“Optimizing the performance of mean-variance portfolios in various markets: an “old-school” approach”

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OPTIMIZING THE PERFORMANCE OF MEAN-VARIANCE PORTFOLIOS IN VARIOUS MARKETS: AN "OLD-SCHOOL" APPROACH

Abstract

The authors study the performance of mean-variance optimized (MVO) equity portfolios for retail investors in various markets in the U.S. and around the world. Actively managed equity mutual funds have relatively high fees and tend to underperform their benchmark. Index funds such as exchange traded funds still charge appreciable fees, and only deliver the performance of the benchmark. The authors find that MVO portfolios are relatively easy to manage by a retail investor, and that they tend to outperform their benchmark or, at worst, equal its performance, even after adjusting for risk. Moreover, they show that the performance of these funds is not particularly sensitive to the frequency at which they are rebalanced so that, in the limit, an investor might have to rebalance his/her portfolio only once a year. This last finding translates into very low trading costs, even for retail investors. Thus, the authors conclude that MVOs offer an easy, cheap alternative to invest in the world's equity markets.

Keywords investments, mean-variance optimization, international markets

JEL Classification G11, G15, G17, G23

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INTRODUCTION

Since its introduction in 1952 by Harry Markowitz, the mean-variance criteria have become the most widely known form of portfolio selection. Due to the simplicity of its underlying theory, as well as its ease of computation, it is taught in every business school, both at undergraduate, as well as graduate levels, and there is a large strand of the literature devoted to its analysis and improvement. However, modern investors rarely entrust their portfolio selection to this venerable methodology, preferring instead to invest either in actively managed portfolio or index funds. In order to demonstrate the value of mean-variance optimization (MVO), we apply this methodology in its simplest form in various markets around the world and find that, at worst, the resulting portfolios obtain the same level of performance as their respective index benchmark and, at best, beat these benchmarks with long-term results that are statistically, as well as economically significant[1].

Over the last few decades, there has been a strong and constant trend in the decline of the value of equities directly held by households, and the surge of household investments in mutual funds and other managed investment vehicles. As of year-end of 2013, there

1 Mean-variance optimization in this paper refers to the use of historical data to generate the stock's covariance matrix and maximize the Sharpe ratio. Other methodologies exist that use techniques such as factor models. These predictive techniques are beyond the scope of this paper.

are 8,974 open-ended mutual funds in the U.S. with combined assets of more than USD 15 trillion, which collectively own 29% of U.S. firms' equity, as well as 1,332 ETFs with assets of more than USD 1.6 trillion. Overall, the share of household financial assets held by investment companies (including mutual funds, ETFs, close-ended funds and UITs) has grown from 2% in 1980 to 22% in 2013[2], with similar trends observable in other countries, as well as in the market for pension funds.

With the average fee charged by U.S. actively managed mutual funds being close to 1% of assets under management, these funds collectively receive revenues of close to USD 43 billion from their investors. And yet, not only is there ample of evidence that active management underperforms its benchmark (Jensen, 1968; Malkiel, 1995; Fama & French, 2010, and others), the very mathematics of active portfolio management implies that, net of fees, these funds must inevitably trail their relevant passive benchmarks, at least on average. Although there is evidence that a small sample of fund managers may be able to outperform its benchmark (see, for example, Avramov & Wermers, 2006; Kosowski et al., 2006; Cuthbertson et al., 2010), the debate is still ongoing in the academic world, and every methodology that purports to identify these 'winners' is complicated and ultimately unreliable.

The case is similar in other countries where, again, actively managed mutual funds tend to underperform their respective benchmarks (see, for example, Białkowski & Otten, 2011; Chan & Yamada, 1997; Gallagher & Jarnecic, 2004; Aggrawal, 2007, and others), and investor fees can be even higher than those in the U.S. market.

Thus, an investor can choose between an expensive, underperforming actively managed fund and a somewhat cheaper fund (mutual fund, ETF or close-ended fund) that tracks the market index.

We explore a third possibility that a retail investor might be able to construct his/her own mean-variance portfolio using simple analytic tools and publicly available information, and maintain that portfolio by rebalancing at a frequency that maximizes risk-adjusted performance, while reducing trading costs. To test this premise, we obtain stock price data from 22 markets (3 U.S. indexes and 19 foreign ones), and conduct a back-test of MVO portfolio optimization over a period of 10 years. We find that, on average, this methodology is superior to indexing, which implies that it also outperforms the locally available actively managed funds. In fact, from 2005 to 2014, there is only one year in which our MVO portfolios trail the market index, and that is in 2008, during the worst period of the recent financial crisis. On average, the MVO portfolios outperform their benchmarks by 5.8% per year over the 10 year period, when rebalancing at monthly intervals. However, even if portfolio rebalancing is carried out once a year, the average outperformance is still 3%. If, in addition, we consider the difference in trading costs between a fund that is rebalanced annually versus an index fund that is rebalanced more frequently, then we can see that we are easily striking down the main arguments against retail investors managing their own funds: performance and costs. The final argument, risk, is also weakened by the fact that, with a few exceptions, our country index MVO portfolios tend to have standard deviation of returns, which are very close to that of their respective indexes.

We explore some potential drawbacks to the application of MVO optimization by retail investors. The main one being that in some cases the resulting portfolios can contain very few assets, which runs counter to the goal of proper diversification. While this does not necessarily imply that these portfolios are far riskier than their benchmarks, we add analyses of portfolios generated adding constraints to the MVO problem. Although performance is somewhat reduced by these constrained solutions, our results remain qualitatively unchanged.

2 "2014 Investment Company Fact Book", Investment Company Institute.

Finally, we perform two additional robustness tests. First, we add the risk-free rate to perform mean-variance optimization in terms of excess returns, as is more commonly applied. This reduces the number of countries in the sample due to lack of an appropriate benchmark sovereign bond to use as a proxy for this rate. While performance decreases marginally, the main conclusions remain unchanged. We also compare the performance of MVO portfolios to that of equal-weighted (EW) portfolios of stocks in each country. DeMiguel et al. (2009) show that a simple portfolio in which all assets have the same 1/N portfolio weight can be more efficient than an MVO portfolio in terms of various metrics. While we do not refute their claims, we do show that MVO portfolio performance is on average higher than that of EW portfolios. Moreover, since even MVO portfolios restricted to a minimum number of assets will always contain fewer stocks than an EW portfolio, trading costs should favor the MVO portfolio.

While we do not claim that simple MVO portfolios are a 'silver bullet' and the best solution available to retail investors, we do show that their performance (in terms of the Sharpe ratio) is higher and costs lower than other equally accessible alternatives, such as indexing and EW portfolio, tested in most markets.

The rest of this paper is organized as follows. Section 1 presents the MVO methodology and the dataset used. Section 2 shows our main results on the performance of MVOs. Section 3 presents results for special cases, and last section concludes.

1. METHODOLOGY AND DATA

The mean-variance optimization (MVO) methodology is based on the maximization of the Sharpe ratio, which is the expected return of the portfolio divided by its standard deviation. In other words, MVO attempts to obtain the highest possible return at the lowest possible risk. There are a number of variations on this methodology that claim to achieve higher levels of risk-adjusted performance, such as the use of Value-at-Risk (VaR) and other indicators of potential loss as optimization restrictions (CVaR, Drawdown, etc.), as well as maximizing risk-reward ratios other than Sharpe, such as the Sortino and Omega ratios (see, for example, Rockafellar & Uryasev, 2000; Chekhlov, Uryasev, & Zabarankin, 2004; Konno & Yamazaki, 1991, and others). We pursue the standard Markowitz approach to mean-variance optimization due to its simplicity and therefore its potential appeal to a large number of investors. A similar methodological approach, together with its theoretical support, can be found in Contreras-Pacheco, Stein, and Vecino (2015).

Let w be a vector of portfolio weights, r a vector of expected asset returns, and Ω an estimator of the variance-covariance matrix of these assets, then a mean-variance optimized portfolio is the one formed by solving for w so that:

$$max \frac{w \cdot r}{w \cdot \Omega \cdot w}, \quad (1)$$

$$\text{s.t.} \sum w_i = 1; w \geq 0. \quad (2)$$

Given this formulation, we are not allowing short sales (portfolio weights must all be positive or zero). This assumption is consistent with retail investor habits. It should also be noted that we are using gross returns and not excess returns, as it is more common. Since this precludes the requirement of data on a risk-free asset for each country, it allows the sample to be larger, as it is not always possible to have these data for all countries studied for the full time series needed. Further tests make use of the standard Sharpe ratio, as explained in section 3.

In order to estimate expected returns and the covariance matrix, we use historical stock returns data. Specifically, we use 5 years of monthly returns.

Finally, we test four portfolio-rebalancing frequencies. We rebalance portfolios at annual, semiannual, quarterly and monthly intervals.

Table 1. Descriptive statistics of country indexes

Country index	Num stocks	Liquidity	Mean price	Index vol 1	Index vol 5
SP100	101	97.79	92.5	2.19	3.65
NASDAQ100	107	98.37	102.76	2.92	4.3
DOW JONES	30	100	88.39	2.44	3.46
BRAZIL	68	98.43	20.81	6.58	5.37
CHILE	93	98.92	2685.95	3.45	3.8
CHINA	50	100	39.55	4.21	5.26
PHILIPPINES	30	99.92	294.16	2.28	4.58
GREECE	60	99.79	6.7	7.77	10.3
INDIA	50	98	907.62	3.58	4.97
INDONESIA	508	73.66	2101.95	1.75	4.46
ISRAEL	102	96.37	9889.17	2.22	3.63
JAPAN	225	99.89	2119.03	4.03	5.29
MALAYSIA	30	99.92	11.01	1.67	2.61
MEXICO	35	100	86.89	3.2	3.3
PERU	25	92.16	6.62	4.43	6.75
POLAND	80	96.45	28.77	3.77	5.12
PORTUGAL	18	94.44	3.7	6.02	5.48
SINGAPORE	30	99.95	9.57	2.36	3.8
SOUTH AFRICA	42	97.62	20830.14	2.31	3.8
SRI LANKA	287	97.57	247.05	3.22	5.91
THAILAND	50	97.33	86.04	3.2	5.06
TURKEY	100	99.14	20.06	6.13	6.81

Notes: Descriptive statistics are presented for each country/index, which provide the sample of stocks eligible to be included in the mean-variance optimized portfolios. The data shown include the number of stocks listed in the index as of the end of 2014 ('Num stocks'), a measure of market liquidity ('Liquidity') obtained as the average of all days in the last year of the sample in which each index stock traded (i.e. there is price information), the mean price of stocks in the last day of 2014 in each local currency, and the volatility of the index expressed as the standard deviation of monthly returns using the past year ('Index vol 1') and the past 5 years ('Index vol 5') of data.

For each rebalancing frequency and period, we use 5 years of past stock data and eliminate stocks from the respective index that do not have a full set of monthly returns for that period. We then obtain the MVO portfolio weights, and proceed to simulate holding that portfolio until the next rebalancing date, when we repeat the process[3].

For each index from which stocks are sampled to be included in the optimized portfolio, this methodology generates 10 portfolios when we apply annual rebalancing, 20 for semiannual, and so on, so that our back test always spans the same 10-year period.

We obtain daily stock and index price from Bloomberg. Data are obtained for 32 countries. However, for some countries, such as Ukraine, Colombia and Egypt, the dataset does not contain the minimum amount of data (15 years) to be included in this study. For others, like Panama, though the time series extends far enough, their markets contain too few stocks that trade actively, which invalidates the methodology and precludes any meaningful computation. Thus, we are left with 22 countries/indexes from which we compute optimized portfolios.

Table 1 shows descriptive statistics of these markets. The first 3 in the table are U.S. indexes, while the remaining 19 are foreign indexes. We tabulate the number of stocks listed in each index at the end of 2014, a measure of liquidity of each index obtained as the percentage of days in which all listed stocks show prices (i.e. trade) during the last year of our sample, the mean price of all listed stocks in the local currency, and a measure of past

3 We have not contemplated a gap in current tests. Time to rebalance a portfolio is a tradeoff between desired speed and execution costs. These costs will increase with the size of each trade, number of trades, and relative liquidity of each traded stock. In our case, portfolios are relatively small (i.e. few trades required to rebalance), and we there is a bias towards more liquid stocks, as these are the components, for example, of the S&P 100, the NASDAQ 100, and other indexes that take include the largest firms in each category. Thus, while a regular mutual fund could rebalance its portfolio in 1 to 2 months, it is conceivable that the portfolios depicted here could be changed faster than that.

index volatility calculated as the standard deviation of index return over the past year and the past 5 years.

By virtue of being included in a representative index, these stocks should have high levels of liquidity, relative to other stocks in the same market not included in the index (when the index does not include all stocks listed in a market). This is true in almost all cases, where our liquidity measure is above 90%, and most of the time close to 100%. The notable exception is Indonesia, with a liquidity measure of only 73.66%. At the same time, we later show that Indonesia is one of the MVO portfolios, which most improve performance when the rebalancing frequency is increased. Thus, we could argue that this performance improvement can be dubious, as trading costs might rise more than in other markets.

One-year volatility varies between indexes from a minimum of 1.75% for Indonesia to a maximum of 7.77% for Greece. However, all one-year volatilities show a reduction when compared to the five-year volatility measure. This is logical, as the five-year measure includes data starting in 2010, when the effects of the recent financial crisis were even more pronouncedly felt than they are now.

Ultimately, we find no relationship between MVO portfolio performance and any of the market variables presented above.

2. PERFORMANCE OF MVO PORTFOLIOS

We employ the methodology and data described in the previous chapter to generate MVO portfolios from the stocks listed in each index in Table 1, and then held until the next rebalancing period. In this chapter, we present our main results as regards to the performance of these portfolios.

Table 2 contains our main performance measures. For each MVO portfolio we compute the annualized mean monthly return, standard deviation and Sharpe ratio, and we compare them with the same statistics for their respective index. Panel A of the table shows the results obtained when rebalancing portfolios at annual frequency, while panels B, C and D contain the performance statistics for portfolios rebalanced at semiannual, quarterly and monthly frequencies, respectively.

The first important observation is that most MVO portfolios obtain a higher level of return than their respective index, and in many cases do so without noticeable increases in their volatility. In fact, some MVO portfolios obtain Sharpe ratios that are close to double that of their index, as is the case for Dow Jones, S&P 100, and China. Two remarkable examples are Portugal and, given the ongoing situation in that country, Greece. These portfolios obtain a positive return, while their respective benchmarks average negative returns, and also positive Sharpe ratios. Additionally, even in cases when the portfolios trail the index in terms of performance, for the most part that difference is small. Exceptions include the portfolios formed with stocks traded in Chile, Poland and Turkey, which invariably trails their benchmark by a considerable distance. As we see later, this underperformance does generate negative and statistically significant alphas for annually and semiannually rebalanced portfolios, but the statistical significance of this measure evaporates at more frequent rebalancing intervals. Moreover, we can also ascertain that the outperformance of the MVO portfolios is very stable. As we can see in the time series plots in Figure 1 (see Appendix), the equal-weighted mean[4] annual MVO excess return is positive in almost every year, with the exception of 2008, the worst year of the recent financial crisis[5].

Second, since increasing the rebalancing frequency means that the information used to generate the portfolios is updated more frequently, we would expect that the performance of high rebalancing frequency portfolios would be higher than that of those rebalanced at lower frequencies. However, there does not seem to be any discernible pattern in the relationship between a portfolio rebalancing frequency and its risk or return. This is important, since, while we do not perform a strict analysis of trading costs in this study, lower rebalanc-

4 We purposefully avoid a value-weighted mean, as it would be dominated by the U.S. indexes.

5 A small dip is also observed for 2014, but we find that this purported underperformance is not statistically significant.

Table 2. Performance of mean-variance optimized portfolios

Country	Portfolio			Index		
	Ret	SD	Sharpe	Ret	SD	Sharpe
Panel A. Annual rebalancing						
DOW JONES	21.88	27.65	0.79	5.14	13.84	0.37
NASDAQ 100	18.84	33.89	0.56	10.04	18.19	0.55
SP 100	13.86	26.52	0.52	4.66	14.30	0.33
BRAZIL	7.91	30.19	0.26	6.66	22.88	0.29
CHILE	–0.13	16.94	–0.01	7.70	13.79	0.56
CHINA	10.67	30.24	0.35	5.18	22.08	0.23
PHILIPPINES	27.86	30.55	0.91	14.68	19.76	0.74
GREECE	3.80	31.43	0.12	–11.50	32.52	–0.35
INDIA	25.46	28.87	0.88	14.72	25.11	0.59
INDONESIA	10.82	37.46	0.29	17.85	22.91	0.78
ISRAEL	8.10	39.55	0.20	7.30	17.66	0.41
JAPAN	3.70	36.49	0.10	4.26	20.53	0.21
MALAYSIA	12.18	21.72	0.56	6.84	12.88	0.53
MEXICO	2.48	31.92	0.08	12.75	17.62	0.72
PERU	5.19	31.47	0.16	14.74	33.18	0.44
POLAND	–11.30	30.02	–0.38	9.80	25.06	0.39
PORTUGAL	1.55	28.88	0.05	–4.50	19.83	–0.23
SINGAPORE	4.56	35.39	0.13	5.33	19.04	0.28
SOUTH AFRICA	23.92	20.12	1.19	14.36	16.98	0.85
SRI LANKA	22.00	27.81	0.79	16.97	23.54	0.72
THAILAND	17.56	30.77	0.57	7.86	23.51	0.33
TURKEY	1.22	34.22	0.04	13.06	28.83	0.45
Panel B. Semiannual rebalancing						
DOW JONES	18.69	28.70	0.65	5.14	13.84	0.37
NASDAQ 100	17.58	38.54	0.46	10.04	18.19	0.55
SP 100	11.53	24.64	0.47	4.66	14.30	0.33
BRAZIL	10.86	32.83	0.33	6.66	22.88	0.29
CHILE	2.68	15.24	0.18	7.70	13.79	0.56
CHINA	11.97	31.67	0.38	5.18	22.08	0.23
PHILIPPINES	27.57	31.75	0.87	14.68	19.76	0.74
GREECE	–0.09	28.70	–0.00	–11.50	32.52	–0.35
INDIA	24.69	27.17	0.91	14.72	25.11	0.59
INDONESIA	7.25	39.53	0.18	17.85	22.91	0.78
ISRAEL	10.87	40.86	0.27	7.30	17.66	0.41
JAPAN	7.68	37.77	0.20	4.26	20.53	0.21
MALAYSIA	13.65	20.78	0.66	6.84	12.88	0.53
MEXICO	8.99	32.62	0.28	12.75	17.62	0.72
PERU	4.20	29.64	0.14	14.74	33.18	0.44
POLAND	–2.63	29.42	–0.09	9.80	25.06	0.39
PORTUGAL	3.10	28.77	0.11	–4.50	19.83	–0.23
SINGAPORE	7.63	34.57	0.22	5.33	19.04	0.28
SOUTH AFRICA	22.01	20.15	1.09	14.36	16.98	0.85
SRI LANKA	15.07	35.80	0.42	16.97	23.54	0.72
THAILAND	10.63	31.39	0.34	7.86	23.51	0.33
TURKEY	–4.15	34.02	–0.12	13.06	28.83	0.45

Table 2 (cont.). Performance of mean-variance optimized portfolios

Country	Portfolio			Index		
	Ret	SD	Sharpe	Ret	SD	Sharpe
Panel C. Quarterly rebalancing						
DOW JONES	23.85	28.45	0.84	5.14	13.84	0.37
NASDAQ 100	18.55	37.32	0.50	10.04	18.19	0.55
SP 100	13.98	23.87	0.59	4.66	14.30	0.33
BRAZIL	9.11	32.23	0.28	6.66	22.88	0.29
CHILE	–1.41	15.08	–0.09	7.70	13.79	0.56
CHINA	13.66	30.66	0.45	5.18	22.08	0.23
PHILIPPINES	28.21	32.23	0.88	14.68	19.76	0.74
GREECE	1.71	30.83	0.06	–11.50	32.52	–0.35
INDIA	23.42	26.03	0.90	14.72	25.11	0.59
INDONESIA	14.66	43.74	0.34	17.85	22.91	0.78
ISRAEL	14.80	38.58	0.38	7.30	17.66	0.41
JAPAN	8.06	34.57	0.23	4.26	20.53	0.21
MALAYSIA	14.68	19.31	0.76	6.84	12.88	0.53
MEXICO	11.29	33.30	0.34	12.75	17.62	0.72
PERU	8.77	30.84	0.28	14.74	33.18	0.44
POLAND	–1.19	30.95	–0.04	9.80	25.06	0.39
PORTUGAL	5.70	27.56	0.21	–4.50	19.83	–0.23
SINGAPORE	6.55	32.53	0.20	5.33	19.04	0.28
SOUTH AFRICA	27.18	20.87	1.30	14.36	16.98	0.85
SRI LANKA	13.36	34.39	0.39	16.97	23.54	0.72
THAILAND	11.57	31.06	0.37	7.86	23.51	0.33
TURKEY	2.91	33.66	0.09	13.06	28.83	0.45
Panel D. Monthly rebalancing						
DOW JONES	23.89	28.83	0.83	5.14	13.84	0.37
NASDAQ100	23.63	37.92	0.62	10.04	18.19	0.55
SP100	14.09	23.38	0.60	4.66	14.30	0.33
BRAZIL	13.02	32.58	0.40	6.66	22.88	0.29
CHILE	0.27	16.38	0.02	7.70	13.79	0.56
CHINA	16.99	30.27	0.56	5.18	22.08	0.23
PHILIPPINES	28.90	32.12	0.90	14.68	19.76	0.74
GREECE	6.17	30.34	0.20	–11.50	32.52	–0.35
INDIA	25.11	25.22	1.00	14.72	25.11	0.59
INDONESIA	20.49	42.23	0.49	17.85	22.91	0.78
ISRAEL	15.12	38.80	0.39	7.30	17.66	0.41
JAPAN	9.46	33.69	0.28	4.26	20.53	0.21
MALAYSIA	16.32	18.96	0.86	6.84	12.88	0.53
MEXICO	12.18	30.60	0.40	12.75	17.62	0.72
PERU	14.38	30.20	0.48	14.74	33.18	0.44
POLAND	3.76	31.38	0.12	9.80	25.06	0.39
PORTUGAL	5.82	27.65	0.21	–4.50	19.83	–0.23
SINGAPORE	8.51	30.74	0.28	5.33	19.04	0.28
SOUTH AFRICA	25.33	20.35	1.24	14.36	16.98	0.85
SRI LANKA	10.71	33.75	0.32	16.97	23.54	0.72
THAILAND	12.43	31.52	0.39	7.86	23.51	0.33
TURKEY	5.90	34.08	0.17	13.06	28.83	0.45

Notes: Portfolios of stocks listed in each index/country are rebalanced at different frequencies: annual, semiannual, quarterly and monthly. Each time a portfolio is rebalanced, portfolio weights are determined by mean-variance optimization using the previous 5 years of monthly stock returns. Independent of rebalancing frequency, the full period time series of monthly returns, 2005 to 2014, are obtained for each portfolio and then used to calculate portfolio performance measures. The measures displayed on this table include the annualized mean monthly return ('Ret'), the annualized standard deviation of monthly returns ('SD'), and the portfolio's Sharpe ratio ('Sharpe'). Both Ret and SD are expressed as percentages. For comparison, the same measures are calculated for each index.

Table 3. Statistical tests of performance of mean-variance optimized portfolios

Country	Alpha		T-test	
	Estimate	T-stat	Estimate	T-stat
Panel A. Annual rebalancing				
DOW JONES	1.22*	1.95	1.24**	2.01
NASDAQ 100	0.74	0.93	0.65	0.82
SP 100	0.65	1.17	0.71	1.28
BRAZIL	0.1	0.19	0.1	0.19
CHILE	–0.54*	–1.67	–0.63*	–1.95
CHINA	0.44	0.78	0.43	0.75
PHILIPPINES	0.83	1.41	0.92	1.59
GREECE	0.95	1.50	1.32*	1.89
INDIA	0.96*	1.78	0.76	1.39
INDONESIA	–0.82	–1.22	–0.52	–0.77
ISRAEL	–0.23	–0.29	0.06	0.08
JAPAN	–0.19	–0.31	–0.05	–0.07
MALAYSIA	0.45	0.93	0.41	0.86
MEXICO	–1.07*	–1.74	–0.8	–1.30
PERU	–0.4	–0.73	–0.73	–1.22
POLAND	–1.67***	–3.03	–1.78***	–3.21
PORTUGAL	0.49	0.83	0.51	0.87
SINGAPORE	–0.11	–0.15	–0.06	–0.08
SOUTH AFRICA	1.26**	2.52	0.68	1.27
SRI LANKA	0.55	1.06	0.36	0.69
THAILAND	0.66	1.49	0.73	1.62
TURKEY	–0.82	–1.38	–0.93	–1.56
Panel B. Semiannual rebalancing				
DOW JONES	1.01	1.52	1.02	1.55
NASDAQ 100	0.66	0.70	0.56	0.60
SP 100	0.52	0.99	0.53	1.03
BRAZIL	0.3	0.50	0.32	0.54
CHILE	–0.21	–0.67	–0.4	–1.21
CHINA	0.54	0.87	0.52	0.84
PHILIPPINES	0.78	1.25	0.9	1.48
GREECE	0.57	0.98	1.01	1.47
INDIA	0.90*	1.93	0.7	1.49
INDONESIA	–1.18	–1.65	–0.79	–1.10
ISRAEL	0.01	0.01	0.28	0.32
JAPAN	0.12	0.19	0.27	0.40
MALAYSIA	0.52	1.19	0.52	1.21
MEXICO	–0.57	–0.91	–0.28	–0.45
PERU	–0.41	–0.76	–0.81	–1.32
POLAND	–0.90*	–1.69	–1.00*	–1.88
PORTUGAL	0.62	1.08	0.64	1.12
SINGAPORE	0.11	0.15	0.18	0.26
SOUTH AFRICA	1.01**	2.12	0.55	1.10
SRI LANKA	0.03	0.04	–0.14	–0.18
THAILAND	0.16	0.32	0.21	0.44
TURKEY	–1.16*	–1.71	–1.38**	–2.01

Table 3 (cont.). Statistical tests of performance of mean-variance optimized portfolios

Country	Alpha		T-test	
	Estimate	T-stat	Estimate	T-stat
Panel C. Quarterly rebalancing				
DOW JONES	1.37**	2.09	1.38**	2.12
NASDAQ 100	0.79	0.86	0.63	0.69
SP 100	0.73	1.41	0.72	1.39
BRAZIL	0.2	0.33	0.19	0.31
CHILE	–0.46	–1.32	–0.74*	–1.94
CHINA	0.69	1.12	0.65	1.06
PHILIPPINES	0.8	1.27	0.94	1.53
GREECE	0.74	1.14	1.15	1.58
INDIA	0.82*	1.94	0.62	1.43
INDONESIA	–0.95	–1.33	–0.23	–0.30
ISRAEL	0.35	0.44	0.57	0.70
JAPAN	0.18	0.33	0.3	0.51
MALAYSIA	0.65	1.57	0.6	1.46
MEXICO	–0.43	–0.67	–0.11	–0.17
PERU	–0.02	–0.03	–0.45	–0.66
POLAND	–0.82	–1.51	–0.88	–1.63
PORTUGAL	0.81	1.44	0.85	1.51
SINGAPORE	0.1	0.15	0.1	0.14
SOUTH AFRICA	1.45***	2.81	0.9	1.65
SRI LANKA	0.03	0.04	–0.26	–0.34
THAILAND	0.26	0.51	0.28	0.57
TURKEY	–0.52	–0.75	–0.79	–1.11
Panel D. Monthly rebalancing				
DOW JONES	1.37**	2.05	1.38**	2.10
NASDAQ 100	1.13	1.21	0.98	1.06
SP 100	0.74	1.47	0.72	1.45
BRAZIL	0.5	0.79	0.49	0.77
CHILE	–0.35	–0.93	–0.6	–1.50
CHINA	0.94	1.54	0.89	1.47
PHILIPPINES	0.9	1.39	0.99	1.56
GREECE	1.08*	1.70	1.51**	2.08
INDIA	0.98**	2.32	0.73*	1.66
INDONESIA	–0.39	–0.54	0.19	0.25
ISRAEL	0.39	0.48	0.59	0.72
JAPAN	0.29	0.56	0.41	0.74
MALAYSIA	0.79*	1.92	0.71*	1.76
MEXICO	–0.26	–0.44	–0.04	–0.07
PERU	0.42	0.71	–0.03	–0.04
POLAND	–0.41	–0.72	–0.47	–0.84
PORTUGAL	0.81	1.43	0.86	1.51
SINGAPORE	0.27	0.41	0.25	0.38
SOUTH AFRICA	1.34***	2.67	0.77	1.44
SRI LANKA	–0.19	–0.25	–0.46	–0.62
THAILAND	0.32	0.61	0.35	0.66
TURKEY	–0.23	–0.32	–0.55	–0.72

Notes: Monthly returns are obtained for all mean-variance optimized portfolios formed from stocks in each index/country for the decade spanning 2005–2014. These returns are then used to estimate to the statistical significance of the portfolio's performance. The first test is the estimation of a market model alpha, where the benchmark market returns used are those of each index. The second is a t-test of the difference between the monthly returns of the mean-variance portfolios and those of their respective benchmark index. For each test we display the point estimate, as well as the corresponding t-statistic. Significance is denoted by ***, ** and * for the 1%, 5% and 10% levels, respectively.

ing frequency mechanically translates into lower execution costs. Thus, if the MVO portfolios can maintain their performance at relatively low rebalancing frequencies, then low trading costs can be added to their virtues.

Finally, the Efficient Market Hypothesis (EMH) would dictate that, if there is indeed an excess return to be had from these MVO portfolios, then investors should invest using this technique until market prices return to equilibrium and it is no longer possible to obtain an excess risk-adjusted return. Following the tenets of the EMH, we would expect to see a negative relationship between the level of competitiveness of the market (as proxied by size) and the outperformance of MVOs. Since the U.S. is the largest and most competitive of the markets in our sample, we would also assume that the MVOs based on the U.S. indexes should be the ones with the lowest performance. However, the MVOs based on all three U.S. indexes fare amongst the best performing of the sample.

In Table 3, we summarize the results of statistical tests performed on the portfolios' returns data. Specifically, we obtain a market model alpha by regressing the returns of the MVO on those of its benchmark index, and we also employ a *t*-test of the difference between the monthly returns of the MVO and those of the benchmark. The table includes both, the point estimate of each test, as well as its respective *t*-statistic.

As we can see in Panels A to D, positive alphas tend to remain significant independently of the rebalancing frequency of their respective portfolios. On the other hand, negative alphas cease to be significant at higher rebalancing frequencies. On average, for monthly-rebalanced portfolios, positive and significant alphas are around 1.2% per month, over a period of 10 years, which means these results are also economically significant.

In general, we see that MVOs tend to outperform their benchmark, while for the most part adding only moderate levels of risk. Moreover, save for a few markets, their performance does not seem much affected by the investor's choice of rebalancing frequency, which allows a would-be MVO investor to drastically reduce the trading costs associated with managing a portfolio by himself/herself. Finally, within certain bounds on minimums, these results do not appear to be sensitive to market variables such as size and liquidity, although the latter should be studied further. As long as the market is large enough and there is enough liquidity for the MVO algorithm to work at all, its results deliver a much needed option to investors looking for an easy and low-cost way to access equity markets.

3. PORTFOLIO STRUCTURE AND FURTHER TESTS

Detractors of the MVO methodology of asset management point to the fact that the resulting portfolios lack diversification.

In Tables 4 and 5, we study the structure of the MVO portfolios in our sample. Table 4 shows the average number of stocks in each portfolio. With few exceptions, most portfolios contain between 3 and 4 stocks, while some average as low as 2. It is interesting to see that the market or index from which these stocks are picked does not seem to influence the size of the resulting portfolio, as we see that, for example, the large U.S. indexes produce portfolios with roughly the same number of stocks than those generated from far smaller market indexes. We also note that neither does the frequency of portfolio rebalancing seem to be related to the size of the resulting portfolio. Thus, we conclude that this is a characteristic inherited from the MVO methodology itself.

Even with a relatively small number of assets, we might still be able to claim a certain acceptable level of diversification if investment capital was spread somewhat evenly amongst them. Table 5 shows the average portfolio weights in our sample. While the overall mean size of portfolio weights (column labeled 'All') is acceptable, ranging from 17% to 52%, the story is quite different if we analyze the highest and lowest allocation in each portfolio separately ('Max' and 'Min', respectively). Save for a few exceptions, the maximum allocation exceeds 70% of the capital invested (with 100% observed in various cases, indicating portfolios with a single asset), whereas the minimum does not exceed around 2% on average, and is often less than 1%.

Table 4. Number of assets in portfolios

Country	All	Annual	Semiannual	Quarterly	Monthly
DOW JONES	2.00	1.90	1.90	2.10	2.10
NASDAQ 100	4.00	4.10	4.00	3.90	3.90
SP 100	3.10	3.30	3.00	3.10	3.20
BRAZIL	3.10	3.50	3.00	3.00	3.00
CHILE	5.00	5.70	4.80	4.80	4.90
CHINA	3.40	3.40	3.50	3.40	3.40
PHILIPPINES	3.20	3.30	3.20	3.20	3.00
GREECE	2.30	2.40	2.20	2.30	2.30
INDIA	3.20	3.00	3.40	3.30	3.30
INDONESIA	4.80	4.40	5.00	4.90	4.80
ISRAEL	3.80	3.70	3.80	3.80	3.90
JAPAN	3.70	4.00	3.50	3.50	3.70
MALAYSIA	2.20	2.20	2.20	2.20	2.10
MEXICO	3.50	3.70	3.50	3.50	3.40
PERU	3.00	2.80	3.30	3.10	3.00
POLAND	3.90	4.00	4.00	3.80	3.90
PORTUGAL	2.40	2.70	2.20	2.20	2.30
SINGAPORE	4.30	4.60	4.30	4.20	4.30
SOUTH AFRICA	2.50	2.70	2.60	2.40	2.40
SRI LANKA	7.50	7.40	7.50	7.50	7.60
THAILAND	3.90	3.80	4.20	3.80	3.90
TURKEY	5.00	4.90	4.80	5.00	5.10

Notes: The number individual of stocks contained in each mean-variance optimized portfolio is averaged over the period of the analysis. Overall means are presented ('All'), as well as means for each portfolio rebalancing frequency ('Annual', 'Semiannual', 'Quarterly' and 'Monthly').

Table 5. Portfolio weights of mean-variance portfolios

Country	Annual			Semiannual			Quarterly			Monthly		
	All	Max	Min	All	Max	Min	All	Max	Min	All	Max	Min
DOW JONES	52.63	100.00	8.20	52.63	100.00	3.56	48.19	100.00	1.03	47.62	100.00	0.48
NASDAQ 100	24.39	74.12	0.58	25.32	100.00	0.20	25.64	100.00	0.17	25.42	100.00	0.12
SP 100	30.30	94.49	1.08	33.33	92.55	0.89	32.00	93.72	0.36	31.33	100.00	0.13
BRAZIL	28.57	100.00	1.44	33.33	100.00	0.40	33.06	100.00	0.40	32.97	100.00	0.12
CHILE	17.54	69.64	0.17	20.83	83.98	0.20	20.94	83.98	0.20	20.30	86.88	0.12
CHINA	29.41	95.19	0.93	28.57	95.90	0.32	29.20	100.00	0.32	29.41	100.00	0.32
PHILIPPINES	30.30	85.13	0.34	30.77	100.00	0.92	31.50	100.00	0.68	32.79	100.00	0.16
GREECE	41.67	100.00	0.31	44.44	100.00	0.38	43.01	100.00	0.38	42.70	100.00	0.28
INDIA	33.33	94.20	0.72	29.85	100.00	0.68	30.53	100.00	0.68	29.93	100.00	0.17
INDONESIA	22.73	55.02	0.48	20.20	68.71	0.50	20.30	68.71	0.50	20.94	68.74	0.22
ISRAEL	27.03	88.37	0.31	26.32	97.12	1.32	26.32	97.12	0.29	25.59	97.12	0.15
JAPAN	25.00	86.41	0.67	28.57	100.00	0.73	28.37	100.00	0.29	26.79	100.00	0.10
MALAYSIA	45.45	100.00	2.47	45.45	100.00	0.48	45.98	100.00	0.33	47.24	100.00	0.21
MEXICO	27.03	83.33	0.45	28.57	95.32	0.84	28.37	96.45	0.82	29.06	100.00	0.33
PERU	35.71	88.57	1.99	30.30	91.08	0.35	31.75	100.00	0.23	32.79	100.00	0.13
POLAND	25.00	85.80	1.36	25.32	85.67	2.12	26.14	93.27	1.01	25.75	93.27	0.12
PORTUGAL	37.04	100.00	0.20	44.44	100.00	0.51	44.44	100.00	0.51	43.01	100.00	0.26
SINGAPORE	21.74	89.89	0.50	23.26	88.21	0.29	23.67	88.21	0.29	23.12	100.00	0.14
SOUTH AFRICA	37.04	100.00	0.91	38.46	100.00	0.25	41.67	100.00	0.25	40.96	100.00	0.14
SRI LANKA	13.51	72.40	0.26	13.42	95.49	0.14	13.42	95.49	0.12	13.17	95.49	0.10
THAILAND	26.32	62.59	4.09	24.10	98.00	0.20	25.97	98.00	0.20	25.70	100.00	0.15
TURKEY	20.41	65.49	0.44	20.83	75.41	0.52	19.80	75.41	0.22	19.70	75.41	0.16

Notes: Portfolio weights are averaged across time for each country and each portfolio rebalancing frequency. Means are shown for the weights of all assets in each portfolio ('All'), as well as for the assets with the highest ('Max') and lowest ('Min') weights in each portfolio.

Table 6. Performance of restricted mean-variance optimized portfolios

Panel A. Performance statistics

Country	Portfolio			Index		
	Ret	SD	Sharpe	Ret	SD	Sharpe
DOW JONES	11.95	14.57	0.82	5.14	13.84	0.37
NASDAQ 100	21.96	25.89	0.85	10.04	18.19	0.55
SP 100	17.29	17.07	1.01	4.66	14.30	0.33
BRAZIL	11.71	24.17	0.48	6.66	22.88	0.29
CHILE	1.64	15.47	0.11	7.70	13.79	0.56
CHINA	15.71	24.61	0.64	5.18	22.08	0.23
PHILIPPINES	20.47	24.23	0.84	14.68	19.76	0.74
GREECE	–0.93	28.64	–0.03	–11.50	32.52	–0.35
INDIA	22.46	24.31	0.92	14.72	25.11	0.59
INDONESIA	20.77	33.92	0.61	17.85	22.91	0.78
ISRAEL	11.98	27.59	0.43	7.30	17.66	0.41
JAPAN	3.25	27.34	0.12	4.26	20.53	0.21
MALAYSIA	11.88	14.84	0.80	6.84	12.88	0.53
MEXICO	19.80	20.75	0.95	12.75	17.62	0.72
PERU	14.31	27.17	0.53	14.74	33.18	0.44
POLAND	–3.99	28.80	–0.14	9.80	25.06	0.39
PORTUGAL	2.60	19.82	0.13	–4.50	19.83	–0.23
SINGAPORE	7.79	23.63	0.33	5.33	19.04	0.28
SOUTH AFRICA	22.36	17.24	1.30	14.36	16.98	0.85
SRI LANKA	7.37	30.56	0.24	16.97	23.54	0.72
THAILAND	18.05	28.47	0.63	7.86	23.51	0.33
TURKEY	2.67	32.11	0.08	13.06	28.83	0.45

Panel B. Statistical tests

Country	Alpha		T-test	
	Estimate	T-stat	Estimate	T-stat
DOW JONES	0.6**	2.48	0.53**	2.12
NASDAQ 100	0.88*	1.76	0.87*	1.77
SP 100	1.02***	3.14	0.96***	2.92
BRAZIL	0.46	1.24	0.39	1.02
CHILE	–0.32	–1.03	–0.48	–1.51
CHINA	0.86**	2.07	0.8*	1.90
PHILIPPINES	0.38	1.08	0.42	1.19
GREECE	0.59	1.15	0.94	1.60
INDIA	0.71**	2.40	0.55*	1.81
INDONESIA	–0.12	–0.23	0.21	0.40
ISRAEL	0.21	0.48	0.36	0.80
JAPAN	–0.14	–0.39	–0.08	–0.23
MALAYSIA	0.43*	1.80	0.39	1.62
MEXICO	0.56*	1.70	0.51	1.60
PERU	0.32	0.84	–0.03	–0.07
POLAND	–1.04**	–2.18	–1.12**	–2.34
PORTUGAL	0.56**	2.32	0.6**	2.44
SINGAPORE	0.19	0.52	0.19	0.52
SOUTH AFRICA	0.81***	2.75	0.57*	1.90
SRI LANKA	–0.51	–0.81	–0.72	–1.16
THAILAND	0.73*	1.93	0.76**	2.02
TURKEY	–0.61	–1.03	–0.81	–1.34

Notes: Portfolios of stocks listed in each index/country are rebalanced every month. Each time a portfolio is rebalanced, portfolio weights are determined by mean-variance optimization using the previous 5 years of monthly stock returns. Portfolio weights are restricted in the optimization model to a maximum value of 20%. The full period time series of monthly returns is obtained for each portfolio and then used to calculate portfolio performance measures and perform statistical tests of this performance. Panel A of this table shows the annualized mean monthly return ('Ret'), the annualized standard deviation of monthly returns ('SD'), and the portfolio's Sharpe ratio ('Sharpe'). Both Ret and SD are expressed as percentages. For comparison, the same measures are calculated for each index. Panel B contains the market-model alpha, where the benchmark market returns used are those of each index, as well as a t-test of the difference between the monthly returns of the mean-variance portfolios and those of their respective benchmark index. For each test we display the point estimate, as well as the corresponding t-statistic. Significance is denoted by ***, ** and * for the 1%, 5% and 10% levels, respectively.

Taken together, these results confirm the claims that, in general, MVO portfolios may achieve high levels of risk-adjusted returns, but do so at the cost of almost insignificantly diversifying the investor's capital, and thus expose her to unnecessary levels of risk.

While various solutions have been proposed to this well-known issue with the MVO methodology (see, for example, Green & Hollifield, 1992), we test the simplest one, which is to impose a single restriction on the maximum acceptable size for portfolio weights. Specifically, we impose a 20% maximum weight restriction[6]. This means that, mechanically, no portfolio can have less than 5 stocks and, even then, capital should be more evenly distributed. The main concern in adding this restriction is that any outperformance the MVO portfolios may have had when computed without the weight restriction might evaporate.

Table 6 summarizes the performance of our restricted MVO portfolios. For brevity, we have included only the data for MVOs rebalanced at monthly intervals, Panel A reports the same data as in Table 2, while Panel B reports the same statistics as are depicted in Table 3.

Comparing the performance of the restricted MVOs to that of the unrestricted sample, we see that, indeed, performance has somewhat decreased. For some, as is the case of the portfolio based on Dow Jones stocks, the resulting return is dramatically lowered. However, that reduction in return is accompanied by an equally dramatic reduction of risk, which causes the Sharpe ratio for this portfolio to remain virtually unchanged. More generally, the number of MVOs that outperform their benchmark in terms of returns remains virtually unchanged, while the risk-adjusted measures give a clear advantage of MVOs over index investing, with ten positive and significant alpha measures, compared to only one negative and significant.

As mentioned in the introduction, the standard formulation of the MVO methodology is based in maximizing a Sharpe ratio based on returns in excess of the relevant risk-free rate of return. For our next test we decide to standardize the rate used in each country portfolio as that of the monthly yield on a 10-year plain sovereign bond. While these data are readily available in developed markets, it is difficult to obtain for smaller markets. Some countries issue these securities infrequently, and so full time series' of returns are not available. Others have started issuing bonds with this description in recent years, so that the full time series for our sample is not available. These issues cause a reduction in our sample to 13 countries[7].

Table 7 shows the performance of MVO portfolios based on excess returns. For brevity, only results for monthly-rebalanced portfolio are shown. As we can see, while generally returns, standard deviations and Sharpe ratios do not change much, there is a noticeable change in the estimations of portfolio alpha. This is due to the use of excess returns in the regression models, which generally lowers performance measures. However, *t*-tests of the difference of returns remain positive, showing the outperformance of MVO portfolios with respect to their benchmarks. Moreover, we once again test the effect of restricting portfolio weights and generating MVO portfolios using excess returns. As can be observed in Table 8, once again the restricted portfolios show a somewhat reduced level of return, but disproportionally lower levels of risk, which in turn translate into more country portfolios significantly beating their respective benchmarks.

Finally, DeMiguel et al. (2009) show that, potentially, a 'naïve', equally-weighted (EW) portfolio can be more efficient than a traditional MVO portfolio. We test the performance of EW portfolios using our sample of country stocks, making sure that the stocks included in each EW portfolio were also eligible to be added to an MVO portfolio. In Table 9, we compare the average monthly returns of each type of portfolio for each market.

6 We opt for that level for the restriction in order to ensure that our sample remains intact. Imposing stronger restrictions, in the form of lower maximum weights, would inevitably reduce the size of our sample of MVOs as, for some countries with small markets, the algorithm fails to find a suitable solution.

7 In some 'borderline' cases, where time series' of 10-year bond data are lacking a few observations, we use interpolation when possible, or we supplement the yields with data from other securities issued by the same government, such as shorter-term bonds (and making reasonable adjustments so that these come close to representing hypothetical rates of a longer-term security).

Table 7. Performance of mean-variance optimized portfolios, excess returns

Panel A. Performance statistics

Country	Portfolio			Index		
	Ret	SD	Sharpe	Ret	SD	Sharpe
DOW JONES	23.89	28.83	0.83	5.14	13.84	0.37
NASDAQ 100	23.54	37.91	0.62	10.04	18.19	0.55
SP 100	14.11	23.37	0.60	4.66	14.30	0.33
BRAZIL	13.09	32.57	0.40	6.66	22.88	0.29
CHILE	0.16	16.23	0.01	7.70	13.79	0.56
CHINA	16.98	30.27	0.56	5.18	22.08	0.23
PHILIPPINES	28.91	32.15	0.90	14.68	19.76	0.74
GREECE	6.16	30.34	0.20	–11.50	32.52	–0.35
INDIA	25.11	25.23	1.00	14.72	25.11	0.59
ISRAEL	15.07	38.79	0.39	7.30	17.66	0.41
MEXICO	12.13	30.59	0.40	12.75	17.62	0.72
POLAND	3.73	31.36	0.12	9.80	25.06	0.39
PORTUGAL	5.83	27.66	0.21	–4.50	19.83	–0.23
SINGAPORE	8.50	30.74	0.28	5.33	19.04	0.28
SOUTH AFRICA	25.34	20.35	1.25	14.36	16.98	0.85

Panel B. Statistical tests

Country	Alpha		T-test	
	Estimate	T-stat	Estimate	T-stat
DOW JONES	1.09	1.64	1.38**	2.09
NASDAQ 100	0.84	0.90	0.98	1.06
SP 100	0.46	0.92	0.73	1.45
BRAZIL	–0.49	–0.77	0.49	0.78
CHILE	–0.85**	–2.30	–0.61	–1.55
CHINA	0.84	1.38	0.89	1.47
PHILIPPINES	0.31	0.47	0.99	1.56
GREECE	0.36	0.57	1.51**	2.08
INDIA	0.33	0.77	0.73*	1.66
ISRAEL	–0.03	–0.03	0.59	0.71
MEXICO	–0.87	–1.48	–0.05	–0.08
POLAND	–0.85	–1.49	–0.48	–0.84
PORTUGAL	0.34	0.60	0.86	1.51
SINGAPORE	0.06	0.09	0.25	0.38
SOUTH AFRICA	0.65	1.29	0.78	1.44

Notes: Portfolios of stocks listed in each index/country are rebalanced every month. Each time a portfolio is rebalanced, portfolio weights are determined by mean-variance optimization using the previous 5 years of monthly stock returns in excess of the risk-free rate of return. Portfolio weights are restricted in the optimization model to a maximum value of 20%. The full period time series of monthly returns is obtained for each portfolio and then used to calculate portfolio performance measures and perform statistical tests of this performance. Panel A of this table shows the annualized mean monthly return ('Ret'), the annualized standard deviation of monthly returns ('SD'), and the portfolio's Sharpe ratio ('Sharpe'). Both Ret and SD are expressed as percentages. For comparison, the same measures are calculated for each index. Panel B contains the market-model alpha, where the benchmark excess market returns used are those of each index, as well as a t-test of the difference between the monthly excess returns of the mean-variance portfolios and those of their respective benchmark index. For each test we display the point estimate, as well as the corresponding t-statistic. Significance is denoted by ***, ** and * for the 1%, 5% and 10% levels, respectively.

Table 8. Performance of restricted mean-variance optimized portfolios, excess returns

Panel A. Performance statistics

Country	Portfolio			Index		
	Ret	SD	Sharpe	Ret	SD	Sharpe
DOW JONES	11.94	14.57	0.82	5.14	13.84	0.37
NASDAQ 100	21.95	25.89	0.85	10.04	18.19	0.55
SP 100	17.28	17.08	1.01	4.66	14.30	0.33
BRAZIL	11.67	24.16	0.48	6.66	22.88	0.29
CHILE	1.55	15.44	0.10	7.70	13.79	0.56
CHINA	15.70	24.61	0.64	5.18	22.08	0.23
PHILIPPINES	20.45	24.24	0.84	14.68	19.76	0.74
GREECE	–1.00	28.63	–0.03	–11.50	32.52	–0.35
INDIA	22.46	24.30	0.92	14.72	25.11	0.59
ISRAEL	11.95	27.59	0.43	7.30	17.66	0.41
MEXICO	19.79	20.75	0.95	12.75	17.62	0.72
POLAND	–4.00	28.80	–0.14	9.80	25.06	0.39
PORTUGAL	2.60	19.82	0.13	–4.50	19.83	–0.23
SINGAPORE	7.80	23.63	0.33	5.33	19.04	0.28
SOUTH AFRICA	22.36	17.23	1.30	14.36	16.98	0.85

Panel B. Statistical tests

Country	Alpha		T-test	
	Estimate	T-stat	Estimate	T-stat
DOW JONES	0.32	1.33	0.53**	2.12
NASDAQ 100	0.6	1.20	0.87*	1.77
SP 100	0.74**	2.29	0.96***	2.92
BRAZIL	–0.53	–1.41	0.39	1.01
CHILE	–0.82***	–2.65	–0.49	–1.54
CHINA	0.76*	1.83	0.8*	1.90
PHILIPPINES	–0.21	–0.59	0.41	1.19
GREECE	–0.14	–0.27	0.93	1.59
INDIA	0.05	0.18	0.55*	1.81
ISRAEL	–0.21	–0.48	0.36	0.80
MEXICO	–0.05	–0.16	0.51	1.60
POLAND	–1.48***	–3.08	–1.12**	–2.34
PORTUGAL	0.08	0.35	0.6**	2.44
SINGAPORE	–0.01	–0.03	0.19	0.53
SOUTH AFRICA	0.12	0.39	0.57*	1.90

Notes: Portfolios of stocks listed in each index/country are rebalanced every month. Each time a portfolio is rebalanced, portfolio weights are determined by mean-variance optimization using the previous 5 years of monthly stock returns in excess of the risk-free rate of return. The full period time series of monthly returns is obtained for each portfolio and then used to calculate portfolio performance measures and perform statistical tests of this performance. Panel A of this table shows the annualized mean monthly return ('Ret'), the annualized standard deviation of monthly returns ('SD'), and the portfolio's Sharpe ratio ('Sharpe'). Both Ret and SD are expressed as percentages. For comparison, the same measures are calculated for each index. Panel B contains the market-model alpha, where the benchmark excess market returns used are those of each index, as well as a t-test of the difference between the excess monthly returns of the mean-variance portfolios and those of their respective benchmark index. For each test we display the point estimate, as well as the corresponding t-statistic. Significance is denoted by ***, ** and * for the 1%, 5% and 10% levels, respectively.

Table 9. Comparison with performance of EW portfolios

Country	MVO	EW	MVO-EW
DOWJONES	0.95	0.61	0.34
NASDAQ 100	1.67	1.13	0.54
SP 100	1.34	0.56	0.78**
BRAZIL	0.93	0.45	0.47
CHILE	0.14	0.39	–0.25
CHINA	1.22	0.74	0.48
PHILIPINES	1.56	1.49	0.07
GREECE	–0.08	–0.7	0.63
INDIA	1.7	1.39	0.32
INDONESIA	1.58	1.14	0.45
ISRAEL	0.95	0.61	0.34
JAPAN	0.27	0.2	0.07
MALAYSIA	0.94	0.83	0.11
MEXICO	1.52	1.23	0.28
PERU	1.12	0.9	0.22
POLAND	–0.34	0.15	–0.49
PORTUGAL	0.21	–0.12	0.33
SINGAPORE	0.63	0.76	–0.13
SOUTH AFRICA	1.7	1.16	0.54**
SRI LANKA	0.59	1.28	–0.69
THAILAND	1.39	1.05	0.35
TURKEY	0.22	1.04	–0.82*
FULL SAMPLE	0.92	0.74	0.18*

Notes: Equal-weighted portfolios are formed with each country's eligible stocks and rebalanced each month. The mean monthly returns of these portfolios ('EW') are compared to those of portfolios obtained through restricted mean-variance optimization ('MVO'). The last column on the table shows the difference between the two, expressed as return of MVO minus returns of EW portfolios ('MVO-EW'). Significance is denoted by ***, ** and * for the 1%, 5% and 10% levels, respectively.

As we can see in Table 9, most MVO portfolios outperform their EW counterpart. On average, for the full sample, the MVOs outperformance is statistically significant, and equivalent to an annualized return of around 2.16% above that of EW portfolios.

While we do not consider this to be absolute proof that MVOs are superior to EW portfolios in all aspects, beyond their higher return it should be noted that, even for restricted MVOs, the number of individual stocks in each portfolio is still low, particularly when compared to a portfolio invested equally in all available securities. This translates into a lower trading cost advantage for MVOs. However, this aspect is not specifically addressed in either this document, nor in DeMiguel et al. (2009).

CONCLUSION

We set out to test the viability of the venerable mean-variance portfolio methodology introduced in Markowitz (1952) as a tool that modern retail investors could use to improve the performance of their investments, over and above that offered by the average actively managed or index equity fund. To do so, we test the performance of portfolios formed by allocating capital to the stocks listed in 22 indexes of 19 different countries and, while varying the frequency at which these portfolios are rebalanced, simulating an investment made continuously over 10 years.

Since the average equity mutual fund tends to underperform its benchmark index, and since our mean-variance optimized portfolios tend to outperform the same index, we conclude that, in terms of risk-adjusted performance, MVO portfolios offer a better alternative to both types of funds currently available in the markets.

We tackle the main concern with MVO portfolios, their low levels of diversification, by adding a simple restriction on the size of resulting portfolio weights to the optimization problem. The results show an adequate increase in the level of diversification of the portfolios, without a noticeable change in our previous conclusions regarding portfolio performance.

Additionally, we test the performance of portfolios formed on an excess-return-based MVO methodology, and compare the performance of MVOs to EW portfolios. We find that the outperformance of MVOs is generally robust to these tests.

Taken together, we believe there is sufficient evidence to support the use of mean-variance optimization as a valid, value-adding tool for retail investors, that is, those investors who are unable to do exhaustive security and market analysis because they are not professionally trained and/or lack the access to some of the proprietary data and models used by professional money managers.

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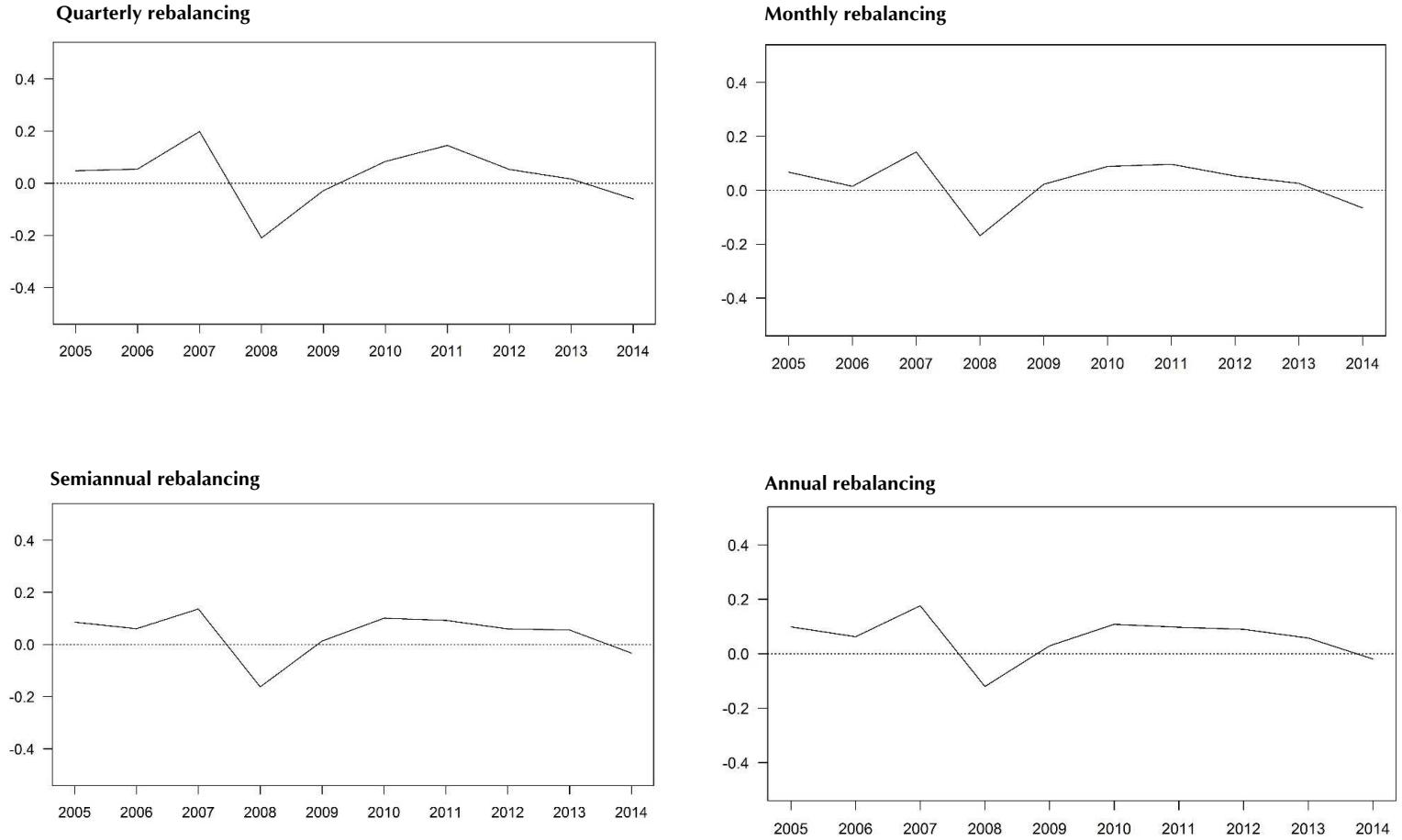
APPENDIX



Notes: MVO portfolios' yearly excess returns are calculated as the return of the MVO portfolio minus that of its benchmark index, and then averaged. The time series of these excess returns is plotted in each pane, for different MVO rebalancing frequency.

Figure 1. Excess return of mean-variance optimized portfolios