component indicates that debt experiences a loss in this transaction with the overall G_L value being negative indicating ODV is not attained by retiring between 50% and 60% debt. Thus, unlike the Hull (2012) finding of a positive value when going from 60% to 50%, we find a negative value when using the new g_L.

Repeating Hull (2012), we go from a 50% debt level to a 40% debt level. For this example, we would not expect to get a positive G_L because we are moving away from the ODV of 0.46. This expectation holds as we get

$$\begin{aligned}G_{L_2}^{E\to D}&=-\$0.008B-\$0.297B+\left(-\$0.208B\right)=\\&=-\$0.153B.\end{aligned}$$

The absolute magnitude of $-\$0.659B$ (when going from 60% to 50%) is greater than that of $-\$0.153B$ (when going from 50% to 40%) with at least some of this due to asymmetry about the optimal leverage ratio where overshooting the optimal is more costly than undershooting. While this overshooting result is consistent with the empirical results of Hull (1999), the difference is less than Hull (2012) for the old g_L. Regardless, we see the possibility of increasing value when equity is lowered, which is central to the underinvestment claim.

4.2.3. *Examination of the notion that leverage increases when wealth is transferred*

Leland (1998) suggests that ODV may increase with asset substitution. In examining this notion by considering the wealth transfer aspect of asset substitution, we find disagreement with the Leland assertion that an asset substitution increases the optimal leverage ratio. Like Hull (2012), we find just the opposite of the Leland assertion when using (11a) and assuming that the wealth transfer component captures an effect similar to an asset substitution effect. For example, we find that $ODE = 0.849$ before adjusting for a wealth transfer and $ODE = 0.786$ after adjusting. These numbers that indicate leverage fall when wealth is transferred are qualitatively similar to the corresponding numbers of 0.673 and 0.627 found by Hull (2012) when using the old g_L. Finally, like Hull, we can confirm the using (11b) would produce the desired Leland results. However, as suggested by Hull, the use (11b) for this situation would be unlikely.

4.2.4. *Debt-equity decision-making with wealth transfers*

In Appendix D, we revisit the Hull (2014a) inlstructional exercise. The numbers used in this exercise for tax rates, costs of capital, and PBR are like those described in Appendix C. Appendix D illustrates V_L for no growth applications (with and without wealth transfer) and growth applications (with and without wealth transfers). The first chart in this appendix plots the relation between V_L and the debt choice (DC) using the old g_L for the four applications. The second chart repeats the first chart but uses the new g_L in conjunction with the constraints given in (8) and (8a).

In examining the two charts, we discover several points of interest. First, as before, we find lower maximum firm $(\max V_L)$ values when using the new g_L, as well as more symmetry around ODE despite using a relative high PBR of 0.35. Second, $\max V_L$ occurs at $DC = 0.5$ for all trajectories except when using the new g_L with a wealth transfer (WT) with growth where $\max V_L$ occurs at $DC = 0.6$. However, were we to use a $G_{L_2}^{Equity}$ equation, $\max V_L$ would occur at $DC = 0.6$ for all eight trajectories. Third, unlike the first chart where there is no RE constraint, the second chart uses the RE constraint and so this constraint prevents a steep drop off for the two growth trajectories. NOTE: Appendix D was updated because Growth (WT) was Growth $(\text{no } WT)$.

Fourth, the second chart reveals that a greater $\max V_L$ occurs when a WT is present for a levered growth situation, but a lower $\max V_L$ for a non growth levered situation with a WT. While these two relations also hold for the first chart, the $\max V_L$ are much more similar in the first chart. Fifth, the constraint given in (8a) is for the situation of non growth. As seen in the second chart, when used with a WT, this constraint kicks in at $DC = 0.9$ with the trajectory ending at $DC = 0.8$ where V_L is $\$9.87B$.

4.3. CSM G_L application with growth, a wealth transfer, and tax change

Hull (2014b) extends the CSM research by incorporating changes in tax rates (ΔTR). There are no detailed examples in Hull for which corrections can be offered using the new g_L and constraints. There is also no instructional paper using ΔTRs for which corrections can be offered. Thus, we create two new ΔTR applications that compare the old g_L with the new g_L.

The results from the new applications are in the two figures in Appendix E. The first figure plots G_L versus DC, while the second figure plots V_L versus DC. Each figure has four trajectories. The first trajectory is for a non growth levered situation with WT and ΔTR with no constraint. The second trajectory is the same as the first but with the constraint given in (8a). The third trajectory is the like the first trajectory but is for a levered growth situation using the old g_L with no constraint. The fourth trajectory is like the third trajectory but uses the new g_L with the RE constraint given in (8).

From the first figure, for the non growth trajectory with ΔTR, WT and no constraint, we have a $\max G_L$ at $\$2.25B$ at $DC = 0.6$. Without a constraint, G_L falls to $-\$1.91B$ at $DC = 0.9$. For nogrowth with the constraint given in (8a), the trajectory is the same except it stops at $DC = 0.7$ when G_L is $\$1.98B$. Thus, at this point there is too much debt exhausting all RE and so external financing is needed. For the growth trajectory with ΔTR, WT the old g_L and no constraint, we have a max G_L at $\$3.84B$ at $DC = 0.6$. Without a constraint, G_L falls to $-\$3.88B$ at $DC = 0.9$. For the growth with the new g_L and the RE constraint given in (8), max G_L is $\$2.13B$ at $DC = 0.6$ where the trajectory ends.

From the second figure, we find V_L results mirror those found in the first figure for G_L. Thus, we can offer the same general conclusions for both figures. First, ΔTR results are like prior results in that the old g_L produces greater max G_L and max V_L values. Second, the constraints do not necessarily affect decision-making in terms of choosing an optimal DC. For example, the non growth constraint has no real affect on decision-making as max $G_L = \$2.25B$ occurs before the constraint kicks in. For growth, it is more difficult to ascertain because the RE constraint sets in when G_L is still rising.

5. DISCUSSION OF RESULTS

In this section, we will offer a brief interpretation of the salient results documented in Section 4. We also call attention to our findings compared to prior research that used the old g_L without constraints.

We interpret the results in Appendix A as follows. Using the new g_L has practical ramification for managers as a lower max G_L/E_U results. This means there is a lower maximum firm value from leverage than suggested by prior CSM growth research. Compared to the results of Hull (2010), we find more symmetry around the optimal leverage ratio consistent with the fact that the decline in trajectories are not as steep when using the new g_L. From Appendix B, we see that the number of violations of the RE constraint increase as the PBR increases. We interpret this as indicating to managers that external financing is needed if they want to maintain larger $PBRs$ with larger DCs. Appendix C directly compares our results with prior research. From the comparisons in the three charts, we can visualize how the g_L correction explains our findings related to lower G_L values with greater symmetry about $ODEs$ and less dramatic falls in firm value.

From revisiting the asset substitution problem, we find that (compared to prior research) it is more difficult to reject the Leland claim that a tax shield effect from debt is greater than an agency costs of debt related to asset substitution. We interpret this as meaning that managers should not underestimate the effect of a tax shield effect compared to an agency effect. From investigating the underinvestment problem, we discover results consistent with the notion that equity can profit by underinvesting. Practically speaking, this means that equity-for-debt transactions can be valuable undertakings by managers. This latter result with the new g_L is like that using the old g_L. When examining Leland' claim that leverage increases when wealth is transferred, we cannot confirm this claim. Our finding using the new g_L is qualitatively similar to prior CSM research. Managers can take notice that a wealth transfer should typically decrease leverage.

From the applications in Appendix D that incorporate a wealth transfer and Appendix E that give new applications with changes in tax rates, we interpret results from these applications as consistent with our prior applications. In summary, once again, we find that using the new g_L renders lower firm values. Regardless, managers can take notice that general conclusions about the optimal DC are similar when using either the old g_L or new g_L. We interpret this as meaning that optimal leverage choices can still be made even if miscalculations about growth rates occur.

CONCLUSION AND FUTURE RESEARCH

In this paper, we further develop the CSM research through the following achievements. First, we offer an important modification to the g_L equation given by Hull (2010). The modification of the g_L equation concerns a correction on the corporate tax adjustment for the variables used in the g_L equation. Second, we introduce a constraint previously missing when a firm grows strictly by internal equity or retained earnings (RE). This RE constraint governs the plowback-payout and debt-equity choices and, in particular, the relation between RE and interest payments (I). When the constraint is violated before the optimal debt-equity choice is achieved, it signals that external financing is needed. Third, a by-product of the RE constraint is a second constraint that governs a no growth situation so that interest payments do not exceed the maximum payout and any gains from leverage that enhance the payout. Fourth, with the g_L correction and two constraints in place, we provide updated applications of prior research along with new applications.

From our applications, we obtain the following results that have practical ramifications for managers. We find lower G_L and V_L values with more symmetry around ODE and less steepness in the fall in firm value. Managers can note that growth is less risky than indicated by prior CSM research. We also show that general managerial decision-making in terms of choosing an optimal debt choice is not materially affected by the g_L correction. Except for larger $PBRs$ and larger DCs, we discover that maximum G_L values are achieved before the RE constraint set in. Thus, managers can often count on internal financing fulfilling their growth needs if that is desired.

The new constraints developed in this paper serve to point out the need for further research to incorporate external financing within the CSM framework. This incorporation should be valuable because it is cheaper and thus has a lower critical point making growth more profitable. We can point out that, while both constraints are not affected on a per share basis, we still cannot rule out the possibility that a different optimal DC might be chosen if we maximize equity on a per share basis. Thus, considering per share values is another subject that future research can explore.

REFERENCES

1. Altman, E. (1984). A Further Empirical Investigation of the Bankruptcy Cost Question. *Journal of Finance, 39,* 1067-1087.
2. Baxter, N. (1967). Leverage Risk of Ruin and the Cost of Capital. *Journal of Finance, 22,* 395-403.
3. Donaldson, G. (1961). *Corporate Debt Capacity: A Study of Corporate Debt Policy and the Determination of Corporate Debt Capacity*. Boston: Harvard Business School.
4. Fischer, E. O., Heinkel, R., & Zechner, J. (1989). Optimal Dynamic Capital Structure Choice: Theory and Tests. *Journal of Finance, 44,* 19-40.
5. Graham, J. (2000). How Big Are the Tax Benefits of Debt? *Journal of Finance, 55,* 1901-1941.
6. Hull, R. M. (1999). Leverage Ratios, Industry Norms, and Stock Price Reaction: An Empirical Investigation of Stock-for-Debt Transactions. *Financial Management, 28*(2), 32-45. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.581.2452&rep=rep1&type=pdf
7. Hull, R. M. (2007). A capital structure model. *Investment Management and Financial Innovations, 4*(2), 8-24. Retrieved from https://businessperspectives.org/journals/investment-management-and-financial-innovations/issue-69/a-capital-structure-model
8. Hull, R. M. (2010). A capital structure model with growth. *Investment Management and Financial Innovations, 7*(4), 55-69. Retrieved from https://businessperspectives.org/images/pdf/applications/publishing/templates/article/assets/3713/imfi_en_2010_04_Hull.pdf
9. Hull, R. M. (2011). Debt-equity decision-making with and without growth. *Managerial Finance, 37*(8), 765-787. Retrieved from https://www.researchgate.net/publication/227430094_Debt-equity_decision-making_with_and_without_growth
10. Hull, R. M. (2012). A capital structure model with wealth transfers. *Investment Management and Financial Innovations, 9*(3), 19-32. Retrieved from https://businessperspectives.org/images/pdf/applications/publishing/templates/article/assets/4736/imfi_en_2012_03_Hull.pdf
11. Hull, R. M. (2014a). Debt-equity decision-making with wealth transfers. *Managerial Finance, 40*(12), 1223-1250. Retrieved from http://www.emeraldinsight.com/doi/abs/10.1108/MF-09-2013-0239
12. Hull, R. M. (2014b). A capital structure model (CSM) with tax rate changes. *Investment Management and Financial Innovations, 11*(3), 8-21. Retrieved from https://businessperspectives.org/journals/investment-management-and-financial-innovations/issue-95/a-capital-structure-model-csm-with-tax-rate-changes
13. Hull, R. M., & Price, D. (2015). Pass-Through Valuation. *The Journal of Entrepreneurial Finance, 17*(1), 82-116. Retrieved from https://digitalcommons.pepperdine.edu/jef/vol17/iss1/4/

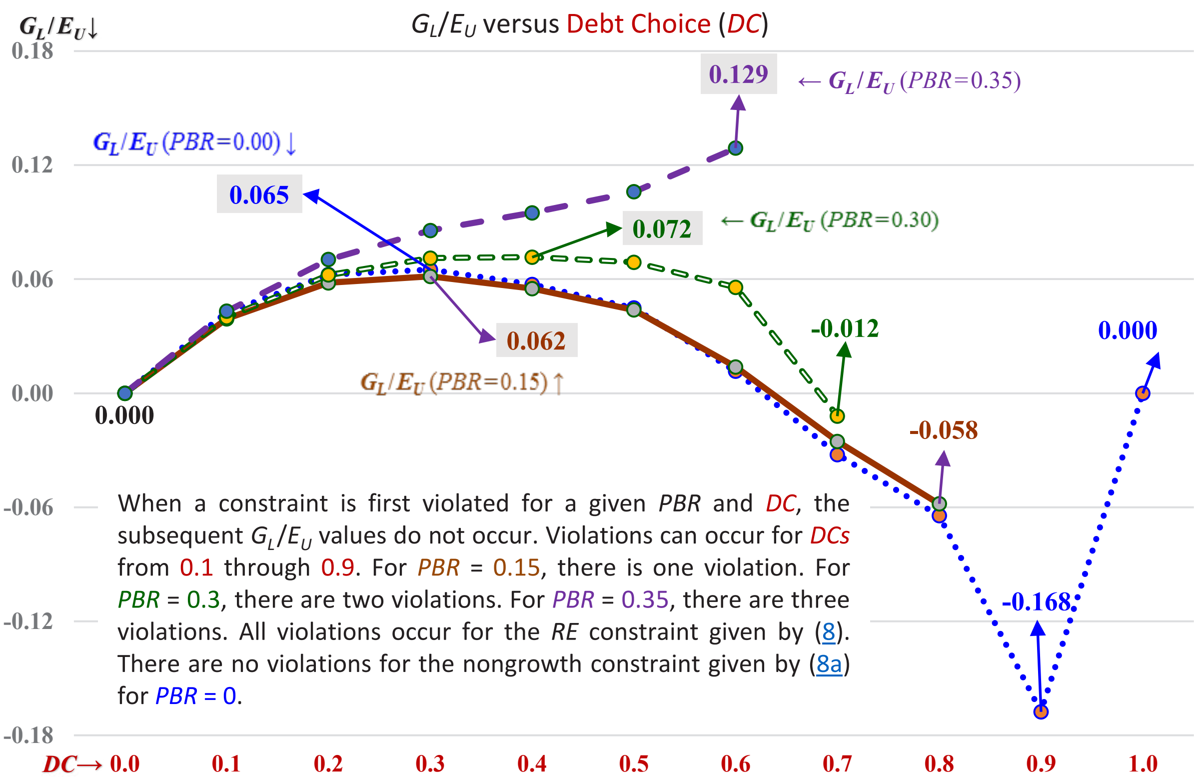
14. Jensen, M. C., & Meckling, W. H. (1976). Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure. *Journal of Financial Economics, 3,* 305-360. Retrieved from https://www.sfu.ca/~wainwrig/Econ400/jensen-meckling.pdf
15. Jensen, M. C. (1986). Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers. *American Economics Review, 76,* 323-329.
16. Kayhan, A., & Titman, S. (2007). Firms' Histories and Their Capital Structure. *Journal of Financial Economics, 83,* 1-32.
17. Korteweg, A. (2010). The Net Benefits of Leverage. *Journal of Finance, 65*(6), 2137-2170.
18. Kraus, A., & Litzenberger, R. H. (1973). A State-Preference Model of Optimal Financial Leverage. *Journal of Finance, 28*(4), 911-922.
19. Leland, H. (1998). Agency Costs, Risk Management, and Capital Structure. *Journal of Finance, 53,* 1213-1243.
20. Miller, M. (1977). Debt and Taxes. *Journal of Finance, 32*(2), 261-275.
21. Modigliani, F., & Miller, M. (1963). Corporate Income Taxes and Cost of Capital: A Correction. *American Economics Review, 53*(3), 433-443.
22. Myers, S. (1977). Determinants of Corporate Borrowing. *Journal of Financial Economics, 5,* 147-175.
23. Myers, S., & Majluf, N. (1984). Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have. *Journal of Financial Economics, 13*(2), 187-221.
24. Pratt, S., Grabowski, R., & Brealey, R. (2008). *Cost of Capital: Applications and Examples.* Hoboken, NJ: John Wiley & Sons, Inc.
25. Van Binsbergen, J., Graham, J. R., & Yang, J. (2010). The Cost of Debt. *Journal of Finance, 65*(6), 2089-2136.
26. Warner, J. (1977). Bankruptcy Costs: Some Evidence. *Journal of Finance, 32*(2), 337-348.

APPENDIX A.

First update of Hull (2010) application using the new g_L

This appendix updates the application of Hull (2010) using the new g_L equation given in (7a). The borrowing costs for the debt choices (DCs) are influenced by Hull (2007) and Pratt et al. (2008). Key values include $T_C = 0.26$, $T_E = 0.05$, $T_D = 0.12$, $r_F = 0.04$, and $r_M = 0.1$, and $\alpha = 0.8$. Tax rates are initial values and change in the predicted fashion consistent with Hull (2014b) as the debt choice (DC) increases in increments of 0.1 for *PBRs* of **0**, **0.15**, **0.3** and **0.35**. A DC reflects the proportion of unlevered equity (E_U) that is retired by debt. The chart illustrates that a no-growth firm $(PBR = 0)$ can have a different DC than a growth firm $(PBR > 0)$. The chart also reveals a swift drop-off in G_L/E_U with too much debt. The highest G_L/E_U value occurs when $DC = 0.6$ and $PBR = 0.35$. G_L/E_U values become negative if DC increases to **0.7** revealing great risk when too much debt is chosen. The four gray-shaded cells with bold print in the below rows correspond to the maximum (max) G_L for four PBR choices and its optimal DC.

Debt Choice (DC)	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
$G_L/E_U\,(PBR = 0.00)$	0	0.043	0.062	**0.065**	0.057	0.045	0.012	–0.032	–0.064	–0.168	0.000
$G_L/E_U\,(PBR = 0.15)$	0	0.039	0.058	**0.062**	0.055	0.044	0.014	–0.025	–0.058	0.000	0.000
$G_L/E_U\,(PBR = 0.30)$	0	0.040	0.062	0.071	**0.072**	0.069	0.056	–0.012	0.000	0.000	0.000
$G_L/E_U\,(PBR = 0.35)$	0	0.043	0.070	0.086	0.095	0.106	**0.129**	0.000	0.000	0.000	0.000

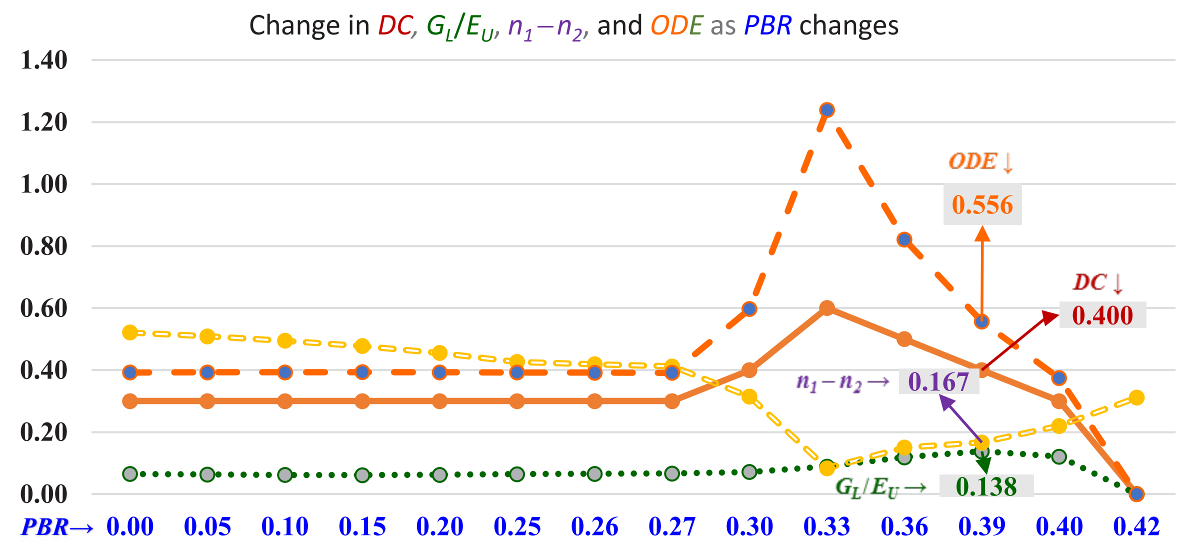


APPENDIX B.

Second update of Hull (2010) application using the new g_L

This appendix updates the application of Hull (2010) using the new g_L equation given in (7a). The borrowing costs for the debt choices (DCs) are influenced by Hull (2007) and Pratt et. al (2008). Key values include $T_C = 0.26$, $T_E = 0.05$, $T_D = 0.12$, $r_F = 0.04$, and $r_M = 0.1$, and $\alpha = 0.8$. Tax rates are initial values because tax rates are allowed to change in expected fashions consistent with Hull (2014b). The chart illustrates what happens as the plowback ratio (PBR) increases. The debt choice (DC) is the optimal choice for a given PBR. Each DC represents the proportion of unlevered equity (E_U) that is retired by debt. Max G_L/E_U is the maximum G_L as a fraction of unlevered equity. The coefficient differential from (15) is $n_1 - n_2$. ODE is the optimal debt-equity ratio that corresponds to the maximum G_L/E_U and thus maximum firm value. The gray-shaded column (with bold print cells) give variable values for the column where the largest max G_L/E_U occurs. The major point illustrated is that plowback-payout and debt-equity decisions are interlinked where firm maximization involves both decisions operating in unison. The number of times a constraint is violated is given in the last row. As expected, for higher $PBRs$, there are more violations. All violations are for the RE constraint given by (8). There are no violations for the non-growth constraint given by (8a).

PBR	**0.000**	**0.050**	**0.100**	**0.150**	**0.200**	**0.250**	**0.260**	**0.270**	**0.300**	**0.330**	**0.360**	**0.390**	**0.400**	**0.420**
Optimal DC	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.400	0.600	0.500	**0.400**	0.300	0.000
Max G_L/E_U	0.065	0.063	0.062	0.062	0.062	0.065	0.066	0.067	0.072	0.089	0.119	**0.138**	0.121	0.000
$n_1 - n_2$	0.521	0.509	0.495	0.477	0.455	0.426	0.419	0.412	0.314	0.084	0.150	**0.167**	0.220	0.312
ODE	0.392	0.393	0.393	0.393	0.393	0.392	0.392	0.391	0.597	1.239	0.822	**0.556**	0.374	0.000
Violations	0	0	1	1	1	2	2	2	2	3	4	**4**	5	6

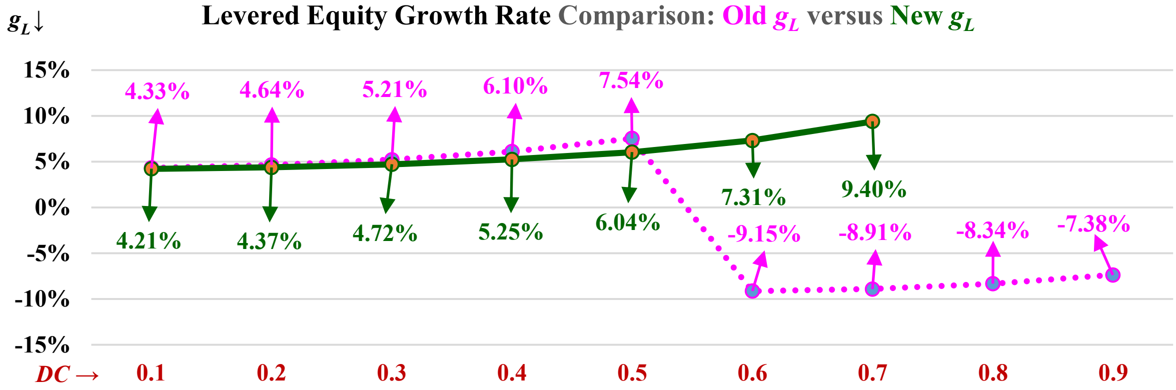


APPENDIX C.

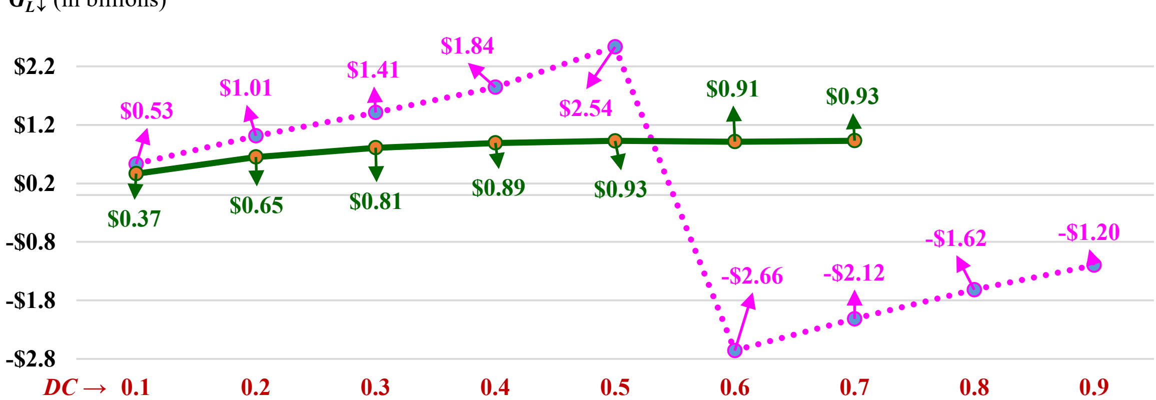
Update of Hull (2011) application: old g_L versus new g_L

This appendix updates the Hull (2011) growth exercise by comparing its "**old**" results (dotted line) with the "**new**" results using the new levered equity growth rate (g_L) given in (7a) and used with the RE constraint given in (8). Key values used by Hull (2011) include $T_C = 0.3$, $T_E = 0.05$, $T_D = 0.15$, $r_F = 0.05$, $r_M = 0.11$, and $PBR = 0.35$. Tax rates do not change. The sequence for costs of debt and equity can differ compared to the prior two appendices with one reason being the change in r_F. This appendix plots g_L, G_L and V_L versus DCs for the old g_L and the new g_L. Each DC is the debt choice that represents the fraction of unlevered equity (E_U) tat is retired by debt.

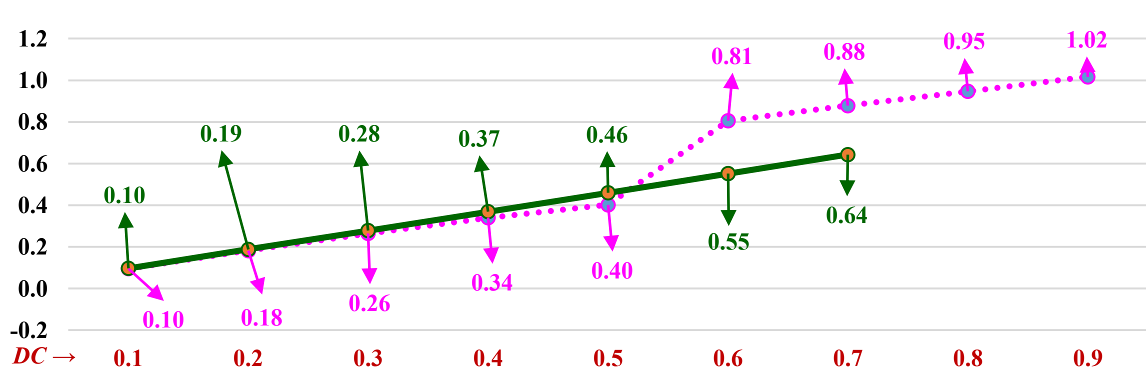
Levered Equity Growth Rate Comparison: Old g_L versus New g_L



Gain to Leverage (G_L) Comparison: Old g_L versus New g_L

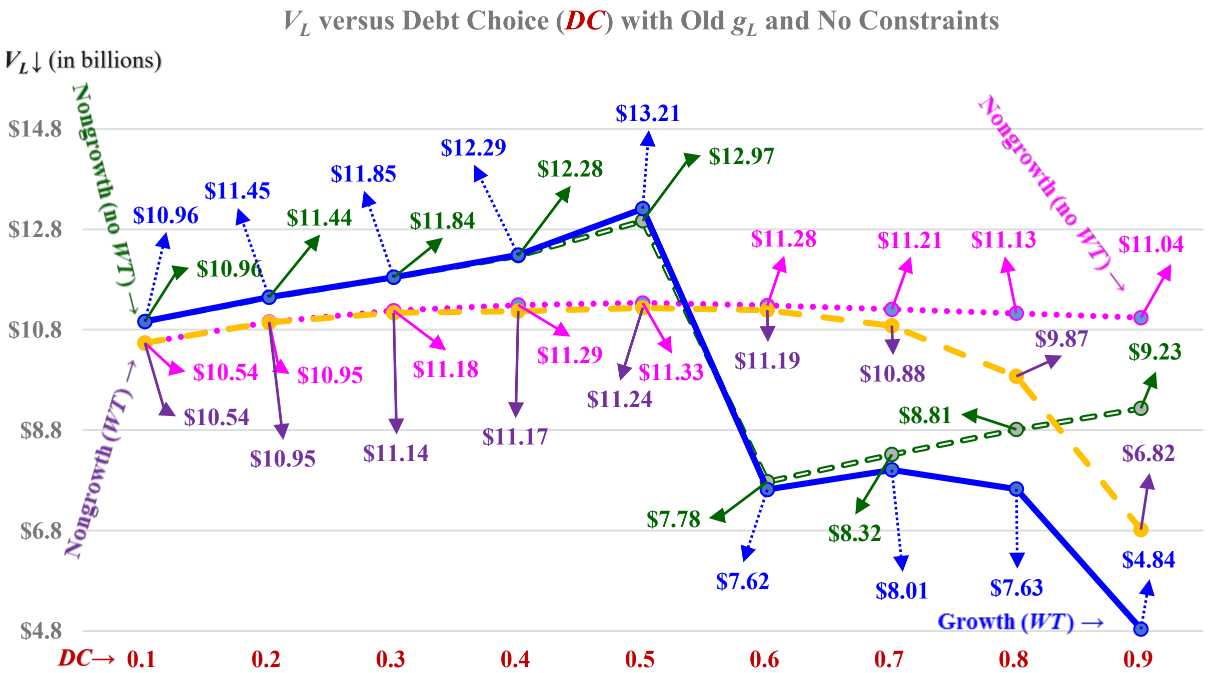


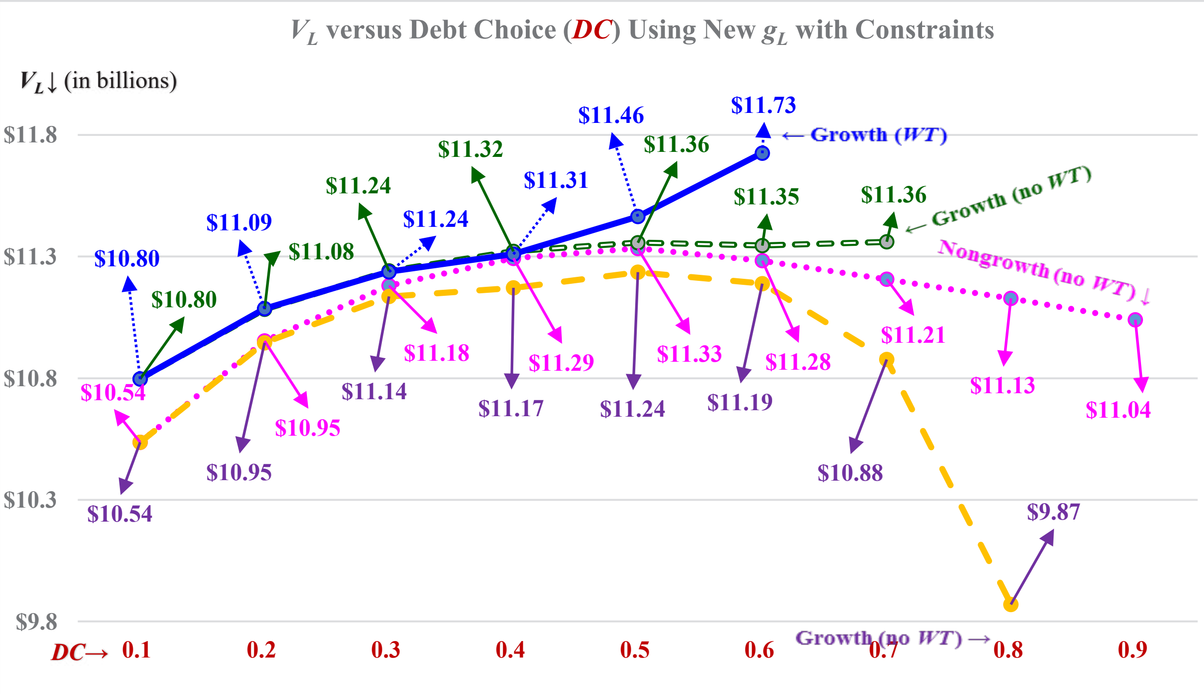
Debt-to-Firm Value Ratio (DV) Comparison: Old g_L versus New g_L



APPENDIX D.

Update of Hull (2014a) application





APPENDIX E.

New Applications using Hull (2014b)