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Idiosyncratic volatility, illiquidity and the expected stock returns: exploring the relationship with quantile regression

Abstract

This paper takes a new look at the trade-off between idiosyncratic risk and expected stock returns in the emerging market. This study examines the properties and portfolio management implication of conditional idiosyncratic volatility in Taiwan Securities Market. The paper finds that idiosyncratic risk has a significantly negative impact on stock returns in the lower state quantile. It is idiosyncratic volatility that drives the forecastability of the stock market. The author shows that positive effect of idiosyncratic risk on expected returns in higher state quantile explains when investors hold sub-optimally diversified portfolios. Further evidence suggests that Fu (2009) and Angelidis, and Tessaromatis's (2009) findings are largely explained by lower state regime with high idiosyncratic risks. Illiquidity also are significantly related to expected return. The author also finds that stock returns are increasing with the level of idiosyncratic risk and decreasing in a stock's liquidity. However, while both illiquidity and idiosyncratic risk play a role in determining returns, the impact of idiosyncratic risk is much stronger and liquidity enhance the expected return.

Keywords: idiosyncratic risk, expected returns, quantile regression, Fama-French 5 factor model, EGARCH.

JEL Classification: C23, G32, L25, M21.

Introduction

Merton (1987) proposed a simple model of capital market equilibrium with incomplete information, illustrating the difficulties faced by investors in their attempts to achieve comprehensive portfolio diversification. This lack of diversification explains the preference of investors for stocks with high average returns when faced with increased idiosyncratic risk. Merton claimed that market segmentation prevents ordinary investors from achieving comprehensive diversification in their portfolios. He also stated that, in cases of information asymmetry, investors will tend to purchase stocks which are familiar to them, with the expectation of high abnormal returns as compensation for purchasing stocks with higher idiosyncratic risk (Douglas, 1969; Lintner, 1965; Tinic and West, 1986; Lehmann, 1990). According to Malkiel and Xu (2002), and Jones and Rhodes-Kropf (2003), institutional investors prefer more volatile stocks due to their higher returns.

Ang et al. (2006) found that the US stocks with high idiosyncratic volatilities tend to have abnormally low returns in the subsequent month. They also found the same results in their investigation of the relation between future returns and past idiosyncratic volatility, across 23 developed markets (Ang et al., 2009). Guo and Savickas (2008) found stocks with high idiosyncratic variance have low CAPM-adjusted expected return in both pre-1962 US and Modern G7 data. The result is consistent with Bali and Cakici (2008), they were using a screen for size, price, and liquidity play critical roles in determining the existence and significance of relation between aggregate idiosyncratic risk and the aggregate stock returns of individual compa-

nies in the USA and the UK. The global idiosyncratic volatility effect is captured by a simple US idiosyncratic volatility factor (Ang et al., 2009).

It is well known that the volatility of stock returns varies over time and with the cross-section of expected stock. The first goal of this study is to utilize quantile regression to investigate risk loadings which are asymmetric across market states. Three important questions are posed: (1) Do the findings suggest a relation between idiosyncratic risk and expected returns in quantile regression? (2) Are the results similar when the stocks are traded in an emerging market like Taiwan? (3) How do the findings obtained from different methodologies including time-series, cross-section analysis, the regime switch model, and quantile regression model are compared?

This study makes three main contributions. Conventional regressions focus on the mean quantile regressions to describe the entire conditional distribution of the dependent variables. Knez and Ready (1997) used a least trimmed squares methodology to examine the results of Fama and French (1992) who demonstrated that firm size is negatively related to average returns. It is assumed that conclusions might be driven by a few extreme sample observations. After eliminating 1 percent of the most extreme returns, Kenz and Ready found a positive and significant relation between average returns and firm size. They established an estimated relation between average returns and firm size that lacked robustness across all data. In addition, the significance of the coefficient of firm size increases as the larger percentages of extreme observations are eliminated from the sample.

Second, an attractive property of the quantile regression estimator is its robustness in the presence of

outlying observations related to the dependent variable (Koenker and Hallock, 2001). While the Ordinary Least Square (OLS) estimator magnifies the effect of outliers, the quantile regression estimator penalizes tail observations, providing a summary of point estimates for calculating the average effect of the independent variables on the 'average firm'.

Finally, the quantile regression approach avoids the restrictive assumption that the error terms are identically distributed at all points of a conditional distribution. Idiosyncratic risk and firm-specific factors in the Taiwan market have a heavy-tailed distribution.

The remainder of the paper is organized as follows. Section 1 describes the measurement of the idiosyncratic volatility of stock as defined in Fu (2009), and gives a literature review. Section 2 introduces the procedures associated with the modeling process. The empirical results are discussed in section 3. The conclusions and suggestions for future research are given in the final section.

1. Literature review

1.1. Idiosyncratic risk matters. Idiosyncratic risk is defined as the risk that is unique to a specific firm, so it is also called firm-specific risk. By definition, idiosyncratic risk is independent of the common movement of the market. Campbell et al. (2001) used firm-level return data to examine the volatility of the value-weighted NYSE/AMEX/Nasdaq composite index and the value-weighted average stock volatility. Goyal and Santa-Clara (2003) investigated the predictability of stock market returns and proposed a new approach to test the presence and significance of a time-series relationship between idiosyncratic risk and expected returns for the aggregated stock market.

Fu (2009) employed the exponentially generated auto regression conditional heteroskedasticity (EGARCH) model and out-of-sample data to estimate expected idiosyncratic volatility and to capture the time-varying property of idiosyncratic risk. The idiosyncratic volatility of stock is computed as the standard deviation of the regression residuals. Fu (2009) pointed that if idiosyncratic risk is highly persistent as following a random walk process, we can simply use the lagged value as an estimate of the expected value, however, there is no reason to presume high persistence in idiosyncratic risk. Ang et al. (2006) proposed a new concept of the idiosyncratic volatility to measure the volatility index (VIX). Aggregate volatility is a risk factor that is orthogonal to existing risk factors. The sensitivity of stocks to aggregate volatility times the movement in aggregate volatility will show up in the residual of the Fama-French model.

1.2. Idiosyncratic risk, illiquidity and expected returns. Spiegel and Wang (2006) used idiosyncratic volatility and illiquidity to explain cross-sectional variations in average returns. Clearly, idiosyncratic risk is important to investors. The results of these studies all reveal a positive relationship between idiosyncratic risk and stock returns. Goyal and Santa-Clara (2003), Bali et al. (2005), and Ang et al. (2006) employed cross-sectional measurements and found a positive relationship between equally-weighted stock volatility and value-weighted portfolio returns in their volatility measures. Wei and Zhang (2005) showed that the results obtained by Goyal and Santa-Clara were driven by data from the 1990s, concluding that trading strategy does not yield positive returns for extended samples.

Ang et al. (2006) estimated a cross-sectional price of volatility risk of approximately -1% per annum, and controlling for size, volume, momentum and liquidity effects. They showed that in their sample, the average return differential between quintile portfolios of the lowest and the highest idiosyncratic risk is about -1.06% per month. For all idiosyncratic volatility measures, Bali and Cakici (2008) found the average return differential on the lowest and the highest idiosyncratic volatility portfolios is very small and statistically insignificant. Ang et al. (2009) indicated that despite a relatively high degree of transparency, stock markets in both the US and the UK are still characterized by higher idiosyncratic risk and lower stock returns. Fu (2009), Peterson and Smedema (2010) and Huang et al. (2010) provided evidence that idiosyncratic volatility is positively related to returns.

Angelidis and Tessaronatis (2009) found the coefficient of the lagged price/dividend ratio to four times larger in the high return-low variance regime than in the low return-high variance regime. State dependent switching was discussed in other studies (Schaller and Van Norden, 1997; Timmermann and Perez-Quiros, 2000; Gu, 2005).

1.3. Recent models and their discussion. Time-series and cross-sectional regression is often used to test the relationship between expected returns and idiosyncratic volatility. Fu (2009), Huang et al. (2010), and Spiegel and Wang (2006) utilized the EGARCH model with a full sample of monthly data and expected returns, to obtain results showing the biased weighting in portfolios (Peterson and Smedema, 2010), as well as for autoregressive models (Chua et al., 2009). Bali and Cakici (2008) used the Fama-French three factor model to evaluate idiosyncratic risk. They established an arbitrage portfolio combining the buying of stocks with high idiosyncratic risk and the selling of those with low idiosyncratic risk. Cross-sectional analysis has been

employed in many studies (for example, Ang et al., 2006; Huang et al., 2010; Fama and French, 1992, 1993), for unbiased estimates of return reversals (Huang et al., 2010).

Angelidis and Tessonatis found there to be a positive and statistically significant relation between returns in the low variance regime, and insignificant relation between idiosyncratic risk and subsequent market returns. They developed a regime switching version of the CAPM and the Fama-French three factor model. They found risk loadings to be asymmetric across market states.

Quantile regression techniques can help to obtain a more complete understanding of the relationship between idiosyncratic risk and expected returns. In recent years, quantile regression has often been used in long-term studies of financial markets involving large samples. Fattouh et al. (2005) used it to investigate capital structure, while Santa-Clara and Valkanov (2003) used it to investigate the relationship between political business cycles and stock returns. Chuang and Kuan (2005) discussed price-volume relationships in the Taiwanese and US stock markets, and Lee (2008) discussed determinants influencing IPO discounts. The quantile regression model, championed by Koenker and Bassett (1978), allows the investigator to characterize any part of the distribution of a variable condition on observable regressors. The quantile regression is a semiparametric

model and makes no hypothesis concerning the distribution of the population. Thus, the estimators all depend on the original distribution of the samples and are free of adverse influence caused by outliers (Kan and Tsai, 2004; Lee and Shen, 2007; Fattouh et al., 2005; Lee, 2008). In other words, the empirical results of items with quantile values 75% or higher are considered equivalent to the empirical results explanatory variables with higher expected returns. The empirical results are related to the impact of idiosyncratic risk and liquidity on return rates. Conversely, the empirical results for items with quantile values 25% or lower are considered equivalent to the empirical results for a lower rate of return. The empirical results are related to the impact of idiosyncratic risk and liquidity on return rates. The quantile regression model can then be used to analyze the impact of idiosyncratic risk and liquidity on expected returns for various rates of return.

2. Research design

2.1. Quantile regression and modeling. In fact, the quantile regression solution $\hat{\beta}_\theta$ is invariant to outliers of the dependent variable that tend to $\pm \infty$. The quantile regression method of estimation was first proposed by Koenker and Bassett (1978). They defined the θ^{th} regression quantile as the solution to the following problem:

$$V_T(\beta; \theta) = \frac{1}{T} \left[\theta \sum_{(t: y_t \geq x_t' \beta)} |y_t - x_t' \beta| + (1 - \theta) \sum_{(t: y_t < x_t' \beta)} |y_t - x_t' \beta| \right], \quad (1)$$

where y_{it} is the dependent variable; x is a vector of regressors; β is a vector of parameters to be estimated; $X_t' \beta_\theta$ denotes the θ^{th} conditional quantile of y_{it} given x_{it} . The θ^{th} regression quantile, $0 < \theta < 1$, is used to solve equation (5).

This is normally written as follows:

$$\min \frac{1}{T} \sum_{t=1}^T x_t' (\theta - I_{(y_t - x_t' \beta < 0)}) = 0, \quad (2)$$

where $I(\cdot)$ is the usual indicator function, $\rho_\theta(\cdot)$, which is known as the “check function”, and is defined as:

$$\rho_\theta(\varepsilon) = \begin{cases} \theta \varepsilon & \varepsilon \geq 0 \\ (\theta - 1) \varepsilon & \varepsilon < 0 \end{cases} \quad (3)$$

The model specifies the θ th-quantile of the conditional distribution of the expected return. By variation of θ_ε , different quantiles can be obtained. The least absolute deviation (LAD) estimator of β is a particular case within this framework. This is ob-

tained by setting $\theta = 0.5$ (the median regression). The first quantile is obtained by setting $\theta = 0.25$, and so on. As we increase θ from 0 to 1, we trace the entire distribution of y , conditional on x . This problem does not have an explicit form, but it can be solved by linear programming methods.

2.2. Conditional idiosyncratic volatility (IVOL).

We next address seasonality in the cross-sectional pricing of idiosyncratic volatility. Following Hung et al. (2010), Fu (2009), Ang et al. (2006) the idiosyncratic risk of an individual stock is measured by controlling for several known pricing factors as follows:

$$R_{i,t} = \gamma_{0,1} + \gamma_{1,t} E_{t-1} [IVOL_{i,t}] + \sum_{k=2}^K \gamma_{k,t} E_{t-1} [X_{kit}] + \varepsilon_{i,t}. \quad (4)$$

In every month, daily excess returns of individual stocks are regressed on the daily Fama-French (1993, 1996) three factors. The SMB, HML, excess returns

on a broad market portfolio ($R_m - R_f$) and the idiosyncratic volatility for stock i are measured as the standard deviation of the residual ε_i after estimating equation (4) using daily excess returns over the past month.

To reduce the impact of infrequent trading on idiosyncratic volatility estimates, a minimum of 15 trading days in a month is required for which TEJ reports both a daily return and non-zero trading volume (Fu, 2009). X_{kit} represents other explanatory variables specific to firm I , in all cross-sectional regressions. These variables are defined as $X_{kit} = [\text{Beta}, \ln ME, \ln BM]$.

In order to capture the time-varying property of idiosyncratic volatility, we also resort to the EGARCH models, because parameter values do not need to be restricted in order to avoid negative variance as other ARCH and GARCH models do. Engle and Ng (1993) suggest that Nelson's EGARCH specification does a good job in capturing the asymmetry of conditional volatilities. The explicit functional forms are as follows.

The conditional distribution of residual ε_{it} is assumed to be normal with a mean of zero and a variance σ_{it}^2 . The objective is to estimate the conditional variance σ_{it}^2 . It is a function of the past p -period of residual variance and q -period of return shocks as specified by equation (5).

2.3. Expected returns, idiosyncratic risk, and liquidity. The expected return is found by incorporating Bali and Cakici's (2008) three-factor model and Carhart's (1997) four-factor model to derive the difference between the payment and non-payment of dividends, as well as Ang et al.'s (2006) five-factor model, which is used to define volatility indices. The resulting model is formulated as follows:

$$R_{i,t} - r_t = \alpha_i + \beta_i(R_m - r_t) + s_i \text{SMB} + h_i \text{HML} + \varepsilon_{it},$$

$$\varepsilon \sim N(0, \sigma_{it}^2)$$

$$\ln \sigma_{it}^2 = a_i + \sum_{l=1}^p b_{i,l} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^q c_{i,k} \left\{ \varphi\left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right) + \nu \left[\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right] - \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \right\}, \quad (5)$$

$$E - R_{1,it} = \beta_0 + \beta_1(R_m - R_f)_{it} + \beta_2 \text{SML}_{it} + \beta_3 \text{HML}_{it} + \beta_4 \text{UMD}_{it} + \beta_5 \text{WML}_{it} + \varepsilon_{it}, \quad (6a)$$

$$E - R_{2,it} = \beta_0 + \beta_1(R_m - R_f)_{it} + \beta_2 \text{SML}_{it} + \beta_3 \text{HML}_{it} + \beta_4 \text{UMD}_{it} + \beta_5 \text{WML}_{it} + \beta_6 \Delta \text{VIX}_{it} + \varepsilon_{it}. \quad (6b)$$

After obtaining equation (6), the *IVOL*, illiquidity (*ILLIQ*), volume by monthly trading (*VOLT*) are adopted as dependent variables to match equation (7), including the market value to equity and book price to market value.

$$E - R_{m,t} = \lambda_0 + \lambda_1 \text{IVOL}_{mt} (E[\text{IVOL}_{mt}]) + \lambda_2 \text{ILLIQ}_{mt} + \lambda_3 \text{VOLT}_{mt} + \lambda_4 \ell \text{ME}_{mt} + \lambda_5 \ell \text{BM}_{mt} + \varepsilon. \quad (7)$$

The control variables are defined below.

2.3.1. Volume (*VOLT*). Lo and Wang (2000) used turnover as a measure of trading volume in a detailed study of the turnover of individual stocks. Llorente et al. (2002) defined the daily turnover of stocks as the total number of outstanding shares, with the daily time series of turnover being nonstationary. We define the turnover ratio using the total number of shares traded, it is obvious that using the total dollar volume normalized by the total market value gives the same results. To avoid the problem of zero daily trading volume, a small constant (0.00000255) is added to the turnover before taking the logs. The resulting series are detrended by subtracting the 200 trading day moving average as follows:

$$\text{VOLT} = \frac{1}{200} \sum_{s=-200}^{-1} \log \text{Turnover} \quad (8)$$

where $\log \text{Turnover}_t = \log(\text{Turnover}_t + 0.00000255)$.

2.3.2. Illiquidity (*ILLIQ*). We predict, as in Bali et al. (2005), that the power of equal-weighted measures of idiosyncratic volatility regarding future market excess returns is driven by a liquidity premium. The market illiquidity measure is similar to that used in Amihud (2002):

$$\text{ILLIQ}_{i,m} = \frac{1}{T} \sum_{t=1}^T \frac{|R_{i,t}|}{\text{VOLT}}, \quad (9)$$

where $T_{i,m}$ is the number of trading days for stock I in month m ; $|R_{it}|$ is the absolute returns for stock i on day t ; and *VOLT* is the dollar trading volume of stock i on day t . The illiquidity measure is also aggregated across all stocks for each month. Liquidity is the control variable for the measure of idiosyncratic risk with stock returns. The idiosyncratic volatility has a time-varying character (Fu, 2009) that is spontaneously influenced by liquidity factors (Amihud, 2002; Ang et al., 2006, 2009). Suppose that the idiosyncratic volatility involves the property of being both time-series and cross-sectional. This can help to test the relationship with stock returns in the Taiwan market. If the realized stock return influences idiosyncratic risk, we can continue studying the relationship using a simultaneous equation to determine the estimation.

3. Results and findings

3.1. Descriptive statistics. The data set collected from the TEJ data base is comprised of 640,992 discrete units from 607 firms for the period of 2000-2011. We used monthly panel data. The study horizon covers a period of over twelve years. Financial

institutions and insurance firms are not included in the study since the balance sheets of those firms have a strikingly different structure than those of non-financial firms. Finally, we drop observations with missing values for either dependent variables or independent variables.

Table 1. Sample distribution by industry

No.	Industry	Firms	Sampling ratio	No.	Industry	Firms	Sampling ratio
11	Cement	8	100%	20	Iron & steel	23	95.8%
12	Food	21	87.5%	21	Rubber	8	88.9%
13	Plastic	19	90.5%	22	Automobile	4	80.0%
14	Textiles	58	96.7%	23,24,30,61	Information & technology	237	86.2%
15	Electrical machinery	29	87.9%	25	Construction	27	84.4%
16	Electrical & cable	10	62.5%	26	Transportation	15	93.8%
17	Chemical	27	93.6%	27	Tourism	6	100%
18	Glass	6	87.1%	29	Consumer goods	11	91.7%
19	Paper	6	85.7%	99	Others	38	100%

Notes: Data is for 2000-2011. Ratios are computed from TSEC and OTC data. The TSEC and OTC firms are classified into different industry groups on the basis of the Securities Identify Codes.

Table 2 provides descriptive statistics of all the variables in our model. The summary statistics of expected returns and individual risk in Taiwan are presented in Table 2. Specifically, we report the cross-firm mean, median, lower and upper quartiles of the λ_{it} estimates, standard deviation, skewness coefficient, Ex. kurtosis coefficients, and Jarque-Bera normality test. The combination of positive skewness ($E_R_1 = 0.3$; $E_R_2 = 0.24$) and high excess kurtosis ($E_R_1 = 8.36$; $E_R_2 = 6.284$) displayed in expected return. $IVOL$ and $E[IVOL]$ display positive skewness and $E[IVOL]$ display high

excess kurtosis (leptokurtosis). Illiquidity are lower than positive Ex. kurtosis and higher than Skewness. It explained that investors prefer kurtosis of return and he constructs his portfolios based on kurtosis. The Jarque-Bera test shows that statistical shocks of either sign are more likely to be present, and that all economics variables are found to have a fatter tail and a sharper central peak than the standard normal distribution. The mean monthly expected returns are 1.27% and 0.67%. The mean $E[IVOL]$ is 9.5 with a standard deviation of 9.71 in the pooled sample.

Table 2. Descriptive statistics of main variables

	25%	Mean	Median	75%	St. dev.	Skewness	Ex. kurtosis	Jarque-Bera
E_R_1	-8.88%	1.27%	1.52%	8.16%	7.17	0.3	8.336	489*** (0.000)
E_R_2	-3.27%	0.87%	0.82%	3.56%	7.28	0.34	6.284	290.6*** (0.000)
$IVOL(Fu)$	-3.4	11.48	8.4	287.1	7.17	0.3	3.3	174293*** (0.000)
$E[IVOL]$	0.002	9.5	6.8	161.1	9.71	2.92	7.26	541268*** (0.000)
$VOLT$	-0.314	1.080	1.844	3.821	46.55	0.608	2.591	2879.5*** (0.02)
$ILLIQ$	14.543	19.022	17.738	28.954	156.4	1.763	1.428	2.341*** (0.000)
$\ln(ME)$	0.81	1.433	1.568	2.921	62.33	1.94	0.6	126.5*** (0.000)
$\ln(BM)$	7.554	8.98	8.59	9.464	14.4	1.55	0.52	2462.6*** (0.000)

Note: 25% – minimum, 75% – maximum. The second column of the normality test lists the p -value. The p -value approaches zero instead of rejecting the hypothesis of normal distribution. $VOLT$ – turnover by trading, $ILLIQ$ – illiquidity. $\ln(ME)$ refers to the natural logarithm of the parenthetical market price to equity (net asset value); $\ln(BM)$ refers to the natural logarithm of parenthetical book value to market value. $VOLT$ defines the daily turnover of stocks as in Llorente et al. (2002). The data sets included 87,408 items for 607 firms listed during 2000-2011. We used monthly data.

Permutation of these orders yields nine different EGARCH models. Therefore, if a stock's idiosyncratic volatility process converges under all nine models, we would have nine estimated conditional idiosyncratic volatilities at month $t+1$. The estimates generated by the lowest Akaike information criterion (AIC) model are chosen. The EGARCH parameters are estimated by using an expanding window of data with a requirement of 30 minimum observa-

tions. Of all the estimates, only 17.19% are yielded by the EGRACH(1,1), 21.88% by the EGARCH(1,2), 4.68% by the EGRACH(2,1), 1.25% by the EGARCH(2,2), 15.6% by the EGARCH(3,1), and 14.1% by the EGARCH(3,2) and EGARCH(3,3) models. In particular, EGARCH(2,2) is the best-fitting model for a smaller number of firm-month observations, and EGARCH(1,2) is the best-fitting model for the largest number of observations.

Table 3. *IVOL* and $E[IVOL]$ by the EGARCH model

Variable	Autocorrelation coefficient								Unit root test	
	α_0	α_1	β_0	γ_0	β_1	β_2	β_3	β_4	ADF	$E[IVOL]ADF$
<i>IVOL</i>	0.58 $t = 9.5$	0.36 $t = 5.98$	0.18 $t = 3.01$	0.23 $t = 3.77$	0.8 $t = 42.41$	-0.34 $t = -21.21$	209.38 $t = 7.24$	0.6×10^{-5} $t = 2.04$	-0.852 $t = -7.63$	-0.853 $t = -7.86$

The best-fitting values are as follows:

$$FUV = \alpha_0 + \alpha_1 + \beta_0 + \gamma_0 + \beta_1 E(r) + \beta_2 \Delta VIX + \beta_3 VOLT + \beta_4 ILLIQ \quad (10)$$

In the estimation model between January 2000 and December 2011, $\alpha_0 = 0.58$ ($z = 9.5$), $\alpha_1 = 0.36$ ($z = 5.98$), $\beta_0 = 0.18$ ($z = 3.01$), $\gamma_0 = 0.23$ ($z = 3.77$), $\beta_1 = 0.8$ ($z = 42.41$), $\beta_2 = -0.34$ ($z = -21.21$), $\beta_3 = 209.38$ ($z = 7.24$), and $\beta_4 = 0.6 \times 10^{-5}$ ($Z = 2.04$).

3.2. Results of quantile regression. We found that in the different quantile groups, *IVOL* and $E[IVOL]$ would tend to produce significantly different effects. Furthermore, negative effects are usually only found for the low quantile items. It can be seen from Tables 4 and 5 that *VOLT* is positively related to expected returns regardless of whether the model used for the calculation is a four-factor or five-factor model. Most of the effects on expected returns are positive at the *IVOL*, except for low quantile items. The $E[IVOL]$ from the four-factor model are positively related to expected returns, but insignificantly negatively related to expected returns for the five-factor models. Illiquidity also mitigated the positive effect with expected returns (ER_1) but showed an insignificant relation with expected returns (ER_2). However, *ILLIQ* is positively related to expected returns under the ER_2 . There is also an insignificant relation with expected returns obtained with the five-factor model and the low quantile items.

It has been shown that the $E[IVOL]$ of individual stocks changes over time and that there is a negative relation between ER and the one-month $E[IVOL]$. Employing the EGARCH models to estimate the $E[IVOL]$, we find a significantly positive relation between ER and $E[IVOL]$ as shown in Table 4. There are three main findings worth mentioning.

The alphas from the time-series regressions on the Fama-French models are significantly negative for quantile regression. Secondly, most of the stocks in the high *IVOL* portfolios tend to be small firms. Third, some stocks with high *IVOL* at month $t-1$ earn positive abnormal returns in the same month. The relationship between average idiosyncratic risk and stock returns of the high quantile group (75%) and the low quantile group (25%) is not discussed in this study, because this issue has been dealt with Ang et al. (2006) and Huang et al. (2010). A quantile regression model is used, which does not hypothesize a normal distribution for this sample. The results show that a negative effect was only found between the low quantile group and stock returns, and that the effect was the most significant.

Generally speaking, except for low quantile items of idiosyncratic risk which are negatively related to expected returns, other items are positively related to expected returns. Supporting our argument that there is an inconsistent relationship with every item of the higher to lower groups, there is mostly a positive effect between idiosyncratic risk and expected returns in spite of the Fama-French factor model. The findings in this paper are consistent with those of Shleifer and Vishny (1997) that there is a negative relation between idiosyncratic risk and excess short expected returns. They explained that speculative investors are risk averse, meaning that investors prefer to invest in stocks with low idiosyncratic risk. Ang et al. (2006) found the reverse effect because of information asymmetry. Oddly, the *IVOL* and $E[IVOL]$ have explanatory power. Illiquidity of the control variables varied according to the *IVOL* and $E[IVOL]$. We, therefore, suggest that the relationship between illiquidity and stock price returns is not linear. This result should be studied further.

Table 4. Quantile regression at 25%, 50%, 75% (with idiosyncratic risk defined as *IVOL* and $E[IVOL]$)

Variables quantiles	E_{R_1}			E_{R_2}			$E_{R_1} E[IVOL]$			E_{R_1}			E_{R_2}			$E_{R_1} E[IVOL]$		
	25%	50%	75%	25%	50%	75%	25%	50%	75%	25%	50%	75%	25%	50%	75%	25%	50%	75%
Cons.	-11.4** -6.81	-2.34** -4.15	6.3** 8.73	-10.2** -4.96	-2.3** -4.51	5** 3.78	-2** -3.74	-2.38** -5.11	4.45** 5.92	-1.97** -10.8	-3.76** -13.24	-0.39** -2.33	-3.39** -16.98	-4.55** -15.2	-0.26 -1.48	-0.53** -8.2	-2.42** -23.6	-0.92** -14.1
<i>IVOL</i>	-0.04** -6.22	-0.01** -3.22	0.03** 4.51	-0.03** 3.44	-0.01** -3.87	0.02** 2.83	-0.05** -4.56	-0.01** -3.26	0.03** 5.17	-0.53** -246.4	-0.51** -5.7	0.7** 3.51	-0.38** -3.52	0.41** 89.1	0.5** 185.6	-0.53** -2.67	-0.51** -6.51	0.7** 3.48
$\ln(ME)$	0.097** 1.99	0.13** 2.84	0.22** 3.35	-0.02 -0.33	0.07 1.544	0.16** 2.83	0.19** 5.22	0.13** 2.49	0.13** 2.17	0.33** 3.41	0.45** 3.82	0.46** 3.20	0.35** 2.86	0.45** 3.11	0.31** 2.99	0.42** 3.6	0.53** 5.7	0.42** 3.95
$\ln(BM)$	0.2** 2.41	-0.007 -0.1	-0.24** -2.84	0.15 1.76	0.003 0.05	-0.07 -0.91	-	-	-	0.19** 8.6	0.17** 4.96	-0.07** -3.55	0.2** 8.4	0.2** 5.66	-0.01 -0.48	-	-	-
<i>VOLT</i>	2.95** 8.58	4.5** 4.9	0.63** 3.18	3.4** 3.62	25.9** 3.34	3.92** 4.25	3.69** 21.3	4.02** 5.25	4.04** 3.03	0.3** 5.05	1.06** 4.11	0.79** 4.26	0.29** 4.37	1.12** 3.42	0.88** 5.21	0.28** 5.07	1.06** 2.98	0.8** 4.41
<i>ILLIQ</i>	-0.009 -0.11	0.006 0.32	0.38 0.154	0.006** 2.65	0.004** 2.49	0.002 0.9	0.007 0.09	0.006 0.32	0.004 0.16	-0.011 -0.64	0.006** 2.47	0.004** 2.46	-0.008 -0.43	0.006** 2.19	0.002 1.27	-0.008 -0.53	0.006** 2.58	0.004** 2.43

Note: Critical values t_{α} , $\alpha = 0.5\%$, mark * ($t = 1.96$); $\alpha = 0.1\%$, mark ** ($t = 2.32$); $\ln(ME)$ refers to natural logarithm of parenthetical market price to equity (net asset value); $\ln(BM)$ refers to the natural logarithm for the parenthetical book value to market value. Default risk refers to corporate debt risk; *VOLT* defines the daily turnover of a stock, as in Llorente et al. (2002).

Table 5. Quantile regression at 5%, 25%, 50%, 75%, 95%

	α_1					$E(R_1)$					α_1					$E(R_2)$				
	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%
<i>IVOL</i>	-5.59***	-2.85***	-2.73**	-1.75**	8.3***	-0.87**	-0.8**	0.48***	1.10**	1.15***	-4.6***	-0.37	-1.08	0.68	0.75**	-0.94**	-0.95***	0.5***	1.07**	1.12***
Z-test	-5.68	-5.15	-2.54	-2.58	4.57	-2.22	-2.39	7.33	2.70	3.47	-8.00	-1.38	-1.34	0.34	2.59	-2.22	-4.84	4.02	2.64	3.86
$E[IVOL]$	-5.6***	-2.85***	-2.77**	-1.75**	8.28***	-0.26**	-2.37**	1.44***	3.27**	3.41***	-1.1***	-4.3**	0.23***	6.42	1.57	-1.11***	-0.86**	0.41	0.88	1.08
Z-test	-5.48	-5.15	-2.57	-2.58	4.72	2.21	-2.38	4.57	2.69	4.67	-2.84	-2.38	3.75	1.32	1.65	-3.34	-2.45	1.34	1.83	1.86
<i>ILLIQ</i>	-1.98**	-0.89***-3.84	-0.92	0.35***	2.34***	-1.45**	0.10***	0.26	0.79**	1.98**	-1.1**	1.05***	0.42***	2.31***	2.89***	-1.55	-1.55***	-1.71	1.91***	-0.58***
Z-test	-2.66		-1.21	6.44	7.55	-2.37	3.28	1.59	2.37	2.37	-2.59	2.84	3.02	4.69	2.78	-1.66	-4.01	-1.76	8.72	-3.4
<i>VOLT</i>	-0.19**	-0.06***	0.83**	0.67**	3.54***	-2.74*	-2.52**	-1.41	2.33***	3.68***	-1.7***	-5.4***	0.65***	7.62***	2.43	-1.01***	-1.73***	0.21**	1.5***	4.9
Z-test	-2.27	-3.77	2.46	2.37	2.46	-1.90	-2.96	-1.38	6.73	4.89	-3.49	-2.79	11.6	3.74	1.76	-5.68	-2.41	2.37	10.41	1.59

Note. This table presents the coefficient estimates and Z-statistics from the panel data regression of individual stock returns. The Z-statistics are reported in parentheses. *, **, and *** denote significance at the 10%, 5% and 1% levels, respectively.

Conclusion and suggestions for future research

From the viewpoint of quantile regression, we determine that groups with low return rates have high idiosyncratic risk. The results indicate that negative expected return have more volatility. The positive influence identified in the high states groups supports Merton's (1987) investor recognition hypothesis. We investigate the pricing of idiosyncratic volatility and demonstrate that conditional idiosyncratic volatility, as defined in Fu, is significantly negatively related to expected returns in the lower states and positively related to expected returns in the higher states. The relation is robust and cannot be dismissed as important pricing factors. In Fama-French four and five-factor cross-sectional regressions, the coefficient on idiosyncratic volatility is negative and significant, even when controlling for illiquidity. Conditional idiosyncratic volatility is generally negatively related to *ER* in the lower states and positively related to *ER* in the higher states.

To summarize, this paper documents interesting results, that the idiosyncratic volatility affects the expected stock returns. Also, the Taiwan stock market can be characterized as the shallow dish type, in that it is subject to global influences as it becomes more accessible to international capital mobility. Future empirical tests of the market integration of emerging markets should take into account the stocks. Furthermore, as in Merton (1987), we postulate that liquidity needs will be realized in the expected stock returns.

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