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Profit participation annuities: a business profitability analysis within a demographic risk sensitive approach

Abstract

The aim of the paper is to analyze the performance of a portfolio of participating life annuities, focusing on the minimum acceptable income level throughout the quantiles of the return distribution. The model, in addition to the necessary consideration of the volatility of financial markets, gives a central role to the impact of the longevity phenomenon. The sensitivity of the portfolio performance to the survival projection, the presence of a break-even point and the time of optimum performance are pointed out, under different hypotheses for the participating quota and with stochastic assumption for the accumulation and the discounting financial processes and for the survival description.

Keywords: participating policies, pension annuities, stochastic interest rates, operating income, longevity, stochastic mortality rates.

JEL Classification: C53, G17, G22, G32.

Introduction

The debate following the guidelines issued by the competent European committees in the insurance sector is still very active, even in anticipation of the new stress test for the sector announced by the European Insurance and Occupational Pensions Authority (EIOPA). The initiative hinges within a general project aimed at preserving stability and transparency of the financial system as a whole, as well as protection of policyholders and pension beneficiaries. The question of solvency and economic performance in the insurance industry is focused on the consideration of a wide range of risk drivers of insurance nature and not (market risks, operational risks, etc.). This situation has made increasingly complex the disposals about solvency, on the one hand, and, secondly, the management policies in terms of achieving the desired performance. The correct reference to risk sources, both in the context of supervision and proactive management as far as the insurance companies are concerned, has a primary role. By virtue of the aforementioned questions, refining the analysis of each risk component is quite crucial, however, that is in light of the new exposure draft that heralds the new IFRS 9 (whose effectiveness has been set on January 1, 2013), which makes the capital ratios related to all business risks. Henceforward the operational problem originates with the estimated measure of regulatory capital apt to the required cover, the reserves amount and – last but not least – the proper management policy, designed to optimize the performance (cf. AAVV, 2009; and Grossi, 2010). The main risk drivers, which the insurer has to front, are the financial risk and the demographic

risk. In this study we will mainly focus on the systematic effect of the uncertainty of the evolution in time of the survival trend, because of the importance of this phenomenon in the case of life/pension annuities. An interdisciplinary research sector during the last decade has given a wide space to the study of the increasing improvement of the survival trend, modeling its evolution on the basis of several main aspects, as for instance age, gender, health and social characteristics, geographical classification (cf. Denuit et al., 2009). The overall improvement of the survival trend, particularly remarkable at adult ages, fosters a climate of uncertainty, especially for life contracts characterized by long duration and strongly exposed to the riskiness of a dangerous underestimation of the future cash flows. The correct evaluation of this risk, together with the evaluation of the financial risk sources, represents the core of each internal model (cf. Coppola et al., 2007; Hari et al., 2008; and D'Amato et al., 2011). The demographic risk originates by the accidental deviations of the real mortality rates from their forecasted values as well as by the systematic deviations of the same rates; the first cause, just because of its accidental connotation, is mitigated by the pooling effect in the case of large portfolio, whilst the second one has decisive consequences on the portfolios suffering its impact.

According to the income perspective, the financial situation of an insurance portfolio is represented, period by period, by means of the business financial result, considered as the financial differential between the total amounts resulting from investments made by the insurer and the cash flows at the end of the period, consisting of the benefits payable and the accruals. This result has a self-consistent value with reference to the business profitability, and it is also an indicator of performance/solvency. So the insurer's period result is a measure of the ability to follow

institutional requests of maintenance of an adequate level of surplus. In this sense, it is essential to measure the ability to absorb the impact of risk sources in terms of competitiveness and achieving a satisfactory performance. It is well known that for a sufficiently large portfolio size, the accidental mortality risk can reasonably be neglected, whilst the consequences of the systematic components need to be considered. So it is equally important to manage the financial risk, which represent a systematic component too. Regarding the financial component, the insurer has to consider the variability of the return on the investment and the discount factor used in calculating the provisions. From a demographic point of view, the uncertainty of the number of survivors at each valuation time is intrinsic; effectively the demographic risk comes from the choice of demographic models which represent the evolution of the survival phenomenon. That representation is more complex because of the thin reference to a market environment, which, as far as existing (as for instance mortality bonds and mortality swaps), is still not enough to completely describe the survival rate as function of the age and the time of valuation.

The aim of the paper is the study of performance indicators apt to measure the impact of the systematic demographic components on the financial results of a portfolio of participating policies, in particular life annuities that participate in the insurer's profits on the basis of a participation rate defined in the contractual scheme. In fact these products still present a wide ranging diffusion, particularly in EU (cf. Coccozza et al., 2011). Their contractual structure varies according to whether the local rules and market require; in our study, in conformity with the guidelines of Coccozza et al. (2011), we will consider a defined participation rate applied to the period financial result.

The paper is organized as follows. In section 1 the contractual profiles are described and the mathematical structures are presented. In section 2 an internal valuation approach is presented by means of accounting indexes. In section 3 financial and demographic scenarios are chosen in order to implement numerical evidences, which are illustrated in section 4.

1. Profit participation annuities

Insurance policies with profit participation constitute a relevant segment within the issue of insurance contracts. They present some contractual distinctions according to the specific country, even if they are characterized by common principles as the guarantee of a minimum rate of return and the

calculation of annual bonus based on return on investment. In Italy, for instance, the structure of profit sharing refers to the differences between income, capital gains and losses (for a deeper understanding here cf. Morgan, 2010). In the following section we will describe the contract under consideration.

1.1. Contract profile. A participating policy allows beneficiaries to participate in the profits according to predefined rules. In particular we are interested in participating life annuities, whose contractual structure is aimed to a participation level referred to the financial result of the period; specifically a participation rate is applied to the aforementioned result, when it reaches a predefined value at least, this implying an embedded option which modify the financial result R_{t+1} referred to the period $[t, t + 1]$. In fact if R_{t+1} , net of the annual quota of the administrative expenses (say γ) is positive, then a bonus equal to a percentage α of $(R_{t+1} - \gamma)$ is added to the provision allocated in $t + 1$, so increasing the future benefits for the policyholders (cf. Coccozza et al., 2011), immediately paid or added to the future instalments.

The architecture of such contracts must balance the attractiveness for policyholders in terms of investment strategy, with the insurer's profitability. Within this context, one must pay attention also to developments and changes for these contracts following the guidelines of Solvency II (cf. also Morgan, 2010); in fact, from the insurer's point of view, the rules involving the SRC (Solvency Capital Requirement) calculation are improving a fortiori the design of less risky products, suitable for lowering the level of capital requirements and, conversely, for improving the global return. Hence, a correct consideration of the impact of the main risk drivers is pressing.

In this framework, although the financial risk component is predominant in the case of participating policies, however in the case of contracts characterized by long durations, it is essential to consider the impact of the systematic components of demographic risk, i.e. the longevity risk, as well as the interrelationships that these risk drivers determine on the economic results. Measuring the impact of the above risk components will provide suitable indications for managerial strategies apt to combine profitability and calibration of the risks to the insurer as well as policyholders' expectations.

Finally the practical implications of a correct ALM strategy will involve the choice of the participation rate (say α) sustainable for the insurer and appealing for the policyholders and that choice is necessarily based on the weighted period results, expressed by means of appropriate financial ratios and balance sheet indexes.

In accordance with the presented considerations, we focus our attention, on the one hand, on the effect due to the financial risk components, i.e. the combined impact of the random movements of the return rates as well as of the interest rates involved in provisions' estimate; moreover, on the other hand, we consider the impact of the demographic risk drivers, framing the evaluation within a best estimate forecast, that is a forecast apt to describe the best estimate of the evolution in time of the accidental deviations of the number of deaths, as well as of the systematic deviations of the mortality rates.

Such evaluation context is obtained by means of stochastic models capturing the forecasted pattern of the survival phenomenon, coherently with the recent international guidelines aimed to avoid the problems due to the unfailing gaps throughout a valuation context lacking in the support of a proper reference market.

1.2. Provisions and financial results within a forward perspective. We assume that at the current time t the insurer obtains information about the random movements of the return rates and the interest rates involved in provisions' estimate, as well as on the number of survivors. In the following we will focus on a valuation process based on assumptions forecasted at the issue time $t = 0$.

The total amount at time $t + 1$ of the provision V_t (referred to the time t) is obtained on the basis of a stochastic accumulation process $\{r(t, s), 0 \leq t \leq s\}$ involving the return rates, as well the provisions are obtained on the basis of a stochastic discount process $\{v(t, s), 0 \leq t \leq s\}$, following a current estimate calculation (cf. Coppola et al., 2007; and Coccozza et al., 2011).

For a life annuity with deferment period T , premium payment until the time τ , annual instalments b_s at the beginning of each year, the stochastic provision at time t is:

$$\begin{aligned} V_t &= \sum_{i=t}^{\infty} (b_i 1_{T \leq i \leq K(x)} - P_i 1_{(i < \tau)}) v(t, i) = \\ &= \sum_{i=t}^{\infty} z_i v(t, i), \end{aligned} \quad (1)$$

where the indicator function $1_{T \leq i \leq K(x)}$ takes the value 1 if $T \leq i \leq K(x)$, $K(x)$ being the random curtate lifetime of an insured aged x at the issue time of the contract, 0 otherwise, whilst the indicator function $1_{(i < \tau)}$ takes the value 1 if $i < \tau$, 0 otherwise. In order to quantify the financial result of the $(t + 1)$ -th accounting period, taking into account the embedded option due to the additional bonus, let us suppose that if the result R_{t+1} given by

$$R_{t+1} = V_t r(t, t + 1) - (b_{t+1} + V_{t+1}) 1_{K(x) > t+1} \quad (2)$$

is greater than γ , the insurer immediately pays the additional bonus $\alpha(R_{t+1} - \gamma)$ to the insured ($0 < \alpha < 1$). Otherwise nothing is recognized except for the standard obligations stated throughout the contract. Hence the total $(t + 1)$ -th financial result TR_{t+1} can be expressed as follows:

$$\begin{aligned} TR_{t+1} &= R_{t+1} - \alpha(R_{t+1} - \gamma)^+ = \\ &= \min(R_{t+1}, (1 - \alpha)R_{t+1} + \alpha\gamma) \end{aligned} \quad (3)$$

Extending the model to a portfolio of different life annuities, we consider a sequence of homogeneous sub-portfolios, say m is their number, identified by common characteristics (age at issue, policy duration, and so on).

So, let us set for the i -th homogeneous portfolio ($i = 1, 2, \dots, m$):

- ◆ n_i = policy duration;
- ◆ $c_i(s)$ = random number of policies at time s
 $\left(\sum_{i=1}^n c_i(0) = c \right)$;
- ◆ x_i = age at issue of the insureds;
- ◆ T_i = deferment period ($0 \leq T_i < n_i$);
- ◆ $b_{i,s}$ = benefit payable to each insured at time s ;
- ◆ $P_{i,s}$ = premium payable by each insured at time s ;
- ◆ $X_{i,s}$ = the flow at time s related to each insured with

$$X_{i,s} = \begin{cases} -P_{i,s} & \text{if } s < \tau_i \\ b_{i,s} & \text{if } s \geq T_i \end{cases}$$

hence, with obvious meaning of the notation:

$$TR_{t+1} = \sum_{i=1}^m R_{t+1}^i = \sum_{i=1}^m \min(R_{t+1}^i, (1 - \alpha)R_{t+1}^i + \alpha\gamma_i), \quad (4)$$

where

$$\begin{aligned} R_{t+1}^i &= \left(\sum_{j=1}^{n_i} X_{i,j} c_i(j) v(t, j) \right) r(t, t + 1) - \\ &- c_i(t + 1) b_{i,t+1} - \sum_{j=t+1}^{n_i} X_{i,j} c_i(j) v(t + 1, j). \end{aligned}$$

2. Insight into internal valuations via accounting indexes

To the aim of performing internal control systems, the risk management needs to avail itself of suitable and well-timed indexes for valuing the insurance business realization. Here we want to focus on the profitability analysis and will consider in particular the Return on Equity (ROE) index, one of the most

popular profitability measures. It defines how much the total amount collected by the insurer to manage a certain insurance business, a policy or a portfolio of policies or several different portfolios, yields in a certain time interval and is extremely simple to be understood, suffice it to say that an increasing behavior of ROE involves immediately an increasing trend of the investors (insureds) interested in the specific business. In the light of the specific structure of the considered index, having at the denominator not the company's total surplus but that part of it supporting the business object of the valuation, even if suitable for all the lines of insurance business, ROE seems to be particularly meaningful in the case of insurance products requiring a large amount of invested surplus and for individual permanent insurance (cf. Easton et al., 2007). It follows that the life annuity, both in its classical acceptance and in its late designs, such as participating life annuity policy, is particularly apt to be synthetically represented in its profitability capacity by means of ROE. The interpretation and the utilization of the ROE index bring the decision-makers towards precise strategic choice.

Broadly speaking, it is defined as the ratio of profits to equity and synthetically expresses the economic yield of the risk capital referred to a specific financial period (cf. Saunders, 2004). ROE is included in the generally accepted accounting principles (GAAP) concept but it is not just a GAAP concept (cf. Beal, 2000). Schematically, ROE shows how many monetary units of net profit the business gets, referred to 100 monetary units of risk capital spent in it, and in this sense summarizes contextually the efficiency and the efficacy of the whole decision-making strategy regarding the specific business line.

The period and the type of business an insurance company can be interested in are extremely flexible. Notwithstanding ROE can apply for different durations, it is often calculated in periods of a single policy year. It is referred to a particular policy or line of policies collected in a portfolio. Insurers often determine the ROE measure for all lines of business and proceed to their comparison through them (cf. Easton and Harris, 2007). Besides the index use for business comparing aims, the ROE is extremely interesting when valued compared with the rate available on fixed investments.

Stopping for a while on the ROE mathematical structure as a ratio, we can observe that the value arises from the profit the insurer gets, at the numerator, and from the equity invested in the particular considered product, at the denominator. The definition of the equity invested in the business turns out to be basic to the aim of the numerical ROE result and its meaning.

Let consider now the ROE related to the specific life annuity contract introduced in section 1. We will refer to a time of valuation $t > T$, considering in this way the profitability index during the accumulation period, that is the period in which the insurer performs his obligations. If P_j is the anticipated annual premium the insured pays at time $j+1$ in the deferment period ($t > T$), we can write:

$$ROE_{t+1} = \frac{TR_{t+1}}{\sum_{j=0}^{t-1} P_j r(j, t)} \quad (5)$$

in which TR_{t+1} is calculated as in equation (3) and the denominator is the fund accrued by the premiums paid by the insureds, calculated on the basis of opportune interest rate assumptions. As we said, the ROE value naturally depends on how the equity is defined. In this formula we mean the return on equity as the rate produced by the sum collected and invested in the considered business line, in the specific case the capital arising from the premiums annually paid, valued at the beginning of the t -th year.

3. Framing within the valuation context

In the following we are going to illustrate the stochastic models selected for the actuarial valuations.

3.1. The financial scenario. We consider two different stochastic interest rate processes for valuing, respectively, provisions and total amounts of the insurer's investments, both in a forward perspective forecasted at portfolio time issue.

As for the provision valuation, made – consistently with the current financial market structure within a fair valuation perspective – by the discounting process $\{v(t, s), 0 \leq t \leq s \leq T\}$, we consider (cf. Heath et al., 1992) the Heath, Jarrow and Morton model (HJM from here on).

This model depicts the evolution of instantaneous forward-rates by means of their instantaneous volatility structures:

$$df(t, T) = \alpha(t, T, f(t, T))dt + \sigma(t, T, f(t, T))dW(t) \quad (6)$$

with the initial condition

$$f(0, T) = f^W(0, T),$$

where $f^W(0, T)$ is the instantaneous-forward curve that the market brings out at time 0.

In equation (6) $W = (W_1, W_2, \dots, W_n)$ denotes an N -dimensional Brownian motion and $\sigma(t, T) = (\sigma_1(t, T), \sigma_2(t, T), \dots, \sigma_N(t, T))$ a vector of adapted processes for $\alpha(t, T)$.

The discrete shape of the forward rates is given by:

$$\Delta(\tilde{f}) = \alpha(t_i, T)(t_{i+1}, T)(t_{i+1} - t_i) + \sum_{h=1}^N \sigma_h(t, T)(\tilde{W}_h(t_{i+1}) + \tilde{W}_h(t_i)) \quad (7)$$

with $h = 0, 1, \dots, N$ and $\Delta\tilde{f} = \tilde{f}(t_{i+1}, T) - \tilde{f}(t_i, T)$, $\tilde{f}(t_i, T)$ being the approximated value of $f(t_i, T)$, $\alpha(t_i, T)$, the drift and $\sigma(t_i, T)$ the volatility function at time t_i .

With regard to the interest rates involved in the accumulation process $\{r(s, t), 0 \leq s \leq t \leq T\}$, we assume its behavior evolves in time according to a Cox-Ingersoll-Ross process, governed by the linear second order stochastic differential equation:

$$dh(t) = -\alpha(h(t) - \mu)dt + \sigma\sqrt{h(t)}dW(t), \quad (8)$$

where α and σ are positive constants, μ is the long term mean and $W(t)$ is a Wiener process.

3.2. The demographic scenario. We illustrate the forecasted evolution in time of mortality rates, focusing the main part of our risk valuation on this uncertainty source; such risk driver shows its basic peculiarity throughout the consequence of the systematic changes in population survival trend. This comes true because of the overall improvement of the socio-economic systems in the industrialized countries during the last two decades. In general the survival distribution is characterized by two phenomena: rectangularization and expansion; the first one lies in an increasing concentration of deaths around the mode of the curve of deaths and the second one consists in the random advancement of the mode of the deaths curve towards the ultimate life time.

Now we assume that the death rates evolve according to a stochastic proportional hazard model, so they are structured as functions of the deterministic anticipated realization of the force of mortality properly adjusted by a multiplicative stochastic factor.

Let $\mu(x + t)$ be the deterministic force of mortality for an individual aged x after t years. Following the basic lines of Di Lorenzo et al. (2002), we assume that the force of mortality evolves in time according to the equation:

$$B(x, t) = \mu(x + t)Y(t), \quad (9)$$

$\mu(x + t)$ being the deterministic baseline of the process and $Y(t)$ a stochastic process (Cox-Ingersoll-Ross type) governed by the following stochastic differential equation:

$$dY(t) = \beta(1 - Y(t))dt + \sigma\sqrt{Y(t)}dW(t), \quad (10)$$

with β and σ positive constants, $W(t)$ a Wiener process, Y_t continuous and positive with the condition $2\alpha \geq \sigma^2$.

Following equation (9), the moving rate determines the reversion of the process towards the long-term value 1, hence, at time $t = 0$, $B(x, t) = \mu(x + t)$, i.e. it coincides with the initial observation.

The translation $Y^*(t) = Y(t) - 1$ allows to turn the process into a continuous centred one, whose discrete shape has the following formalization (cf. Deelstra, 1995):

$$\tilde{Y}^*(t) = \sum_{i=0}^{t-1} \phi^i \sigma_a \sqrt{1 + \frac{2\phi}{1+\phi} \tilde{Y}_{t-i+1}^* a_{t-1}}, t = 1, 2, 3, \dots \quad (11)$$

in which the a_t 's are standard normal variables for all t , independent on $\{a_i | i \leq t-1\}$, $\phi (> 0)$ is an estimation of the drift rate and $\sigma_a > 0$ is the diffusion parameter.

In the following we propose a scenario analysis that considers different assumptions about the evolution of the life expectancy by calibrating the estimated parameters in equation (9) taking into account the improvement in mortality trend (cf. Coppola et al., 2012).

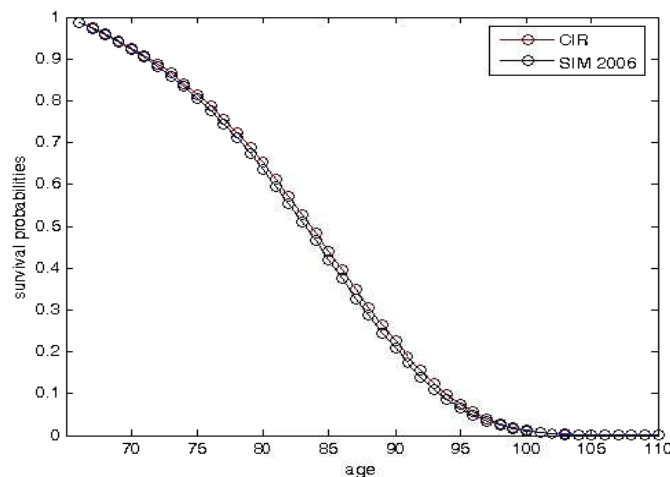


Fig. 1. Survival probabilities of a male aged 65 by SIM 2006 and by the stochastic mortality model in equation (9)

4. Numerical evidences

4.1. Portfolio description and basic assumptions.

To show the descriptive potential of the proposed tools, in this section we illustrate some numerical applications. We refer to a cohort of 1000 immediate participating life annuities consisting in 10 annual anticipated benefits paid to the insureds in case of life for a single premium paid by the insureds at the issue time. We assume that all annuitants are 65 years old. The pure premium paid by each insured is calculated at a technical rate of 2% and the survival probabilities are obtained by the stochastic mortality model described in section 3.2, in particular setting the long-term mean of the process described by equation (10) equal to 1. As showed in Figure 1, in this case we obtain a projection very close to the deterministic baseline of the process. The Company invests the collected premiums in the market and the global rate of return from investments is described by the CIR stochastic process, as in equation (8). The reserve valuation at each time is made discounting the benefits by means of the HJM stochastic process in equation (6).

In order to calibrate the parameters of the HJM process, following the basic lines of D'Amato et al. (2011), we refer to a dataset of EURIBOR and EURIRS over the period from January 2002-March 2009, with several maturities (0-year, 1-year, 3-year, 5-year, 10-year), obtaining, for each maturity, the corresponding implicit forward rate curves (see the Appendix for a discussion of the D'Amato et al.'s (2011) work). The parameters of the CIR accumulation process are calibrated referring to the annual yield, over the period from January 2002-March 2011, on a basket of Treasury Italian Bonds (BTP) listed on the electronic bond and government bond market (MOT) and having a residual maturity greater than one year. The estimation parameters procedure refers to D'Amato et al. (2011). As pointed out in several papers (see for example Di Lorenzo et al., 2006), age intervals around 65 are particularly sensitive to improvements in the survival probabilities due to the longevity influence. In line with a scenario analysis approach, we will assume three different future survival descriptions taking care of the longevity phenomenon, improving with time and decisive in valuations concerning life connected contracts, especially if with long durations. We perturb the long-term mean of the stochastic component in the force of mortality process in equation (10) obtaining three different survival probability tables with increasing levels of projection (cf. Coppola et al., 2012). In particular:

- ♦ Minimum projection, obtained by setting the long-term mean equal to 1.
- ♦ Medium projection, obtained by calibrating the parameter so that the expected lifetime duration

of the individual aged 65 increases from 18 to 20 years.

- ♦ Maximum projection, obtained by calibrating the parameter so that the expected lifetime duration of the individual aged 65 increases of 22 years.

4.2. ROE computation in a simulation framework: a quantile analysis. The event consisting in ROE values lower than the expected ones with a specified confidence level can be studied in the quantile assessment. Here we quantify the tail event resorting to a simulation procedure for obtaining an empirical distribution of the ROE values. The implementation of the stochastic simulation procedure on equation (5) has been made simulating 10000 ROE values checked at different times. Fixing the ROE evaluation time t , the simulated $\{ROE_t\}_s$, with $s = 1, 2, \dots, 10000$, can be treated as the sample we use for estimating the associated q_ε quantiles. We choose $\varepsilon = 99\%$, hence we consider the case of having a ROE lower than q_ε with a probability of 1%. In the following figures the ROE simulated paths for different times of valuation are reported, each group of paths according to a specified survival projection. The graphs are referred to a participating quota $\alpha = 40\%$.

The simulated ROE graphs, as clear in Figures 2, 3 where we report, as an example, the cases of minimum and maximum projections, show an increasing volatility when t increases and at the same time we cannot observe any connections between ROE volatility and projection level. In the following tables we report the quantile ROE values having chosen a confidence level of 99% in the previously specified sense.

In Tables 1, 2, 3 the values expressing the ROE quantiles q_ε generally do not present a trend significantly depending on the level of the projection. We can notice a tendency to reach higher values towards the end of the contract.

Table 1. Minimum projection

	q_ε	Min simulated ROE	Max simulated ROE
$t = 2$	-2.19%	-0.17%	0.15%
$t = 4$	-0.33%	-2.1%	2.5%
$t = 6$	-2.17%	-4.8%	5%
$t = 8$	0.11%	-7.9%	8%
$t = 9$	2.68%	-4%	4%

Table 2. Medium projection

	q_ε	Min simulated ROE	Max simulated ROE
$t = 2$	-0.37%	-0.17%	0.15%
$t = 4$	1.47%	-2.9%	2.5%
$t = 6$	0.48%	-4.2%	4.8%
$t = 8$	3.36%	-8%	8.5%
$t = 9$	1.18%	-10%	7%

Table 3. Maximum projection

	q_ε	Min simulated ROE	Max simulated ROE
$t = 2$	-3.01%	-0.17%	0.15%
$t = 4$	0.18%	-1.1%	1%
$t = 6$	1.15%	-3.4%	4%
$t = 8$	3.44%	-15%	15%
$t = 9$	4.21%	-15%	17%

4.3. Expected ROE. The analysis goes on deepening the expected ROE values in different longevity scenarios and for different participating quotas α , in the same stochastic financial hypotheses. The expected values are reported in the following Tables 4, 5 and 6.

Looking at these three tables, it is directly noticeable a decreasing trend of the expected ROE when the participating quota increases.

We can observe in particular that the ROE expected values decrease when the projection level becomes higher: this is due to the fact that, when the projection increases, the insurer liabilities suffer a worsening with respect to the demographic scenario assumed for the premium calculation. The ROE values naturally are influenced by this circumstance and show a decreasing trend.

Tables 4, 5 and 6 show the presence of a break-even point during the third contract year, when the expected ROE sign becomes positive. This happens

in any projection hypotheses and for any participating quota value.

Finally we observe the presence of a maximum for the expected ROE in $t = 6$, this meaning that the business in force registers an optimum performance value in $t = 6$. This happens in any projection hypotheses and for any participating quota value and can induce to think that it occurs apart from them and is most likely due to the financial operations involved in the numerical analysis. In particular we want to point out the influence of the dynamic interaction between the two processes representing the accumulation and discounting operations. In this order of ideas the optimum expected ROE values showed in $t = 6$ can be considered as a time point in which the effects on the ROE of the perspective and retrospective valuations are in a sort of equilibrium point. The expected ROE trend for different participating quota is better viewable in Figure 2 in which the highest ROE values are evidently referred to the lowest participating quota and the largest differences among the values at different participating quota are noted in $t = 6$, for each projection assumption. Finally, Figures 3 and 4 represents some examples of the simulated ROE values checked at the specified times and in the minimum and maximum projection hypotheses.

Table 4. Expected ROE – minimum projection

$E[ROE]$	$\alpha = 20\%$	$\alpha = 40\%$	$\alpha = 60\%$	$\alpha = 80\%$
$t = 1$	-3.4466%	-3.4466%	-3.4466%	-3.4466%
$t = 2$	-3.2120%	-3.2120%	-3.2120%	-3.2120%
$t = 3$	2.7579%	2.2307%	1.7034%	1.1762%
$t = 4$	2.8352%	2.2841%	1.7331%	1.1821%
$t = 5$	2.7953%	2.2502%	1.7051%	1.1600%
$t = 6$	3.7018%	2.9156%	2.1294%	1.3433%
$t = 7$	2.8303%	2.2589%	1.6874%	1.1160%
$t = 8$	2.1137%	1.7190%	1.3243%	0.9296%
$t = 9$	1.1903%	1.0255%	0.8607%	0.6959%
$t = 10$	1.1682%	1.0182%	0.8682%	0.7183%

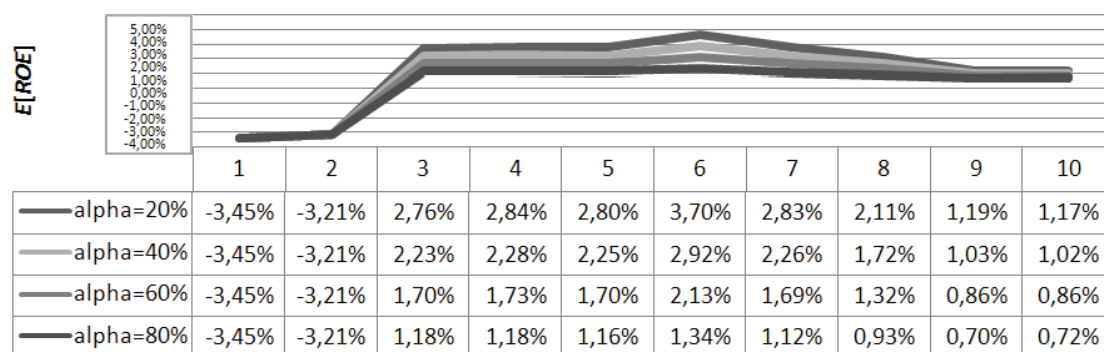
Table 5. Expected ROE – medium projection

$E[ROE]$	$\alpha = 20\%$	$\alpha = 40\%$	$\alpha = 60\%$	$\alpha = 80\%$
$t = 1$	-3.5298%	-3.5298%	-3.5298%	-3.5298%
$t = 2$	-3.5248%	-3.5248%	-3.5248%	-3.5248%
$t = 3$	2.7410%	2.2180%	1.6950%	1.1719%
$t = 4$	2.8089%	2.2644%	1.7200%	1.1755%
$t = 5$	2.7581%	2.2223%	1.6865%	1.1507%
$t = 6$	3.6552%	2.8807%	2.1062%	1.3316%
$t = 7$	2.7695%	2.2133%	1.6570%	1.1008%
$t = 8$	1.4429%	1.2159%	0.9889%	0.7619%
$t = 9$	1.1130%	0.9675%	0.8220%	0.6765%
$t = 10$	1.1097%	0.8155%	0.7224%	0.6384%

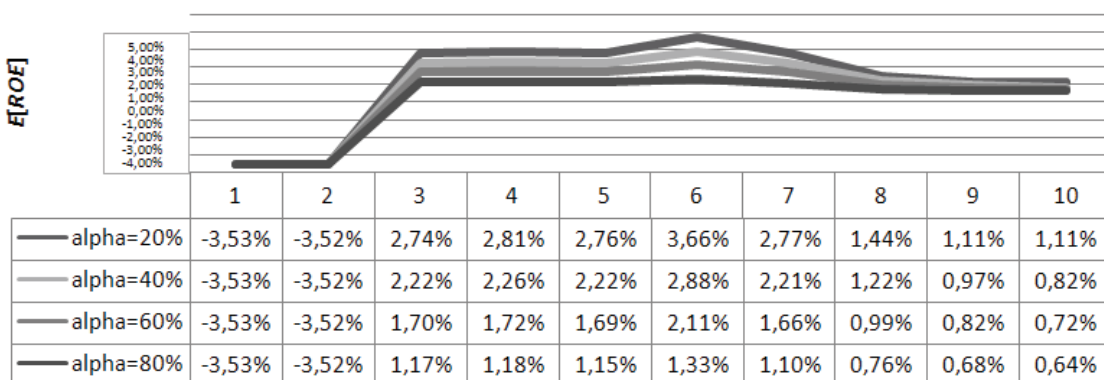
Table 6. Expected ROE – maximum projection

$E[ROE]$	$\alpha = 20\%$	$\alpha = 40\%$	$\alpha = 60\%$	$\alpha = 80\%$
$t = 1$	-3.7374%	-3.7374%	-3.7374%	-3.7374%
$t = 2$	-3.5290%	-3.5290%	-3.5290%	-3.5290%
$t = 3$	2.7346%	2.2132%	1.6918%	1.1703%
$t = 4$	2.7988%	2.2568%	1.7149%	1.1730%
$t = 5$	2.7439%	2.2116%	1.6794%	1.1472%
$t = 6$	3.6372%	2.8672%	2.0971%	1.3271%
$t = 7$	2.7450%	2.1949%	1.6448%	1.0947%
$t = 8$	2.0047%	1.6372%	1.2698%	0.9024%
$t = 9$	1.0821%	0.9443%	0.8066%	0.6688%
$t = 10$	1.0629%	1.7254%	0.6682%	0.6254%

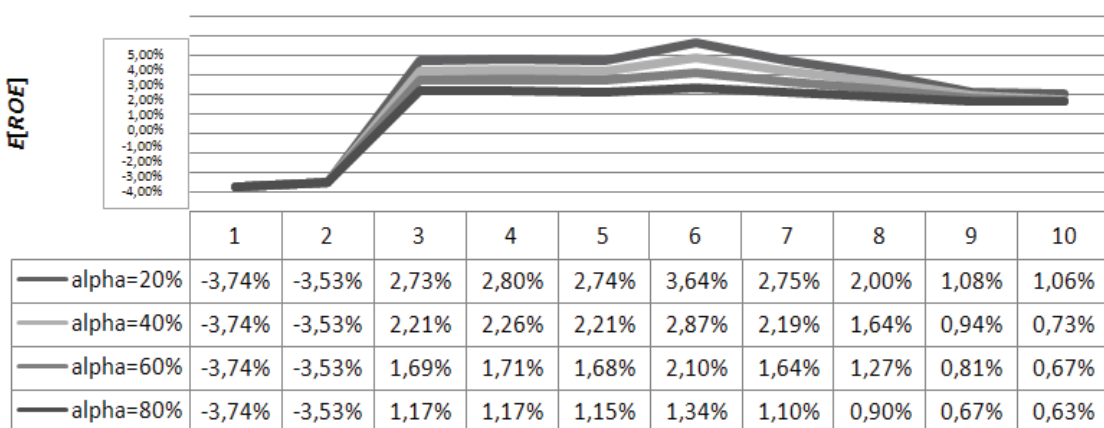
Expected ROE – minimum projection



Expected ROE – medium projection



Expected ROE – maximum projection

Fig. 2. Expected ROE at different participating quota α and projections

Concluding remarks

The paper is focused on an evaluation approach of insurance products primarily based on balance-sheet indicators; in particular, annuities with profit participation for the insureds are considered, since these products are well suited to the demand of customers who meet a lengthening of their life expectancy and hence unavoidable financial requirements related thereto. Estimating ex-ante profit for the accounting period (and indicators related) in the light of the most important risk components can adequately address the insurers management policy, in terms of investment decisions concerning premiums and reserves, but also provides useful suggestions for product designing.

In a period marked by heavy turbulence in financial markets and by deep structural changes in the dynamic evolution of the insured cohorts, the key topic is a well-balanced and careful composition of insurers profit expectation and sufficient guarantees, meeting the needs of policyholders. The new demographic trends in industrialized countries inevi-

tably open the way to long-term financial problems, increasingly burdensome in the current financial crisis. The challenges posed by these scenarios and the network of regulatory constraints which must be taken into account require careful consideration of the impact of risk components.

In this study, we sought to provide a methodological approach, which, in addition to the necessary consideration of the volatility of financial markets, gives a central role to the impact of longevity. In this sense, we measured the period performance through the dynamics of portfolio return, identifying a sort of minimum acceptable income level, by analyzing the quantiles of its distribution. Moreover we deepened the expected values of the portfolio return for practical purposes. The sensitivity of the portfolio performance to the survival projection, the presence of a break even point and the time of optimum performance have been pointed out, in different hypotheses for the participating quota and with stochastic assumption for the accumulation and discounting financial processes and for the survival description.

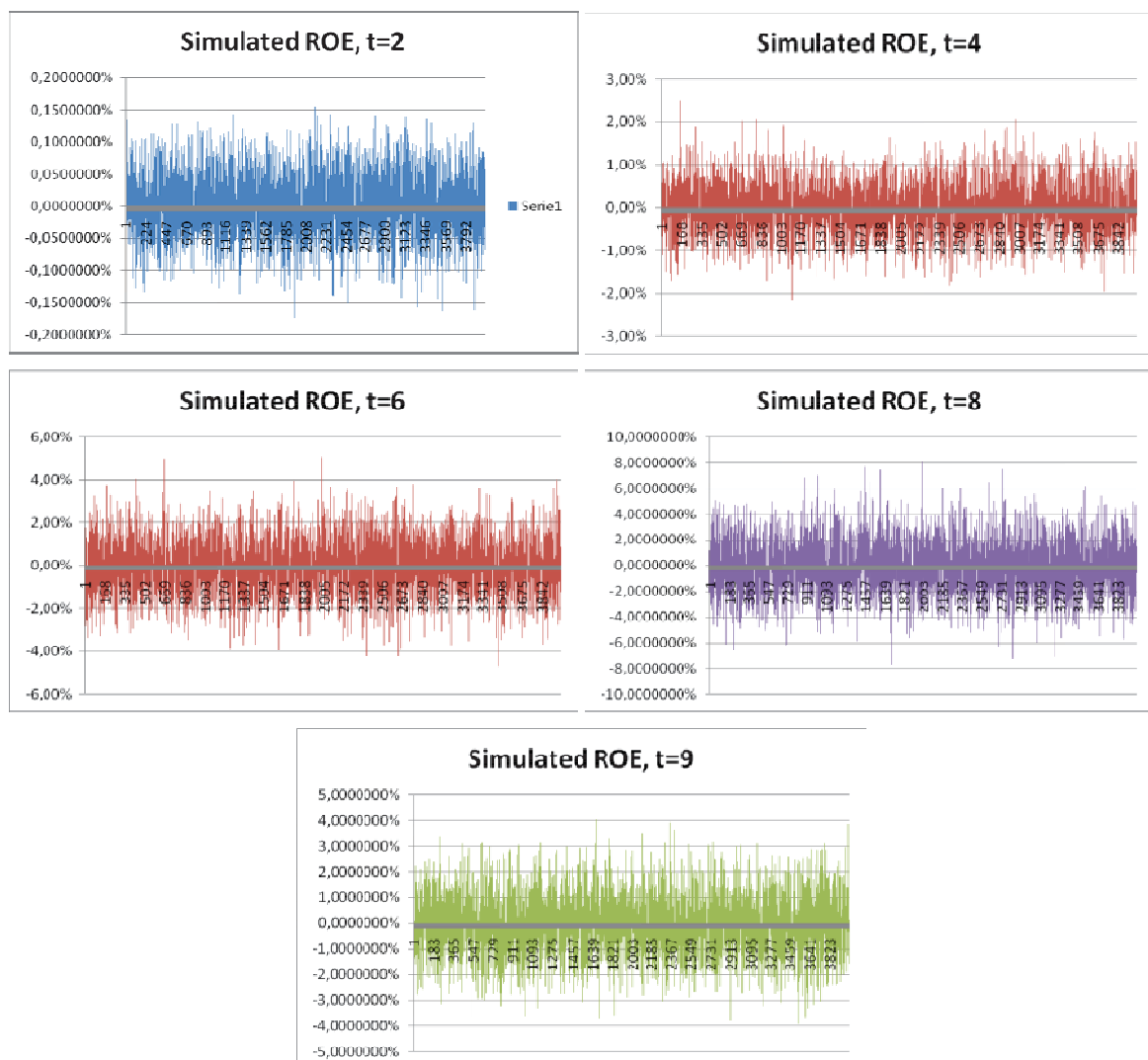


Fig. 3. Simulated ROE – minimum projection

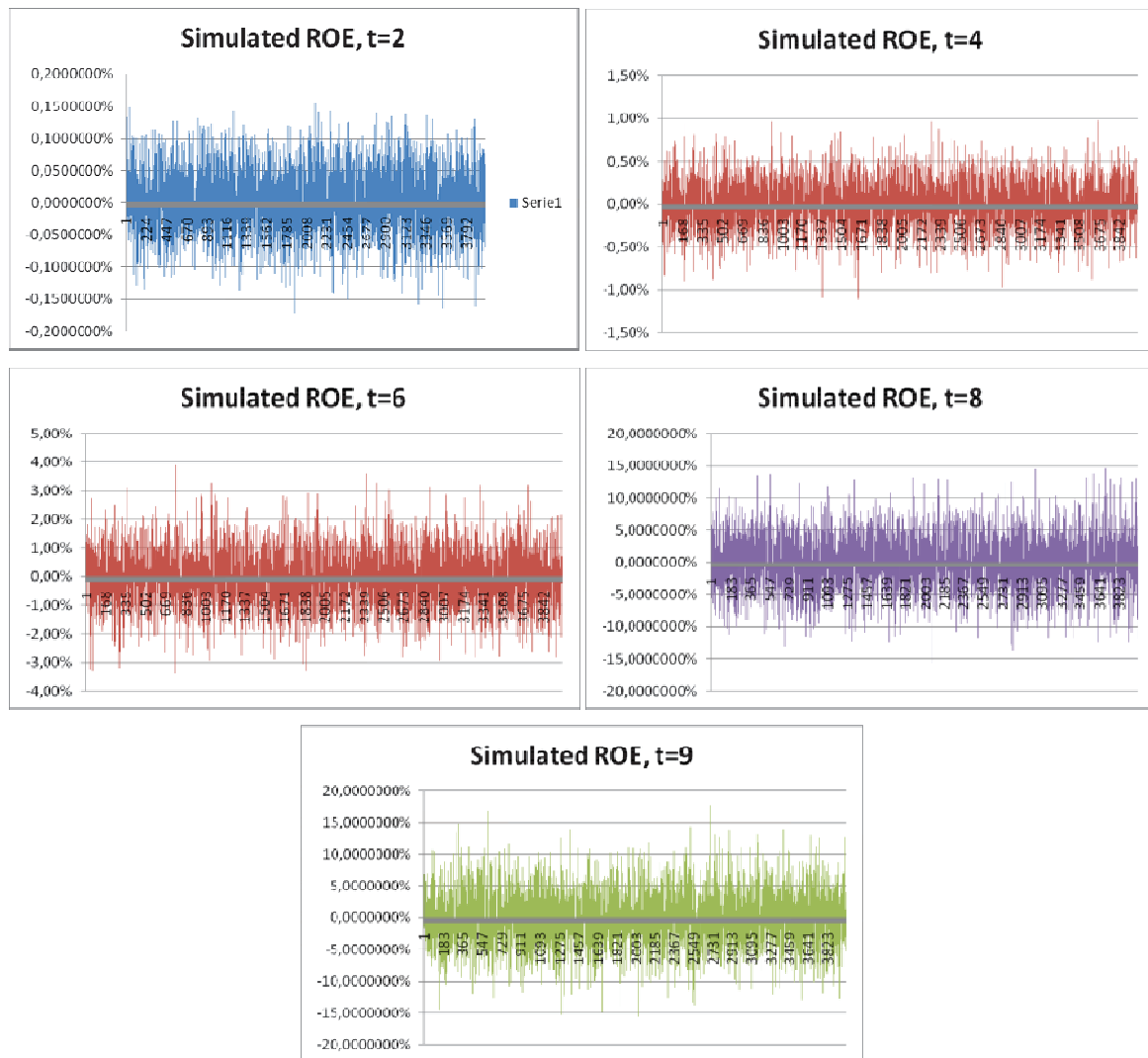


Fig. 4. Simulated ROE – maximum projection

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Appendix. The calibration of the HJM model

The Heath, Jarrow and Morton model (HJM) is considered one of the interest rate models that better manages to describe the evolution in time of the forward rate curve. In this Appendix we intend to deep its calibration by means of some details about the procedure.

The first step consists in the consideration of the interest rate series observable in the market at the analysis starting point. In our case we used the time series of Euribor rates ranging from January 2002 to March 2009, from which we calculated the instantaneous forward rates. The assumptions underlying the HJM model involve that both the drift and the diffusion coefficient in the instantaneous forward rate stochastic differential equation depend on the volatility structures, as the following equation, characteristic of the HJM model, clearly shows:

$$df(t, T) = \left(\sum_{k=1}^n \sigma_k(t, T) \int_t^T \sigma_k(t, s) ds \right) dt + \sum_{k=1}^n \sigma_k(t, T) dz_k(t),$$

where $\sigma_k(t, T) = \frac{\partial \sigma_k^*(t, T)}{\partial T}$, $\sigma_k^*(t, T)$, is the instantaneous volatility due to the k th risk factor and $z_k(t)$ is the Wiener process associated to the k th risk factor.

To design the forward rate curve we need the information contained in the variance covariance matrix of the risk factors, assumed to be stochastically independent. In particular in the empirical matrix the volatilities are proportional to the eigenvalues, estimated on the basis of a Principal Component Analysis based on historical data. On this basis we proceeded with the model calibration. As in D'Amato et al. (2011) and in Coccozza et al. (2010), and taking into account the results obtained by Jarrow (1996), Rebonato (1998), Willmott (1998), and James and Webber (2000), one can say that the primary sources of randomness in the forward rate curve are the random changes in the slope of the curve in their turn followed by random twists in the curve itself and in the curvature of the yield curve. On account of this, the forward rate equation for a three-factor HJM model, if the instantaneous forward rates are discretized by an Euler scheme, can be expressed as follows:

$$\tilde{f}(t_{i+1}, T) = \tilde{f}(t_i, T) + \alpha(t_i, T)(t_{i+1} - t_i) + \sum_{j=1}^3 \sigma_j(t_i, T) [\tilde{W}_j(t_{i+1}) - \tilde{W}_j(t_i)]$$

$$i = 0, 1, \dots, N$$

with $\tilde{f}(t_i, T)$ the value of the approximation of $f(t_i, T)$ at the discretization time t_i and $\alpha(t_i, T)$ and $\sigma(t_i, T)$ the drift and the volatility functions of the forward rates at time t_i , respectively. Under the equivalent martingale measure $\tilde{W}_j(t_i)$ is a Wiener process. We applied the calibration procedure to this three-factor HJM model. In order to obtain an estimation of the forward rate volatility structure, we applied the Fourier cross-volatilities estimator as proposed by Malliavin et al. (2007) upon the calibration of HJM.

Referring to this paper, Matlab software codes have been implemented and Monte Carlo algorithm based on 10,000 simulations has been performed, the starting point being the estimated volatilities. In this way we derive the implicit forward rates curves related to different forward periods (0-year forward that corresponds to the initial forward curve, 1-year forward, 3-year forward, 5-year forward, and 10-year forward).