“Identifying explosive behavioral trace in the CNX Nifty Index: a quantum finance approach”

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IDENTIFYING EXPLOSIVE BEHAVIORAL TRACE IN THE CNX NIFTY INDEX: A QUANTUM FINANCE APPROACH

Abstract

The financial markets are found to be finite Hilbert space, inside which the stocks are displaying their wave-particle duality. The Reynolds number, an age old fluid mechanics theory, has been redefined in investment finance domain to identify possible explosive moments in the stock exchange. CNX Nifty Index, a known index on the National Stock Exchange of India Ltd., has been put to the test under this situation. The Reynolds number (its financial version) has been predicted, as well as connected with plausible behavioral rationale. While predicting, both econometric and machine-learning approaches have been put into use. The primary objective of this paper is to set up an efficient econophysics' proxy for stock exchange explosion. The secondary objective of the paper is to predict the Reynolds number for the future. Last but not least, this paper aims to trace back the behavioral links as well.

Keywords econophysics, Hilbert space, Reynolds number, artificial neural network, heuristics

JEL Classification G1, G02, C45

INTRODUCTION

Researchers studying stock markets can be broadly classified into traditionalists or financial economists who have largely applied Gaussian random walk based statistical methods in studying specific stock market phenomena, and econophysicists who have, instead, applied conceptual theories of physics, including quantum physics, to study stock market behavior.

Recently, a group of researchers (Ausloos, Jovanovic, & Schinckus, 2016) have studied apparent contradictions in the two approaches adopted by financial economists and econophysicists. While both employ quantitative analysis and modeling, the former are seen to be adopting the top-down methodological approach involving a positivist attitude where the researcher works with the premise that reality or knowledge exists and is distinct from the phenomena being studied. In contrast, the latter adopt the opposite bottom-up methodological approach based on empirical data analysis that attempts to study micro situations through data and, where possible, generalize observations resulting from the analysis. The authors, however, concluded that indeed, both approaches have much in common though there are differences in the vocabulary and their usage.

Applications based on econophysics have been widely used in the global context to define symmetry, chaos, direction, and stability. Portuguese researchers (Bentes & Menezes, 2012) investigated seven stock markets across the globe to primarily question effectiveness of the generalized autoregressive conditional heteroskedasticity (GARCH) as an appropriate measure to quantify volatility. Entropy as an alternative measure of volatility was introduced in their study. Specifically, Tsallis, Renyi, and Shannon's entropies were used. While all three are information entropies and not deterministic thermodynamic entropies, their study concluded entropy to be a better measure of volatility in stock markets than the traditional GARCH. Though all the three entropy measures returned directionally similar results, Shannon's entropy was preferred over the other two, given higher accuracy levels of results returned by this measure.

Polish researchers (Jakimowicz & Juzwiszyn, 2012) applied the principles of rotary movement of objects around a central position of equilibrium found commonly in nature to measure volatility of stock prices. Borrowed from the field of fluid mechanics, their initial study was designed to investigate stability of markets or equilibrium positions of stock prices through the study of vortices of economic vectors taking price, volume, and time as dimensions. Specifically, they developed on the Kaldor's cobweb model, hitherto a two-dimensional model. Being a working paper, their (first) study in 2012 was inconclusive. In the sequel paper published in 2014, they highlighted the absence of equilibrium in the long run, implying equilibrium is reached only in the short or very short run. Using the Reynolds number signifying the quantum explosion (market breaching the normal volatility level), they calculated the Osborne Reynolds number for the Warsaw Stock Exchange (WSE) to find explosion and stability possibility. The model used was $R^3 = PQT$, where P was the index value, Q the volume traded, and T the time. Their attempt was to apply orthogonal projection of rotational trajectory to movements in stock prices on the WSE. Their study, however, was limited to the national Polish stock market. The WSE hosts the Polish benchmark Wig 30 Index and currently (as of December 22, 2016) lists 488 domestic stocks. Though the WSE is rated as one of the larger Central European stock markets, globally, they are not part of the 16 stock markets spread across three continents that make up the US$ 1 trillion club accounting for 87% of all market capitalization.

The study on computed financial turbulence by analyzing the cash flow viscosity of US stocks came up in the early 2000 (Los, 2004). This study was based on an earlier study (Mandelbrot, 2004), which revealed that physical turbulence and financial turbulence are similar. Indeed, Cornelis A. Los was the first to introduce the Reynolds number to the finance domain calling it "the Reynolds number for uniform cash flow". The study clearly portrayed the Reynolds number as a wavelet for determining turbulence in cash flow.

Series of studies that resulted in the publication of several books (Bouchaud, 1994; Sornette, 2003) expounded the market crash theory to explain the process of bubble formation, their growth and peaking, and bubbles bursting. Bouchaud's study was based on the research of events both prior to and post the crashes across markets and geographies. The study, which was profound in its reach resulting in 660 publications, covered both efficient market hypothesis (exogenous dynamics) and reflexivity of markets (endogenous dynamics). Furthermore, Sornette's study was focused on three aspects, namely frequency of crashes, distribution of drawdown and large crashes (as outliers), that is studied the anatomy of bubbles (displacement, credit creation, euphoria, financial distress, and revulsion). The study also established the correlation between positive feedback and creation of bubbles when he observed in his 2003 study that five of the six markets exhibited long lasting bubbles with asset values increasing more than 15 times their fundamental values. Mathematically, this translates into finite-time singularity. The study also attempted to explain anti bubble or deflation of prices and negative feedback in the Japanese markets in 1990.

Mantegna and Stanley (2000) furthered Bachelier's study after a gap of 95 years to showcase that scaling behavior of power law can be applied to financial markets. Scaling law is conditioned exponential decay and the study investigated the scaling of probability distribution of the S&P 500 using non-Gaussian methods. While scaling was seen in the S&P 500, interestingly, it was also found that the scaling exponent remained constant over the six-year study period from 1984 to 1989. The study established the possibility of scaling in economic systems originally seen and proved in thermodynamics and statistical probability distributions. Some Nigerian researchers (Olanipekun, Michael, & Oluwaseun, 2012) have borrowed quantum well as in quantum mechanics in Hilbert's space and applied it to stock markets. While vector space is traditionally infinite, their study made it finite in the context of the Nigerian stock market and used the Newton-Raphson model to predict price in a converging capital market. Similarly, Portuguese, Romanian, Chinese, and Polish researchers have studied the quantum well behavioral pattern of stocks (within defined circuit filters) in a well-defined finite-dimensional Hilbert space, but the logical continuation of their research in the Indian context is missing.

Two Chinese researchers (Zhang & Huang, 2010) defined the relation of Chinese stocks in a defined circuit filter with a quantum well, but did not continue it further. Similarly, Racorean (2015) found out that security prices are behaving like quantum particles within range bound markets. Cotfas (2013) also did his rate of return modeling inside a well-defined finite-dimensional Hilbert space. The logical continuation of their research is directional symmetry or effect of martingales in the time series, which will determine the predictability of that series.

Nonetheless, directional symmetry and effect of martingales in the time series depict the same story whatsoever. This, to a layman, will determine the predictability of a series of stock prices. For financial market stability, Nigerian researchers (Olanipekun, Michael, & Oluwaseun, 2012) have conducted the finite barrier model of complex systems using Newton-Raphson statistics, but they did not use entropy measurement to gauge stability of the markets. While Portuguese researchers (Bentes & Menezes, 2012) have used Shannon's entropy for volatility prediction, little or no research work has been found in the Indian context.

As all researches preceding this study are in a way linked through this research, we intend to present a combined picture of the work done so far by researchers from Portugal, China, Poland, and Nigeria. In fact, all the other researches were done under specific economic conditions as an independent quantum, and this study makes it a continuous rational quantum string. The cardinal difference is the definition of Reynolds number in this work. Besides this, all behavioral gaps will be identified and validated as per relevant behavioral theories. It is therefore believed that this work will be an addition to the existing body of knowledge.

If symmetry or chaos is observed to be high, it should logically be reflected through the higher Reynolds number. As the Reynolds number will go northwards, the stability should logically head south, and therefore, by calculating the Reynolds number, a clear picture of market stability would emerge, which would be immensely useful to large investors, including foreign institutional investors (FIIs), foreign portfolio investors (FPIs), and qualified institutional buyers (QIBs), or similar types of investors.

From various literature reviews, it is evident that though, stock exchange volatility, stability and prediction of volatility are of paramount importance for FIIs, FPIs, QIBs, and large investors, as similar work has not yet been undertaken in the Indian context. Even in the international context, partial works have been done, but the logical loop has not been completed. There exists the opportunity, which we intend unearthing through this work. To keep the logical loop running, a mix of econometric, econophysics and, lastly, machine learning tools will be used to validate all rationales, which is difficult to achieve staying within the domain of traditional finance.

1. LITERATURE REVIEW

Modern quantitative finance is largely driven by stochastic models, derived from the fundamental physical science theories and analogies. Physicists have started to use economic interpretations and finance-economic researchers have started to explain their research questions based upon physics equations, based on concepts that laid their foundation long back. Louis Bachelier, a noted physicist and mathematician, initiated and constructed a "theory of speculation" ("théorie de la spéculation") in his doctoral thesis in mathematical sciences in 1900 (Jovanovic, 2012). His work primarily suggests a theoretical and analogical connection between stochastic theory (random walks or Brownian motion) and financial theories. However, Bachelier's theory for option pricing linking Brownian motion faced harsh skepticism at the time. Though the work was forgotten for almost a century, his innovative approach is considered as a landmark in econophysics.

1.1. Quantum finance

Quantum theory is used to model secondary financial markets (Schaden, 2002). Consequently, quantum finance is one of the most contemporary subjects used in latest econophysics. For the first time, it was used in 2001 (Ilinski, 2001) to draw the analogy between quantum field and portfolio (as a financial field). Path integral and differential manifold were effectively used as the control points to explain the change of financial markets after the gauge transformation. In simple terms, a gauge could be easily described as a coordinate system, which in turn is a function of comparative location, with respect to a base space. Thus, a gauge transformation could well be described as a change of coordinates, defining such locations, and a gauge theory stands out as a model for some physical systems to which gauge transformations could be applied. Quantum mechanics application to the stock market has been carried out recently on a theoretical and mathematical platform. The stock exchange itself here is a macro-system, which absorbs all possible impurities of a micro-system like a stock. Stocks do behave (Zhang & Huang, 2010) like corpuscles (minute quanta particles) that traverse in a straight line with a finite velocity. Again the price fluctuates inside the market, showing clear similarity to a wave property. So, a clear wave-particle dualism is visible and present at the micro-system. This holds the link very firmly that the stock will be featured in quantum mechanics theory as well.

1.2. Hilbert space

In Hilbert space, dimensions are numerous and their lengths are infinite. However, in this case, the entire length is finite (as in stock market, the circuit filter is defined and fixed) and defined as d_o. Probability of the stock or index price between a and b (the index price or stock price a and b correspond to the lower circuit and the upper circuit, respectively) at time t is $P(t)$. The state of the index/stock exchange or single security before trading would ideally behave like a wave packet, constructed as superposition of its innumerable micro-states with different prices at different time. So, at what state the individual security is, once the trading starts, falls into the territory of quantum indeterminacy. However, purchase and sale happen on various tangible or intangible parameters directly or indirectly influencing the traders. These kinds of trading process can be coined as a physical measurement or probably an observation. Owing to this trading, the state of that micro-system (security or stock) will have a certain state function, i.e. a price. The probability density of the stock or index at time t is $I\psi(\gamma,\ t)I^2$.

Probabilistic return for a vector in Hilbert space (finite) is denoted as

$$P(t) = \int_a^b I\psi(\gamma,t)I^2 dp. \tag{1}$$

1.3. Quantum well

Quantum well at the core has been found to be quite similar to the stochastic finance. The entire work on the quantum well and stocks moving inside the well could result in predicting the symmetry, chaos, predictability, and explosion. The process starts with the boundary definition of the finite quantum well (Zhang & Huang, 2010):

$$d_o = \gamma_o \cdot 20\%, \tag{2}$$

where d_o is the width of the finite quantum well.

The width has been defined as a multiplication factor on previous closing price with the circuit filter gauge, applied in the market. If both the upper filter and the lower filter are 10%, then the filter gauge is 20%. Inside that defined space (i.e. Hilbert space), stocks and securities showcase their wave-particle dualism. γ_o is the previous day closing price of a stock or an index. $\gamma' = \gamma - \gamma_o$, where γ is the current price of the stock or index, so γ' is the gap between two closings. Also, γ' works like a Hermitian operator, to be more precise, as a price operator.

1.4. Schrödinger and Brownian applications

The glaring research warts of Black-Scholes option pricing model (Bouchaud, 1994) have been exposed by the econophysicists for many decades. Detailed work on European call options, based on MATIF, CAC40, and FTSE100 using Brownian stochastic process, ARCH, and Levy process, showed that residual risk is still there. So, risk corrected option prices came out of this extensive work. One outstanding study found importance of trading in determining the asset price individually and collectively. The researcher considered securities and cash held by investors as the wave functions to construct the finite Hilbert space (Schaden, 2002). Most of the studies for constructing the stochastic models for stock markets (for prediction purpose) overlooked one important aspect, ignoring the cause of market fluctuations. As the price of the traded security is newly negotiated every time it is traded, it generates the volatility on which the model is based. Even in sophisticated stochastic models, the coherent effects of trading are not incorporated; thus, generating an error while measuring equilibrium while predicting future price.

Nowadays, the scientists from the domain of physics are implementing ideas of universality, networks, game theory, and intra-agent interaction models to regular financial issues both from corporate finance and from investment finance. They are using these for financial crashes, the movement of stock exchanges and currency exchange rate fluctuations, share price evolution, etc.

2. METHODOLOGY OF THE RESEARCH

In this innovative piece of research, we have considered opening, closing, day's high, day's low, and volume of CNX Nifty from February 15, 2000 to December 7, 2015. The total number of dataset is 47,016. The entire work has been conducted in two parts. The first part consists of calculation of the relative volatility index (RVI) and ease of movement (EMV), thus confirming the Reynolds number. The second part consists of stationarity (the augmented Dickey-Fuller, ADF, test) and martingale (the variance ratio, VR, test) checks on the Reynolds number time series, in order to satisfy the probability of future predictions. Further, we want to predict it with the use of GARCH and feed forward, back propagating classical artificial neural network as well. This will show the results obtained from two different methodologies (one econometric and the other machine learning). The frequency, pattern, and upper values of the Reynolds number (in the first part of research) are analyzed, whereas the predictability aspect along with the final model to predict the Reynolds number in future is taken into account in the subsequent part (the second part of research). This piece of work is cobbled carefully, with econophysics and econometrics, nestled comfortably in the arms of behavioral finance.

2.1. Relative volatility index

An innovative study conducted in the Republic of Serbia showed that the application of the optimized moving averages convergence divergence (MACD) and the RVI indicators of technical analysis in decision making process on investing the financial market contributes truly in enhancing profit while investing in dynamic securities (Eric, Andjelic, & Redzepagic, 2009). The RVI came in the early 1990s (Dorsey, 1993) as an evolution from the relative strength index (RSI). The RVI was used to determine the direction of the stock price volatility. Instead of the absolute change in closing price in the RSI, the RVI was measured on the basis of the price deviation for those specified days (generally 10 days). During the course of his study, Dorsey found that the extensive use of technical oscillators as functions is insufficient. According to him, most of the oscillators that were used to find the movement of a stochastic time series are nothing but the "repacked" version of the existing oscillator. It has been found that since the rationale is the same behind these oscillators, the accuracy of prediction is not improved due to information duplication.

$$\acute{r} = 100\, \acute{\ddot{\upsilon}} / (\acute{\ddot{\upsilon}} + \delta), \tag{3}$$

where $RVI = \acute{r}$, $\acute{\ddot{\upsilon}}$ – Wilder's smoothing of USD, and δ – Wilder's smoothing of DSD.

Welles Wilder developed the Welles Wilder's smoothing average (WWS) that is a part of the RSI indicator usage.

USD = If close > close (1) then SD, S else 0; 10 day SD is in use.

DSD = If close < close (1) then SD, S else 0; 10 day SD is in use.

S = Specified period for the standard deviation of the close (as per Dorsey's suggestion, it should be 10 days).

N = Specified selected smoothing period (as per Dorsey's suggestion, it should be 14 days).

2.2. Ease of movement

Richard Arms, the famous technical analyst, won the prestigious Market Technicians Award way back in 1995. Arms (1996) is famous for the Arms Index and certain innovative indicators including EMV, equivolume, and volume cyclicality. EMV calculation and concept is derived from the concept of viscosity. The calculation is basically a ratio, with numerator being "distance" and denominator being "box ratio".

$$\text{Distance} = \varrho = \frac{\text{High} + \text{Low}}{2} - \\ - \frac{\text{Prior High} + \text{Prior Low}}{2}.$$

Volume and current high-low range form the box ratio, which is quite similar to equivolume charts.

$$\text{Box ratio} = \text{Ч} = \frac{\text{Volume}/100{,}000{,}000}{\text{High} - \text{Low}},$$

$$\text{EMV} = \frac{\text{Distance}}{\text{Box ratio}} = \\ = \frac{\frac{\text{High} + \text{Low}}{2} - \frac{\text{Prior High} + \text{Prior Low}}{2}}{\frac{\text{Volume}/100{,}000{,}000}{\text{High} - \text{Low}}} = \frac{\varrho}{\text{Ч}}. \tag{4}$$

If the high and the low are quite close to each other in absolute value, then box ratio will be higher. This means that the EMV may be lower. However, if the volume is very low (thinly traded stocks), then box ratio will be very low.

2.3. Osborne Reynolds number: from physics to finance

Fluid transmission from a laminar into a turbulent flow was a significant problem statement in fluid mechanics that was solved to a great extent in 1883 (Reynolds, 1901). Osborne Reynolds demonstrated that a delta or change of fluid flow can be represented by a unit less number (the Reynolds number). When an object passes through a fluid, in a specified outer boundary, it experiences forces against the flow called as drag (R and T). According to Reynolds, the pressure drag (R) and the viscosity drag (T) are represented as (Reynolds, 1883):

$$R = C\frac{\rho\vartheta^2}{2}S, \tag{5}$$

$$T = B\eta\vartheta l. \tag{6}$$

where B and C are dimensionless constants, ϑ is certain mean velocity of the fluid, ρ density of the fluid, S cross-sectional area of the object (perpendicular to the direction of the fluid flow), η fluid viscosity, and l linear dimension of the object.

$$\frac{R}{T} = \frac{C\rho\vartheta^2 S}{2B\eta\vartheta l}. \tag{7}$$

So, assuming $C = 2B$ and $S = 1^2$.

$$R_e = \frac{\rho\vartheta l}{\eta}. \tag{8}$$

So, the final version of the Reynolds number has momentum indicators as numerator and viscosity indicators as denominator. The RVI is similar to momentum of particles having wave-particle dualism inside a defined quantum well, placed inside a finite Hilbert space. The EMV is similar to viscosity of particles having wave-particle dualism inside a defined quantum well, placed inside a finite Hilbert space. So, interestingly, the RVI as numerator and the EMV as denominator redefine the Osborne Reynolds number for financial markets and stock exchanges.

Rotary trajectory in a defined Hilbert space is a hydrodynamics domain specific work. However, the similarity with stocks, displaying their wave-particle dualism in a finite Hilbert space (within circuit filters) encouraged researchers to develop an econophysical analogue of the stock market Reynolds number (Jakimowicz & Juzwiszyn, 2015). The turbulence factor in a financial market, its frequency of occurrence and its pattern, and, last but not least, its prediction capability was a missing link in the existing body of knowledge. The Reynolds number derived for stock markets can be used for symmetry prediction (having similar and low values in a narrow range for a long period), volatility prediction (having rapid changes in the number within a short spun of time), and to identify and quantify the future numbers, and thus developing a warning notice for the investors as well.

In this research, the Reynolds number has been represented as:

$$R_e = (100\,\text{ΰ} / (\text{ΰ} + \delta)) / (\varrho/\text{Ч}). \quad (9)$$

2.4. Checking for the presence of unit root in the Reynolds number in Nifty

Trend stationary will take the study one step closer to being predicted. If the null hypothesis stays firm, then the presence of unit root will confirm non-stationary. The equation of the augmented Dickey-Fuller test is below:

$$y(t) = \mu + \delta y(t-1) + \sum_{i=1}^{p} \varphi i \Delta y(t-i) + \varepsilon(t), \quad (10)$$

$y(t)$ is the time series of the Reynolds number, calculated on CNX Nifty, μ is the intercept, p is the maximum number of lags, $\Delta y(t-i)$ is the differentiated lag coefficient for i lags, and φ is the error term. Interestingly, it has been found on CNX Nifty that the rational bubble is absent (Ghosh, 2016), which means that the null hypothesis stays and prediction possibility becomes bleak. However, this study is indirectly related to the one referred above, since the underlying of Reynolds number is CNX Nifty on both the occasions. The finding seems logical, as rational behavior of all market participants (who receive information at the same time) in unidirectional manner can cause bubble (all are either buying together or selling together). Whereas Indian stock exchanges rarely observed rational behavior among market participants, the chances of finding rational bubble come down.

2.5. Checking for the presence of martingale in the Reynolds number in Nifty

According to Samuelson (1965), stock prices unlike other stochastic counterparts cannot be predicted and, thus, a positive martingale is present in the time series. However, he commented about the Reynolds number martingale test for the purpose of financial prediction. Since there has not been such study conducted so far, the testing of a martingale becomes critical. If the time series has been found with a martingale, then the conclusive prediction mechanism fails.

2.6. GARCH prediction

A couple of decades ago, Duan (1995) argued that the majority of the predictive models often used to model asset returns and volatility could well be termed as different variants of various GARCH models and their modified equations. Whether it is asset price or commodity price or for that matter any stochastic time series, GARCH has a clear advantage, as in-built volatility is embedded into that system (as a reflection of combined coefficients of short-term and long-term memory). That is why GARCH has been preferred over normal continuous time series models (Herwartz, 2017; Matías et al., 2010).

Currently in this study, h_t corresponds to the conditional time series of the Reynolds number; α_0 has been denoted as an intercept; p and q are the non-negative integers based on lags, ε is an error generated due to unnatural explosion and various factors in the said time series. The conditional variance model of GARCH (p, q) is mentioned below:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}. \quad (11)$$

In this research, $i = j = 1$, which means RN (1), is used to predict RN in real time. So, thus the equation changes to

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}. \quad (12)$$

2.7. Back propagating artificial neural network prediction

Generally feed-forward artificial neural network (ANN) with iteration loop of back propagating errors is a combination of interconnected perceptrons, which are processing units called neurons. Neurons are usually structured in many layers. The first layer is called input layer, where initial signal reaches control variables. The last layer is called the output layer, where the final decision is depicted, so generally it is a single neuron. The middle layer is called hidden layer, which sometimes is seen as a black-box. Neurons of each layer generate feature vector in all possible finite directions within the specified neural structure.

$$a_j^{(l)} = \sum_{i=1}^{n} w_{ij}^{(l)} x^i + b_j^{(l)}. \quad (13)$$

Let us assume that $x_1, ..., x_n$ are the various inputs of neuron j at hidden level l. This equation is a representation of activation of the neuron $a_j^{(l)}$ at level l. The activation $a_j^{(l)}$ is then transformed by the usage of a non-linear differentiable function to give the output of neuron j at level l.

Aided by the rational and common usage of neural network architecture used in social sciences and natural sciences, simple gradient scheme is used for repetitive updating of weight and bias parameters (Boyd & Vandenberghe, 2004).

$$\left[w^{(t+1)}, b^{(t+1)}\right] = \left[w^{(t)}, b^{(t)}\right] - \eta \nabla E\left(w^{(t)}, b^{(t)}\right), \quad (14)$$

where η is known as learning rate of this network and is greater than zero. Also, $w^{(t)}$ and $b^{(t)}$ represent the weight and bias vectors at repetition t. $E(.)$ is the local error function that has to be minimized by back propagation using numerous iterations. ∇E is the gradient of E.

Generally, it has been noted by several eminent researchers that if the learning rate is significantly smaller, then the gradient descent method will converge to a local minimum (Boyd & Vandenberghe, 2004). However, no literature or working paper has so far found a cardinal value or range of possible learning rate, so that the gradient descent converges to local minimum (Bishop, 2006).

Mathematically it can be mentioned that a classical ANN with a binary firing pattern could well be described as an artificially networked computing system. This system is comprised of a directed graph with an additional defined structure (McCulloch & Pitts, 1943).

Generally a binary alphabet $A_2 = \{0, 1\}$ is attached to each perceptron or neuron that classifies the neural activity. In this case, 0 corresponds to non-firing neural state and 1 corresponds to a firing neural state.

The firing patterns of an ANN can thus be expressed by the set of all binary strings of length *N*: $A_2^N = \{S_1, S_2, ..., S_N : S_k \in A_2, k = 1, 2, 3, ..., N\}$.

The strength and type of each neural link is expressed with a real valued weight as S, with a specified activation threshold. A transfer function in this case indicates the state transfer of the neurons. In this work, we have considered [2, 5, 1] feed forward and back propagating classical form of artificial neural network. The input perceptrons or neurons are the two lags of the Reynolds number time series (stochastic). The output is singular, depicting the predictive mode of Reynolds number.

3. STUDY OUTCOME AND DISCUSSION

This graph depicts Nifty level 7,000 as the local maxima and the Reynolds number is taking a sharp decline around 5,500 levels. The rationale behind this plot came from the observation of VIX, due to purchase or sale in Nifty, resulting variable being Reynolds number. Since CBOE VIX

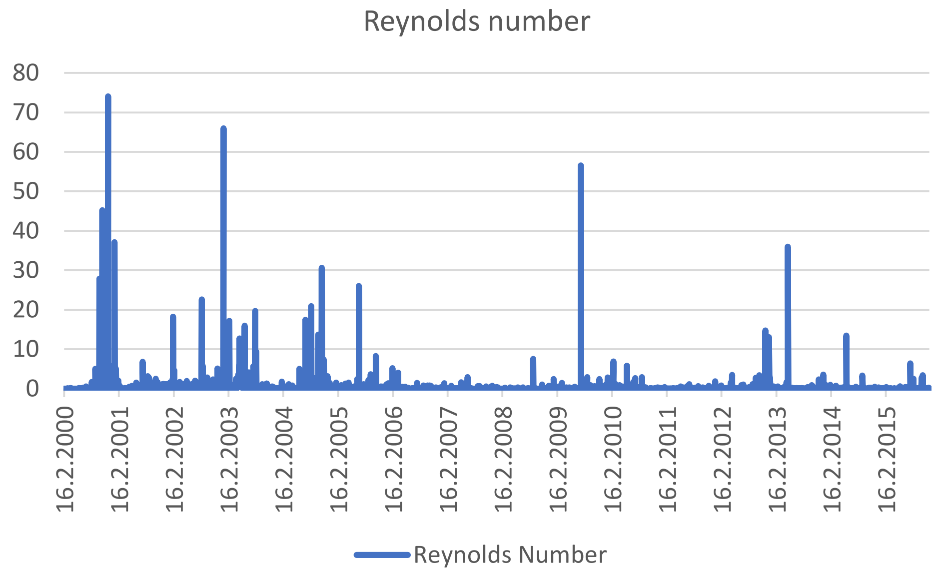


Figure 1. The Reynolds number over a period of 16 years, clearly coming down in amplitude and frequency

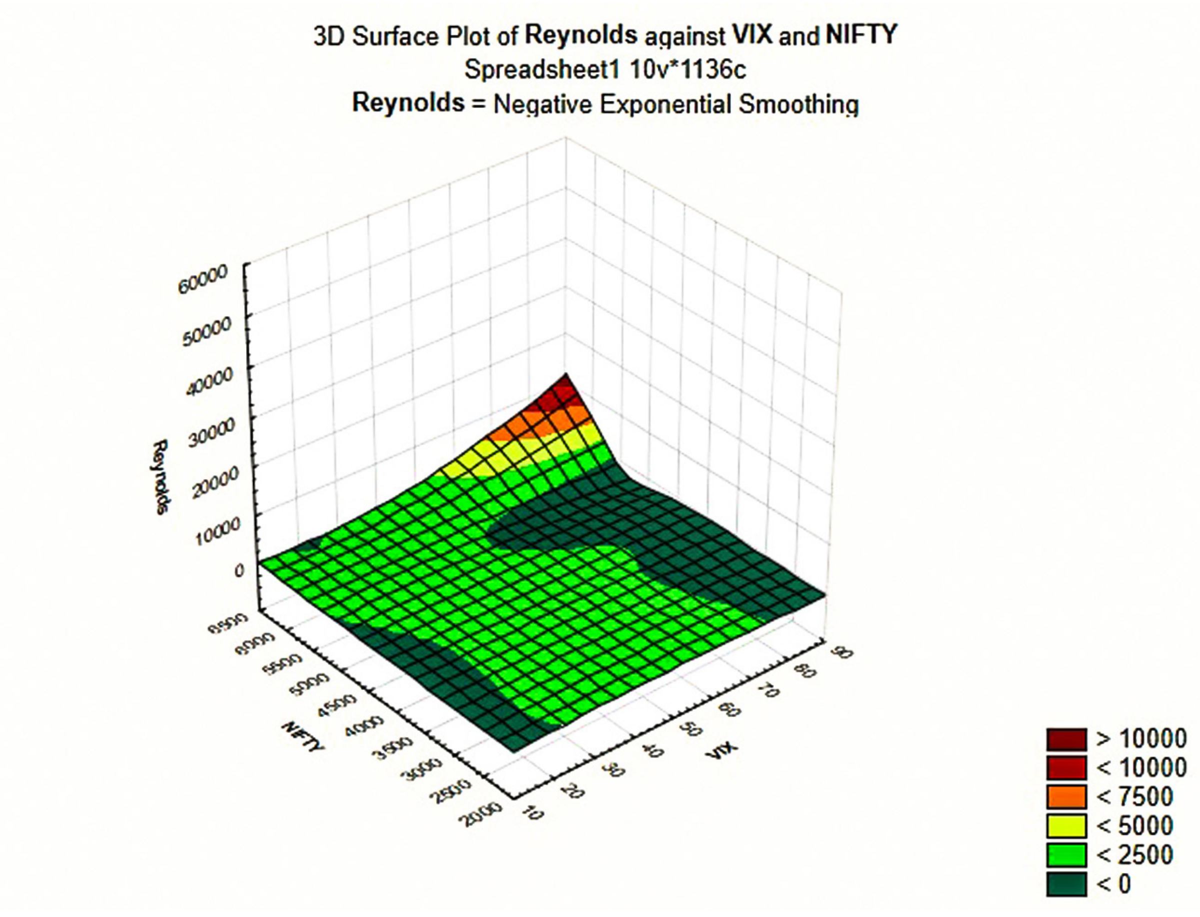


Figure 2. The Reynolds number is plotted with CNX Nifty and the volatility indicator CBOE VIX in the second zone (2007 to 2011); local maxima is observed at higher levels of CNX Nifty

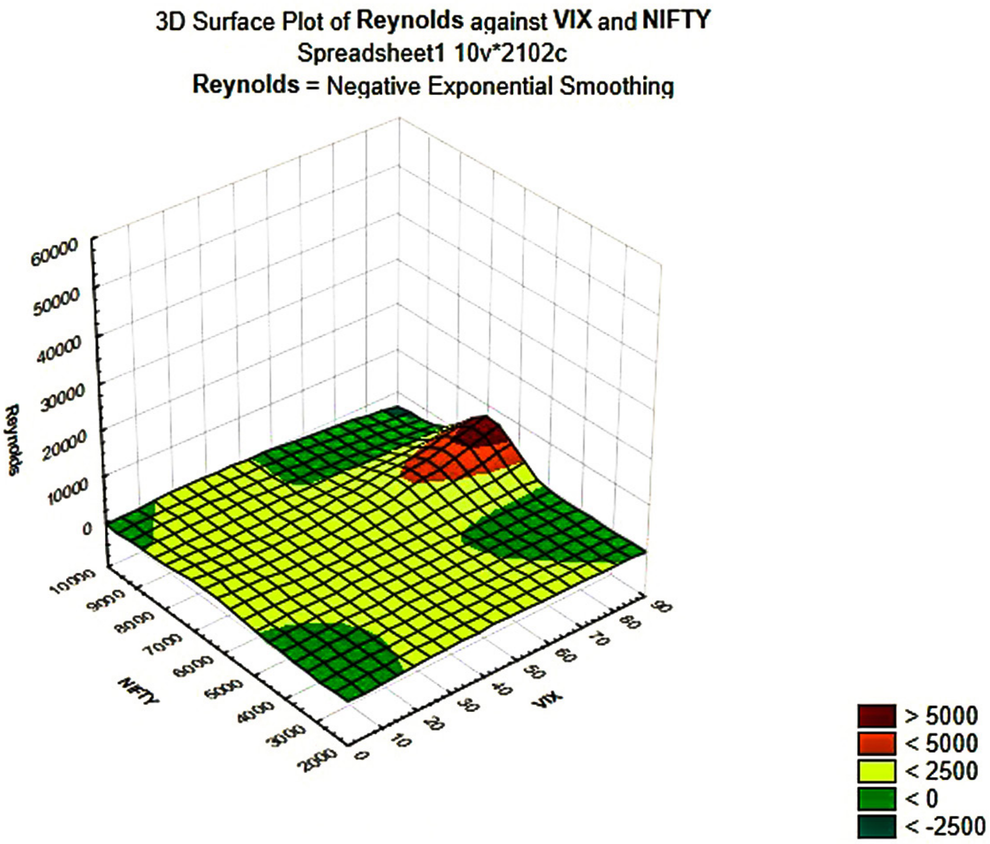


Figure 3. The Reynolds number is plotted with CNX Nifty and the volatility indicator CBOE VIX in the second and third zone combined (2007 to 2015); local minimum is observed at higher levels of CNX Nifty

came to existence for Indian stock exchanges only from mid-2007, similar study could not have been carried out for the first zone. The first zone falls in between 2000 and 2006. According to Figure 1, both the frequency and the amplitude of the higher Reynolds number were found to be substantially higher. It can be observed that even at all other levels, the Reynolds number is either zero or lower than zero. The basic theory of this depiction came from a fact that market participants observe CBOE VIX and accordingly buy and sell in CNX Nifty. Their actions get reflected in the Reynolds number and thus it varies. The following plot will capture a combination zone of 2007 to 2015.

This graph depicts that explosion has been spotted between Nifty levels of 7,000–8,000. However, it has been observed that although VIX level is almost constant across Nifty levels of 2,000 to 10,000 explosion in the Reynolds number does happen in a narrow band. Referring to equation 9, it can be concluded that either RVI is higher, or EMV is lower or both conditions are true at the same time in the narrow band of Nifty (from 7,000 to 8,000).

H0: The Reynolds number has a unit root.
Ha: The Reynolds number has trend stationary.
Lag length: 0 (Automatic – based on SIC, max lag = 29).

ADF test on the Reynolds number proves that it does not have a unit root. The presence of unit root would have been an indication that this stochastic series (the Reynolds number) does not have the stationary trend. However, test results prove the presence of the stationary trend, ensuring that it follows a definite pattern in a specified trend that is stationary in nature. AIC, SC, and HQ criteria depict that the volatility in this stochastic series (the Reynolds number) is small and moves in a narrow band as well. DW statistic of 2.01 ensures that the stochastic series has no-autocorrelation whatsoever.

Table 1. The augmented Dickey-Fuller test

Critical level, %	T-statistics	Probability	Occurrence, %
1	–3.96	0	100
5	–3.41	0	100
10	–3.12	0	100

Table 2. Robustness measures

Adjusted R2	0.49	AIC	4.69
Log Likelihood	–9091	SC	4.70
F-statistics	1876	HQ	4.69
Prob. (F)	0	DW	2.01

However, the ADF test with t-stat value of –3.41097 (at 5% critical level) falls true for almost 49% of the time during this predictive attempt.

Now, the next question that comes is whether this (the Reynolds number) can be predicted in future. So, the martingale theory has been used in practice.

H0: Cumulated RN is a martingale.
Ha: Cumulated RN is not a martingale.
Sample: 13,917.
Included observations: 3,917 (after adjustments).
Heteroscedasticity robust standard error estimates.
User-specified lags: 4 8 16.

P value being zero, the *H0* will be rejected. Martingale in a stochastic series means that past cannot determine future. No martingale means that past can determine future. Also, it can be noted here that the occurrence of this probability is substantially increasing in higher periods. This ensures that the prediction is both plausible and possible in the long run. Standard error in this process increases marginally though. Further GARCH modeling has been carried out for the prediction of the Reynolds number in future.

Dependent variable: RN.
Method: ML – ARCH (Marquardt) – Normal distribution.
Sample (adjusted): 2/16/2000; 12/01/2015.

$$\mathrm{GARCH} = \mathrm{C}(3) + \mathrm{C}(4) \cdot \mathrm{RESID}(-1)^2 + \\ + \mathrm{C}(5) \cdot \mathrm{GARCH}(-1) + \mathrm{C}(6) \cdot \mathrm{RN} + \mathrm{C}(7) \cdot \mathrm{RN}(1).$$

AIC, SC, and HQ criteria depict that the volatility in this stochastic series (the Reynolds number) is small and moves in a narrow band as well (1.85 approx.). DW statistic of 2.186 ensures that the stochastic series has almost no autocorrelation whatsoever. The GARCH test with *t*-stat value of 0.411631 (at 5% critical level) is found to be quite accurate indirectly.

$$\mathrm{RN} = 0.411354569952 + \\ + 0.115352618049 \cdot \mathrm{RN}(1). \tag{15}$$

Figure 4 ensures the accuracy as Maximum Absolute Error (MAE) and Root Mean Square Error (RMSE) are considerably on the lower side (0.53 and 2.538, respectively).

Table 3. Martingale test

Joint test	Value	Df	Probability	Occurrence
Max $\lvert z \rvert$ (at period 16)	5.27	3917	0	100%

Table 4. Individual tests

Period	Var. ratio	Std. error	z-statistic	Probability	Occurrence, %
4	1.069418	0.030242	2.295382	0.0217	97.83
8	1.150573	0.041957	3.588724	0.0003	99.97
16	1.303377	0.057526	5.273723	0	100.00

Table 5. Lag equation

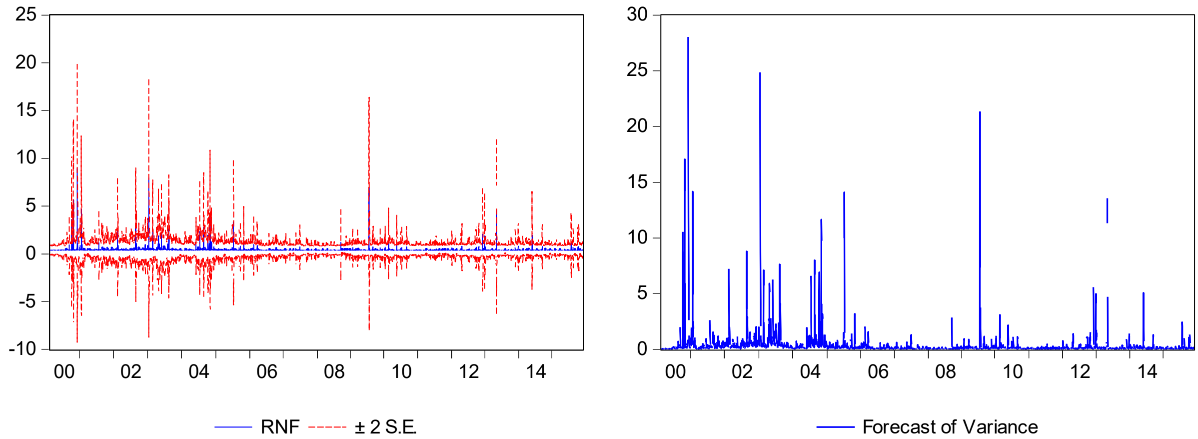
Variable	Coefficient	Std. error	z-statistic	Prob.
C	0.411355	0.007315	56.23779	0
RN(1)	0.115353	0.018736	6.156839	0

Table 6. Variance equation

Variable	Coefficient	Std. error	*z*-statistic	Prob.
C	0.027595	0.007889	3.497939	0.0005
RESID(–1)^2	–0.001564	0.000247	–6.321046	0
GARCH(–1)	0.411631	0.032424	12.69523	0
RN	0.15887	0.018468	8.602495	0
RN(1)	0.375003	0.036519	10.26873	0

S.E. of regression	2.539213	Mean dependent var	0.372921
Sum squared resid.	25235.92	Akaike info criterion	1.854792
Log likelihood	–3624.683	Schwarz criterion	1.866005
Durbin-Watson	2.18625	Hannan-Quinn criter.	1.858771

In fact, the Theil's inequality coefficient, which does not depend on the Lorenz curve and unlike Gini follows a different measurement standard, is found to be quite high (0.82). This means that there are more clusters and the explosive exponents of the Reynolds number in Nifty are well spaced out and identified in certain zones in this study.



Forecast: RNF
Actual: RN
Forecast sample: 2/16/2000 12/02/2015
Adjusted sample: 2/16/2000 12/01/2015
Included observations: 3916
Root Mean Squared Error: 2.538565

Mean Absolute Error: 0.583052
Mean Absolute Percent Error: 6671.399
Theil Inequality Coefficient: 0.820386
Bias Proportion: 0.0.001030
Variance Proportion: 0.775573
Covariance Proportion: 0.223397

Figure 4. GARCH forecasting of the Reynolds number in Nifty

Table 7. Neural network output

	Model Fit	Predictions
No of Obs	3116	779
Max Positive Error	0.7178	0.7189
Maximum Absolute Error	0.503	0.508
Root Mean Square Error	2.68	1.73

4. CLASSICAL NEURAL NETWORK OUTPUT

Forward feed and back propagation based ANN ensures the accuracy as Maximum Absolute Error (MAE) and Root Mean Square Error (RMSE) are considerably on the lower side. In fact, RMSE comes further down while predicting (from 2.68 to 1.73), where MAE is keeping a quasi-constant value.

$$\begin{aligned} RN = &-0.362436 + N2 \cdot 2.86683 - \\ &-N2^2 \cdot 1.88634. \end{aligned} \tag{16}$$

$$\begin{aligned} &N2 = -0.0275156 + RN(1), \\ &\text{cubert} \cdot 1.04399 - RN(1), \\ &\text{cubert}" \wedge 2 \cdot 0.23779. \end{aligned} \tag{17}$$

Table 8. Comparative intercepts of GARCH and neural network

Neural Intercept (Absolute)	GARCH Intercept (Absolute)
0.362436	0.411355

Both GARCH and neural intercepts are found to be very close to each other (0.36 and 0.411) on an absolute basis. Neural equation (16) does not have the lags of the Reynolds number. On the contrary, it features a neural variable N2, defined further by equation (17). This indicates the presence of behavioral factors.

CONCLUSION

The Reynolds number has shown explosive characteristics in three specified zones in this research. The first zone starts in 2000 and finishes in 2006. The first zone witnessed 18 occurrences of explosion (the Reynolds number > 10). The events that occurred in those specified days in the first zone were classified into three time zones again, namely January 2003, October 2004, and July 2005.

According to the exchange report (BSE, 2003), the screen based retail trading in G-Secs was initiated on January 16, 2003. The Securities and Exchange Board of India (SEBI) also specified completely new eligibility criteria for inclusion of stocks for introducing stock options and futures in nine additional scripts on January 31, 2003. So, these two events, most likely the first rather than the second, could trigger the explosive pattern in the Reynolds number. However, it has to be noted that the two events that are mentioned did happen in BSE and not in NSE. So, a clear coupling of the indices is visible in this case. In October 2004, the Reserve Bank of India (RBI) discarded both 7 day and 14 day repo rate (Pathak, 2011). Thus only overnight repo rate was available for the banks for their liquidity. Due to the torrential rain, NSE was functional partially on 26th and 27th July 2005 (Economic Times, 2005), yet due to the substantial FII flow, it touched an all-time peak that time. All these can be termed as structural changes, where the use of heuristics is spotted (Shiller, 2000). So, the rationality behind explosive behavior in the Reynolds number could be explained.

The second zone is a comparatively smaller zone from 2007 to 2011. The second zone witnessed one occurrence (the Reynolds number > 10). July 2009 witnessed Satyam scam (Economic Times, 2009) that was created by the fraudulent auditing practices allegedly done in cooperation with auditors and chartered accountants. Misrepresentation and suppression of facts happened to all its stakeholders, regulators, exchanges, and investors. Trading was stopped several times and merger happened immediately after this event but the FII flows were strong during this phase. During October 2010, USD 6.4 billion worth shares were bought by FIIs, which constituted almost 25% of the total calendar year inflow by FIIs in 2010. Unidirectional fund flow in Nifty was the cardinal reason for the Reynolds number to move up sharply. Representative bias (Ritter, 2003) or giving too much weight on recent information or experience have been spotted.

The last zone is even smaller with the year 2013 and some small periods in and around that year. The interesting point to be noted here is that both the cluster of explosive Reynolds numbers and their pick points are steadily decreasing. The third zone witnessed four occurrences (the Reynolds number > 10).

Ben Bernanke's (FED Chairman) (Bloomberg, 2013) declaration on May 22, 2013 investigated the concern on the probable early tapering of the quantitative easing program by the US FED. FIIs were spotted to exit riskier markets (on credit basis) and started to enter US bonds. Market touched selling freeze a couple of times in each trading session in Nifty, which is why the EMV was very low, thus the Reynolds number went up sharply. FIIs have been observed to follow the "timid choice" (Kahneman & Lovallo, 1993) or considering this case unique and not related to the global scenario. Again, they were found to have taken "bold forecasts", based on these as well, and thus moved the funds back to US bonds.

The first zone saw the Reynolds number going past 70 marks, when the second zone saw it going past 50 and the third zone saw it going past 30. The pattern is steadily decreasing and with lesser density of picks post 2006. The stochastic time series featuring the Reynolds number could well be predicted as it is both trend stationary and without the presence of martingales. Prediction is quite accurate using both GARCH and forward feed and back propagation based on ANN. The analogy of physics and fluid mechanics, when coupled with the technical oscillators such as RVI and EMV, does produce a number that could predict event based or information based explosions from a long drawn continuous stochastic process.

Implications for the investors and policy makers

Investors (especially the retail ones) lose most of their wallet share in the events of extremities (Black Swan events). They lack the wallet size to average out even when the prices hit the rock bottom. Financial Reynolds number will be a significant indicator for them to exit the market prior such a colossal catastrophe. Though the frequency of financial Reynolds number in the vicinity of 10, steadily decreased over the years, yet it would be prudent for the investors to stay away from the bourses during such a period. On the other hand, whenever it tends to zero or near zero, the volatility will be extremely low, hence it would provide a safe zone for trading. Policy makers too could use the financial Reynolds number as an indicator of market volatility and treat any value closer or above 10 to be significantly volatile zone, thus cautioning the investors well in advance.

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