

# “The predictive power of forward rates: a re-examination for Germany”

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## The predictive power of forward rates: a re-examination for Germany

### Abstract

Although this study is related to the expectations hypothesis for the term structure of interest rates, the main focus is the predictive power of lagged forward rates for future spot rates. The authors use German monthly swap rates for the period from 1995 to 2007 to analyze whether forward rates contain useful information that could be used when making predictions. The results, although indicate cointegration among spot and lagged forwards rates, do not suggest supportive evidence in favor of forward rates as useful forecasting tools. Only a model containing six lagged forward rates was able to beat the naive forecast.

**Keywords:** expectations hypothesis, term structure of interest rates, forward rates, predictive power.

**JEL Classification:** C32, E43, G17.

### Introduction

The term structure of interest rates has been a subject to extensive research in the past decades. While the first papers appeared in the 40s of the 20<sup>th</sup> century, studies seeking to find an evidence helping to explain the term structure can hardly be counted. As a result of this research, the expectations theory of the term structure emerged, containing three versions: the pure expectations theory, the liquidity theory, and the preferred habitat theory. All three versions have been intensively tested. Despite a great number of papers in this field, an unambiguous conclusion still cannot be drawn.

Testing the expectations theory in many of its forms is complicated by the fact that special assumptions regarding the expectations formation process are necessary, with the rational expectations being the most common. However, this joint hypothesis results in ambiguity when interpreting tests results. Either the expectations theory does not hold or the expectations are formed in a way different from that assumed.

To overcome these difficulties, our paper attempts to avoid the necessity of assuming some particular form of economic agents' expectations. Instead, we concentrate on the issue, whether forward rates contain any predictive power with respect to the future spot rates.

Within the numerous contributions on the term structure theories, tests applying UK or US data are clearly dominating. Most authors find evidence against the expectations theory. As for Europe, evidence is still limited. We extend the existing literature employing recent data ranging from 1995 to 2007. In addition, due to new developments in econometrics the testing methodology has changed significantly. We employ cointegration analysis and

the error correction model in order to test the forecasting ability of lagged forward rates.

The remainder of the paper is organized in the following way. Section 1 covers several term structure theories and summarizes major findings of previous research. Section 2 conducts a preliminary data analysis and enlightens the methodology employed. In Section 3 cointegration analysis and the error correction model are used to analyze the information content of forward rates. Finally, the last Section summarizes our results.

### 1. Theoretical framework

In this section we present the essence of the expectations theory of the term structure. Firstly, we outline several common testing procedures as well as the model used in our study. Subsequently, the theoretical foundation is followed by a summary of existing empirical evidence.

**1.1. The expectations hypothesis of the term structure.** The expectations theory of the term structure of interest rates is comprised of several forms: the pure expectations theory, the liquidity theory, and the preferred habitat theory. In its pure version which was originally proposed by Irving Fisher in 1896, the expectations theory assumes that the term structure is determined entirely by the expectations of future short-term interest rates. In contrast to the pure expectations theory, two other forms of the theory state the existence of some additional factors explaining the term structure. Consequently, they are referred to as biased expectations theory.

Liquidity theory<sup>1</sup> stresses uncertainty connected with long-term securities and assumes that market participants demand a liquidity or risk premium for holding a longer-term security. According to this

theory, the shape of the yield curve is determined by the expectations of future interest rates plus a liquidity premium. The longer is the bond maturity, the higher the liquidity premium should be. Forward rates, implied by the term structure, are therefore no longer unbiased predictors of the future short-term rates as, in addition to the expectations of the future short-term rates, they also contain a liquidity premium.

Likewise, the preferred habitat theory<sup>1</sup> asserts that the yield curve is formed by expectations and a risk premium. However, this premium does not rise uniformly with maturity. Within this form of the expectations hypothesis, investors do not necessarily prefer shorter-term securities. Instead, the theory assumes investors to have different preferred investment

$$r_t^n = \left[ (1+r_t^{0,1}) \cdot (1+E_t r_{t+1}^{1,2}) \cdot (1+E_t r_{t+2}^{2,3}) \dots (1+E_t r_{t+n-1}^{n-1,n}) \right]^{1/n} - 1, \tag{1}$$

where  $r_t^n$  is the rate of return on a bond with maturity  $n$  and expectation terms denote the expectations of future one-year short-term rates on an investment starting in  $t + i$ ,  $i = 0, 1, \dots, n$  periods from now.

The pure expectations theory states that forward rates reflect expected future interest rates:

$$f_t^{t+i,t+i+m} = E_t r_{t+i}^m, \tag{2}$$

where  $f_t^{t+i,t+i+m}$  stands for the forward rate determined today for a contract starting in  $t + i$  and ending in  $t + i + m$  and  $r_{t+i}^m$  is the future spot rate for a contract starting in  $t + i$  which lasts for  $m$  periods. Equation (2) implies forward rates to be unbiased predictors of future spot rates. Another characteristic feature of the pure EH is that bonds of different maturities are treated as perfect substitutes. Therefore, an investor with five-year investment horizon will be indifferent between buying a bond with the maturity of five years or rolling over five one-year bonds. From equation (1), implicit expectations of market participants about future expected short-term rates can be derived as:

$$E_t r_{t+2}^{2,3} = \frac{(1+r_t^3)^3}{(1+r_t^{0,1}) \cdot (1+E_t r_{t+1}^{1,2})}, \tag{3}$$

$$f_t^{t+i,t+i+1} = \frac{(1+r_t^{i+1})^{i+1}}{(1+r_t^i)^i}. \tag{4}$$

horizons or habitats. If supply and demand for a given maturity range do not match, a risk premium is required to induce market participants to buy bonds outside their maturity preference or habitat. All three versions of the expectations theory were subjects to intensive testing. In order to enlighten these testing procedures, we first state the pure expectations theory in mathematical terms. The pure expectations hypothesis asserts that long-term spot rates are equal to the geometric mean of current and expected future short-term rates. The slope of the term structure then reflects current expectations of market participants regarding future short-term rates. If short-term rates are expected to rise, the yield curve will have a positive slope. The pure expectations hypothesis (EH) can be stated as:

In the empirical literature, a linearized version<sup>2</sup> of formula (1) is widely applied:

$$r_t^n = \frac{1}{n} \sum_{i=0}^{n-1} E_t r_{t+i}^1 + v^{n,1}. \tag{5}$$

Equation (5) states that the  $n$ -period interest rate is explained by the simple average of the current and future expected one-period<sup>3</sup> rates plus a constant risk or term premium. Equation (5) represents a weaker version of the EH. In the pure expectations hypothesis the term  $v^{n,1}$  is absent. The next step is to subtract the term  $r_t^1$  from both sides of the equation (5) and rearrange terms to receive the following:

$$r_t^n - r_t^1 = \sum_{i=1}^{n-1} \left( 1 - \frac{i}{n} \right) E_t (\Delta r_{t+i}^1) + v^n. \tag{6}$$

According to equation (6), the spread between long-term and short-term interest rates can be explained by the difference in the expected future one-period interest rates plus a term premium. As expectations of market participants are not known, a typical assumption is that expectations are formed rationally:

$$E_t r_{t+i}^1 = r_{t+i}^1 + \varphi_{t+i}. \tag{7}$$

With equation (7) a testable version of the EH is obtained:

$$r_t^n - r_t^1 = \alpha + \beta (r_{t+i}^1 - r_t^1) + \eta_t. \tag{8}$$

In this framework, the pure EH is tested by estimating equation (8) and testing the null hypothesis ac-

<sup>1</sup> This version of the expectations hypothesis was initially proposed by Modigliani and Sutch (1966).

<sup>2</sup> Under the approximation  $\ln(1+r) \approx r$ .

<sup>3</sup> This expression can also be generalized to  $m$ -period short rates.

ording to which  $\alpha = 0$  and  $\beta = 1$ . If the null hypothesis is rejected but significance of  $\beta$  is confirmed, this result is usually interpreted as evidence of forward rates having explanatory power.

For testing purposes the forward-spot spread approach is also frequently adopted. This approach is similar to equation (8), the only difference constitutes the term in brackets on the right hand side. Instead of the difference between the future short-term rates, the forward-spot spread is applied:

$$r_t^n - r_t^1 = \alpha + \beta(f_t^1 - r_t^1) + \eta_t. \tag{9}$$

According to this formulation, the spread between the long- and short-term spot rates can be explained by the forward-spot spread. Using equation (2), it can also be directly tested if forward rates can predict future spot rates:

$$r_{t+i}^m = \alpha + \beta f_t^{t+i,t+i+m} + \phi_t. \tag{10}$$

Then the parameters of the null hypothesis are  $\alpha = 0$ ,  $\beta = 1$  and  $\beta = 1$  for the pure and biased expectations theory, respectively.

Then we concentrate on testing how well forward rates can predict future short-term spot rates. Thus, as a first step we test equation (10). In addition, we check if forward rates lying farther in the past contain any explanatory power with respect to future spot rates<sup>1</sup>. In other words, if  $r_t^1$  is today's one-year spot rate then not only the forward rate one period before  $f_{t-1}^{1,2}$  might have some predictive power, but also forward rates of the preceding periods such as  $f_{t-2}^{2,3}, f_{t-3}^{3,4}$ , etc. The number of lagged forward rates was chosen to be six. Therefore, we consider six models each containing an additional lagged forward rate as a predictor of future spot rate:

$$r_t^1 = f(f_{t-1}^{1,2}, f_{t-2}^{2,3}, f_{t-3}^{3,4}, f_{t-4}^{4,5}, f_{t-5}^{5,6}, f_{t-6}^{6,7}). \tag{11}$$

The model according to formula (11) can be represented in the following way:

$$r_t^1 = \beta_0 + \beta_1 f_{t-1}^{1,2} + \beta_2 f_{t-2}^{2,3} + \beta_3 f_{t-3}^{3,4} + \beta_4 f_{t-4}^{4,5} + \beta_5 f_{t-5}^{5,6} + \beta_6 f_{t-6}^{6,7} + \xi_t. \tag{12}$$

Table 1 provides an overview of the models considered in our study, where  $r_1$  and  $f_i$  denote the one-year spot rate and a forward rate  $i$  periods before, respectively.

Table1. Forward rate models

Model	Variables included
1	$r_1, f_1$

<sup>1</sup> This model was initially proposed by Gischer (1997).

2	$r_1, f_1, f_2$
3	$r_1, f_1, f_2, f_3$
4	$r_1, f_1, f_2, f_3, f_4$
5	$r_1, f_1, f_2, f_3, f_4, f_5$
6	$r_1, f_1, f_2, f_3, f_4, f_5, f_6$

**1.2. Previous empirical findings.** First formulations of the expectations theory appeared already in the end of the 19<sup>th</sup> century. However, the theory has been fully developed only in the 30s of the past century. First tests of the expectations theory date back to the 70s and were conducted using US data employing simple linear regressions. Since that a great variety of tests were performed which tested different implications of the theory, and applied different methods and maturities. A great majority of these studies, however, concentrates on US data. The literature on the term structure can be divided into those who use the term spread and those who apply the forward spot rate for testing the expectations theory.

Early studies for the US undoubtedly reject the EH and find poor explanatory power of forward rates as well as term spreads. Among the authors are Hamburger and Platt (1976), Fama (1976), Shiller, Campbell and Schoenholz (1982), Mankiw and Summers (1984). The results of Fama (1984) for the period from 1959 to 1982, although reject the EH, suggest some predictive power of the forward-spot spread. Later, Mankiw and Miron (1986) use US data from 1890 to 1979 to test whether the slope of the yield curve can predict changes in the spot rates. Whereas their study documents little predictive power of the spread for the period after 1915, the EH proves to be consistent with the data before 1915. They attribute poor performance of the expectations theory after 1915 to the increased role of the Federal Reserve System.

Fama and Bliss (1987) analyze the information content of long-term forward rates from 1964 to 1985 and conclude that they exhibit significant predictive power especially for longer forecasting horizons which, according to the authors, can be explained by mean reversion of spot rates. In contrast, Jorion and Mishkin (1991) also use the forward-spot spread approach for the period from 1973 to 1989. However, they conclude that the information content of the spread is poor and report significant predictive ability only at the five-year horizon. Campbell and Shiller (1991) adopt the vector autoregression (VAR) approach to test the EH with the yields on US treasury bills for a variety of maturities. They assert that the term spread only has significant forecasting ability with respect to changes in short-term spot rates, but not in long-term spot rates.

Despite that fact that some of the above studies confirm that some explanatory power in forward rates or short-long spreads is present, they in general statistically reject the restrictions of the EH. Tests of the expectations hypothesis considering European data, although limited to only few studies, are in general more supportive to the expectations theory of the term structure.

For Germany, there have been only a few studies. Hardouvelis (1994) analyzed the ability of the term spread to predict changes in both long-term and short-term rates for a variety of countries including Germany, Italy, France, the USA, Canada, and Japan for 1953 to 1992. Although the pure EH is rejected, he reports significant coefficients for the short-term spot rate model for all countries with the exception of Germany and the USA. However, when regressing the change in the long-term spot rate on the term spread, his study finds little forecasting ability and documents negative slope coefficients for all countries except of Italy and France thereby supporting the “sign puzzle” received in previous studies. The negative sign, however, disappears if instrumental variables are introduced for all countries with the exception of the US.

Gerlach and Smets (1997) test the predictive power of the term spread with respect to changes in short-term spot rates for 17 countries including Germany. Their study provides quite striking results which are considerably in favor of the pure EH. In almost 70 percent of all regressions the null hypothesis that the beta coefficient equals one cannot be rejected. Moreover, in 50 percent of all cases even the joint hypothesis  $\alpha = 0$ ,  $\beta = 1$  cannot be rejected. This is by far the most supportive result for the pure EH.

In contrast, the study of Jondeau and Ricart (1999), who applied both term spread and forward-spot spread approach to German, French, UK and US data, could not provide such a strong support of the theory. In general, their study for 1975 to 1997 rejected the EH for Germany and the US. Moreover, in the regression of forecasting changes in the long-term spot rate, negative slope coefficients were obtained for both countries. In contrast, the EH is generally supported by French and UK data as  $\beta = 1$  could not be rejected.

Boero and Torricelli (2002) use estimated German term structure data for the period from 1983 to 1994 to test the models specified by equations (5) and (6). They report that the long-short spread as well as the forward-spot spread are good predictors for the future short-term spot rates. In contrast, long-short spreads show little forecasting power with

respect to future changes in long-term spot rates. The latter result is consistent with previous findings for the US. However, although the information content is poor, in German data at least the direction of changes in long-term spot rates can be predicted.

The study of Dominguez and Novales (2002) is of particular interest for our study as they examine the ability of forward rates to predict future spot rates for a variety of interest rates using data in levels and not the spreads. They analyze the data set ranging from European to US and Japanese interest rates for the period from 1978 to 1998 and present evidence that forward rates can explain future spot rates to a significant extent. Moreover, in their study even unbiasedness of forward rates cannot be rejected.

As described above, in general the pure EH as well as its weaker versions were not confirmed by empirical investigations. This result is especially pronounced for US and UK data. As for Europe, evidence is rather inconclusive providing, however, more support for the EH.

The usage of different data sets for different countries and maturities has resulted in a variety of contradicting findings. As strong evidence supporting the EH could not be found, this gave strike to further research. Many authors address one difficulty connected with the interpretation of test results, namely, the necessity to assume some particular expectation formation process. Consequently, either the expectations theory does not hold or the expectations are formed in a different way. Some further hypotheses suggest to explain a general failure of the expectations theory include the overreaction hypothesis, i.e., that long-term spot rates over- or underreact regarding the expectations of future short-term spot rates. Mankiw and Miron (1986) suggest that the inability of the expectations theory to reliably predict future spot rates could be due to the existence of a time-varying term premium. However, with numerous tests and contradictory results, the question that most of the studies seek to answer is whether the term structure can be used to predict future spot rates.

The goal of our paper is not to once more test the EH but to examine whether information helping to predict future interest rates can be extracted from forward rates. Regarding the above mentioned difficulties, in our paper we only test the expectations theory in an indirect way by determining whether forward rates from past periods, which reflect expectations of market participants in the respective periods, can be used to predict future short-term spot rates.

## 2. Econometric methodology

The choice of an appropriate econometric procedure strongly depends on data properties. As the empirical literature mainly applies spreads to test the EH, the problem of non-stationary data was not so pronounced. If spreads are employed, standard regression could be applied for estimating regression coefficients. For our study, which uses data in levels and not the spreads, it is important to examine time series properties of the data before deciding on the most appropriate econometric method. Therefore, subsection 2.1 is devoted to data description and their main characteristics. After that we outline the econometric techniques adopted in our study in subsection 2.2.

**2.1. Preliminary data analysis.** The data set employed in our study consists of monthly swap rates for maturities between one and six years over the period from 1978 to 2006. The period of the financial crisis that occurred in 2007 deliberately is not a part of our analysis for several reasons. At first, including this period could possibly lead to a structural break in the data, which would affect the time series analysis and would require a different examination technique. Additionally, significant credit spreads

were observed in Germany during the financial crisis. Compared to only a few basis points in the pre-crisis period, credit spreads of more than 100 basis points were observed during the crisis. Significant credit spreads would bias our empirical results. Finally, as a result of severe liquidity problems, the German swap market experienced a dramatic breakdown during the crisis, so that the German Central Bank had to act as the interbank market in this time. Under such circumstances, the assumption of an efficient swap market is not valid for the crisis period. As the main purpose of our study was to examine a well-functioning fixed-income market, we restrict our analysis to the pre-crisis period.

Table 2 gives an overview of the basic data characteristics. Already at the first glance it is apparent that forward rates systematically overestimate future spot rates and this effect increases with the lag of forward rates. The difference between lagged forward rates and realized spot rates ranges from 0.5 percent for  $f_1$  to almost 2.5 percent for  $f_6$ . Thus, it seems doubtful that forward rates can serve as good predictors of future spot rates. Standard deviation of lagged forward rates, however, lies considerably under that of the spot rate and decreases with an increasing lag of forward rates.

Table 2. Descriptive statistics, sample of 1978-2006

Variable	Mean	Median	Max	Min	SD	Skewness	Kurtosis	JB
$r_1$	5.627	4.950	13.949	2.008	2.546	0.790	3.015	36.316
$f_1$	6.116	5.552	12.499	2.238	2.308	0.428	2.248	18.870
$f_2$	6.589	6.352	12.114	2.646	1.979	0.168	2.422	6.500
$f_3$	6.878	6.875	11.810	2.967	1.793	0.001	2.567	2.730
$f_4$	7.500	7.507	11.588	4.061	1.616	0.026	2.466	4.184
$f_5$	7.835	7.801	12.047	4.352	1.526	0.275	2.833	4.816
$f_6$	8.071	8.034	12.372	4.635	1.422	0.288	3.161	5.199

It is worth noting that real data for monthly swap rates is only available starting from November 1994. Before that point in time swap rates were estimated using linear interpolation. Because of such a long and mixed data set, we firstly conduct a breakpoint test to identify possible structural breaks. This is essential as testing for unit roots in a data set contaminated by structural breaks can result in misleading conclusions, for example that the data is non-stationary although it is in fact stationary but structural breaks are present<sup>1</sup>.

As reported in Table 3, the results of Quandt-Andrews tests, which does not require a specification of the breakpoint date, indicate multiple breakpoints in the data in 1983, 1992, 1993, and 1995.

Table 3. Quandt-Andrews breakpoint tests, sample of 1978-2006  
( $H_0$ : No structural breaks within data)

Model	Critical value	Test value	Date
$r_1, f_1$	17.5	45.31*	1983M03
$r_1, f_1, f_2$	28.6	65.08*	1995M04
$r_1, f_1, f_2, f_3$	22.7	51.29*	1993M02
$r_1, f_1, f_2, f_3, f_4$	22.1	48.77*	1983M07
$r_1, f_1, f_2, f_3, f_4, f_5$	18.5	43.61*	1992M10
$r_1, f_1, f_2, f_3, f_4, f_5, f_6$	15.2	36.89*	1992M10

Note: \*Indicates rejection of the hypothesis at the one percent significance level.

Although we expected the test to find a break in the period from 1991 to 1992, i.e., when the German reunification took place, two more breaks were identified, namely, in 1983 and 1995 in which there were no regime changes or other reforms that could cause such a break. The last break occurred in April 1995 and for this year the value of the test statistics

<sup>1</sup> Perron (1989) was the first to investigate the implications for unit roots in the presence of structural breaks. He could reject the null hypothesis of a unit root in a majority of cases if structural breaks are considered.

is also the largest. Consequently, we chose a sample starting in May 1995 and ending in October 2006 for our analysis. This subset is of special interest for us as it is free of structural breaks. An additional motivation for this choice is the fact that the sam-

ple from 1995 to 2006 is composed of real data. Although we possess data up to October 2007, we do not consider data for the last 12 months in our analysis to evaluate an out-of-sample performance of our model. Series plots are shown in Figure 1.

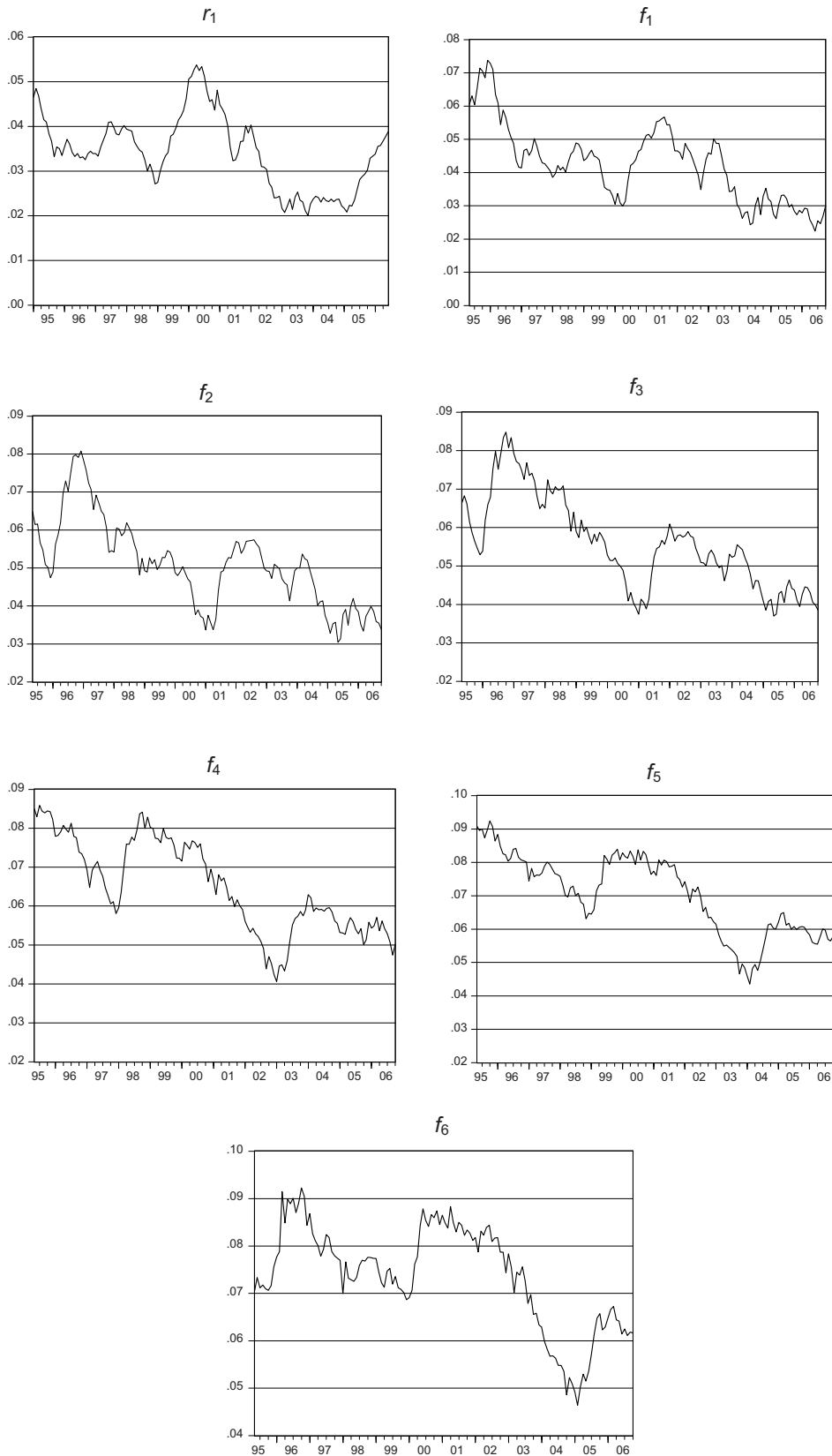


Fig. 1. One-year spot and lagged forward interest rates, sample of 1995-2006

The issue of stationarity has received great attention in the past two decades. A stationary process is characterized by mean, variance, and autocovariance which are time-independent. Formally, time series  $y_t$  is (weakly) stationary if the following conditions hold:

$$E(Y_t) = \mu_Y \text{ for all } t,$$

$$\text{Var}(Y_t) = \sigma_Y^2 \text{ for all } t,$$

$$\text{Cov}(Y_t, Y_{t+k}) = \gamma_k \text{ for all } t \text{ and } k,$$

where  $\mu_Y$ ,  $\sigma_Y^2$  and  $\gamma_k$  are constants. In general, ordinary least squares (OLS) estimation is only justified when the data exhibit constant mean and variance. Non-stationary time series or time series having a unit root, however, are characterized by means and variances that are time-dependent. Thus, false inferences from conventional statistics could be drawn when OLS techniques are employed. Estimating non-stationary time series with OLS leads to meaningless results or “spurious” regression<sup>1</sup>. Conventional tests statistics cannot be applied as their asymptotic distributions are non-standard under non-stationarity.

There is an ongoing discussion on the time series properties of interest rates. Using the Dickey-Fuller<sup>2</sup> (DF) test has been standard practice to test for the presence of a unit root in the empirical literature which generally resulted in the inability to reject the null hypothesis that interest rates are non-stationary time series. Suppose that the time series is represented by a first-order autoregressive process, or AR(1):

$$r_t = \mu + \rho r_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2). \quad (13)$$

The DF test, which aims at checking whether  $\rho < 1$ , proceeds in the following way:

$$\Delta r_t = \mu + (\rho - 1)r_{t-1} + \varepsilon_t = \mu + \mathcal{G}r_{t-1} + \varepsilon_t, \quad (14)$$

where  $\mathcal{G} = \rho - 1$ . The null hypothesis of the DF test is  $\mathcal{G} = 0$ , i.e., the time series has a unit root. Equation (14) is estimated using OLS and the obtained  $t$ -values are compared with the critical values reported by Dickey and Fuller. Of course, the autoregressive process can also be of a higher order. In this case, the DF

test is augmented by additional lags and is referred to the Augmented Dickey-Fuller (ADF) test:

$$\Delta r_t = \mu + \mathcal{G}r_{t-1} + \sum_{i=1}^{\rho} \varphi_i \Delta r_{t-i} + \varepsilon_t. \quad (15)$$

The number of lags in equation (15) has to be selected appropriately in order to capture the nature of the process, but at the same time also not to include too many lags as this would lead to the loss of degrees of freedom. Commonly, the Akaike or the Schwarz information criterion is used to select the number of lags. Other popular tests are the Philips-Perron<sup>3</sup> (PP) test and the Dickey-Fuller GLS test which both represent a modified DF test. If, according to the above stationarity tests, a time series has to be differenced  $d$  times in order to become stationary, it is said to be integrated of order  $d$  denoted  $I(d)$ . If a series turns out to be stationary and does not require differencing, it is referred to as integrated of order zero  $I(0)$ .

It was common to consider interest rate data as well as many other macroeconomic data, to be an  $I(1)$  process<sup>4</sup> based on the ADF test, i.e., data become stationary after first-differencing. However, some authors question non-stationarity of interest rates<sup>5</sup>. Standard techniques, such as ADF or PP tests, employed to detect the presence of a unit root have been criticized because of their low power, i.e., their inability to distinguish between a pure unit root and near unit root data generating process<sup>6</sup>. A shortcoming of the ADF and PP as well as DF GLS tests is that they are based on a unit root assumption and thus, unless there is very strong evidence against the null hypothesis, it tends to be accepted.

A new procedure for testing for stationarity was suggested by Kwiatkowski, Philips, Schmidt and Shean (1992). The unit root test, which is known as the KPSS test, is considered to be more powerful as its null hypothesis is a stationary process instead of a unit root process. This test is, therefore, less likely to reject stationarity. In order to obtain as accurate results as possible concerning the time series properties of our data, we employ several unit root tests including those whose null hypothesis is the absence of a unit root. The results are reported in Table 4.

Table 4. Unit root tests, sample of 05/1995-10/2006

	Level				First difference			
	ADF	PP	DF-GLS	KPSS	ADF	PP	DF-GLS	KPSS
$r_t$	-2.22	-2.09	-1.33	0.540**	-5.79*	-9.42*	-3.93*	0.142
$f_t$	-2.05	-1.87	-0.77	0.884*	-9.84*	-9.87*	-3.54*	0.048

<sup>1</sup> See Granger and Newbold (1974; 1977).

<sup>2</sup> See Dickey and Fuller (1979).

<sup>3</sup> See Philips and Perron (1988).

<sup>4</sup> See, among others, Campbell and Shiller (1987), Hall et al. (1992), Shea (1992), Zhang (1993), Tabak (2009).

<sup>5</sup> See Wu and Chen (1996), Wu and Chen (2000), Beechey et al. (2009).

<sup>6</sup> Lanne (2000) and Beechey et al. (2009) suggest an alternative technique to test for stationarity in case of near-integrated processes.

Table 4 (cont.). Unit root tests, sample of 05/1995-10/2006

	Level				First difference			
	ADF	PP	DF-GLS	KPSS	ADF	PP	DF-GLS	KPSS
$f_2$	-1.33	-1.77	-0.47	0.919*	-9.72*	-9.82*	-2.48**	0.037
$f_3$	-1.13	-1.40	-0.57	1.015*	-10.78*	-10.89*	-4.05*	0.050
$f_4$	-1.48	-1.60	-0.06	1.083*	-11.28*	-5.88*	-11.33*	0.060
$f_5$	-1.56	-1.60	-0.10	1.050*	-12.35*	-12.33*	-11.49*	0.082
$f_6$	-1.36	-1.39	-1.45	0.730**	-14.12*	-13.86	-2.47**	0.131

Notes: \*, \*\* and \*\*\* denote the rejection of the null hypothesis at the 1%, 5% and 10% level, respectively. The null hypothesis of the KPSS test is a stationary process. All other tests assume a unit root process. The information criterion used in DF tests is that of Schwarz, as the usage of the Akaike criterion resulted in a higher number of lags. KPSS test with Bartel kernel was applied.

According to Table 4, unit root tests deliver quite uniform results in our sample. When variables are expressed in levels, the null hypothesis of a unit root process fails to be rejected by all tests for all time series. The most interesting is comparison between the traditional unit root tests whose null hypothesis is presence of a unit root from one side and the KPSS test from the other side. However, we observe that even this more powerful test rejects the null hypothesis of a stationary process at least at the five percent significance level. In case of  $f_1, f_2, f_3, f_4,$  and  $f_5$  the null hypothesis is rejected even at the one percent significance level. Therefore, we find more evidence that interest rates are in fact the best described by a unit root process.

Regarding to the order of integration of our data, when interest rates are expressed in first differences, ADF and PP test both reject the null hypothesis of a unit root process at the one percent significance level. The DF GLS test supports this result at the one percent significance level for all cases excluding  $f_2$  and  $f_6$  where rejection is at the five percent significance level. The KPSS test reinforces the conclusion obtained from the previous tests. It fails to reject the null hypothesis of a stationary process. Thus, we consider all our time series to be a first-difference stationary process<sup>1</sup>.

**2.2. Cointegration and the error-correction model.**

The ordinary least squares technique is not appropriate in case of integrated time series. One possible approach in case of an  $I(1)$  process is to take series in first differences. Suppose we want to estimate the following equation:

$$r_t = \beta_0 + \beta_1 f_{t-1} + \phi_t \tag{16}$$

If  $r_t$  and  $f_t$  are  $I(1)$  series, then one can estimate:

$$\Delta r_t = a_0 + a_1 \Delta f_t + \phi_t \tag{17}$$

to achieve stationarity. However, this procedure is not very popular as critics of this approach point out that the “long-run” information is ignored<sup>2</sup>. A possible solution to this problem would be to incorporate some kind of long-run information into the above formula and estimate the following expression:

$$\Delta r_t = \delta_0 + \delta_1 (r_{t-1} - \hat{\beta}_1 f_{t-1} - \hat{\beta}_0) + a_1 \Delta f_t + \phi_t \tag{18}$$

Equation (18) incorporates long-term as well as short-term parameters and is referred to as an error correction model (ECM). The term in brackets,  $r_{t-1} - \hat{\beta}_1 f_{t-1} - \hat{\beta}_0$ , is an error correction term whereas the coefficient  $\delta_1$  measures the speed of adjustment to correct for this error. A shortcoming of this approach is that, although two difference terms are stationary, the error correction term is a linear combination of non-stationary variables and is, therefore, also non-stationary.

One way to solve this problem is to apply the concept of cointegration which was introduced by Granger (1987). If two time series are  $I(1)$  but a linear combination of them can be found that is stationary, then these series are said to be cointegrated. This result can also be extended for the case of more than two series and higher order of integration. If  $X_t$  is a vector of  $I(d)$  variables and a vector  $\alpha \neq 0$  exists such that linear combination  $\alpha'X_t \sim I(d-b)$ ,  $b > 0$ , then the components of the vector  $X_t$  are cointegrated of order  $d, b$  denoted  $CI(d, b)$ <sup>3</sup>. The vector  $\alpha$  is called the cointegrating vector,  $\alpha'X_t$  is a vector of error correction terms.

Stock (1987) demonstrated that if variables are cointegrated, the OLS estimator of the cointegrating vector,  $\hat{\alpha}$ , will be “super consistent”, i.e., it will converge to the true parameter value at a faster rate than the OLS estimator of a regression involving

<sup>1</sup> We also analyzed the whole sample of 1978-2007 for stationarity and found similar results.

<sup>2</sup> For further details, see Davidson et al. (1978).

<sup>3</sup> Flores and Szafarz (1996) provide an enlarged definition of cointegration and show that cointegration may also arise among the series with different order of integration.

stationary variables. However, its distribution will be non-standard and therefore conventional statistical inference cannot be applied. One crucial implication of cointegration, known as the Granger representation theorem<sup>1</sup>, is that if the series are  $CI(1,1)$  an ECM will be a valid representation of the data. The error correction representation is appealing because it only contains stationary variables as the term in brackets in equation (18) will be stationary under cointegration.

Currently there are several tests available to detect cointegration. In case only two time series are to be estimated, the Engle-Granger (1987) two-step estimation procedure can be applied. As a first step, equation (16) is estimated using OLS and the error term is calculated:

$$e_t = r_t - \hat{\beta}_0 - \hat{\beta}_1 f_{t-1}. \quad (19)$$

Subsequently, the error term from the regression is tested for stationarity:

$$e_t = \rho e_{t-1} + v_t. \quad (20)$$

In case of a stationary error term the series are said to be cointegrated. The Engle-Granger two-step procedure, applied to a multivariate case, can no longer guarantee the uniqueness of the estimated cointegrating vector as there can exist  $n - 1$  linear relationships in case of  $n$  variables involved.

Jonansen (1988, 1991) and Johansen and Juselius (1990) developed a technique that enables detection and estimation of multiple cointegrating vectors. This procedure is frequently applied to test for cointegration. At first, a vector autoregression of order  $k$  is estimated:

$$X_t = \Pi_1 X_{t-1} + \dots + \Pi_k X_{t-k} + \varepsilon_t, \quad (21)$$

where  $X_t$  is a  $p$ -dimensional vector of  $I(1)$  series,  $\Pi_i$ ,  $i = 1 \dots k$  is a  $p \times p$  matrix of coefficients and  $\varepsilon_t$  is a  $p \times 1$  vector of error terms that are independently identically distributed (i.i.d.). The above expression can also be represented in the error correction form:

$$\Delta X_t = \Gamma_i \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + \varepsilon_t, \quad (22)$$

$$\Gamma_i = -I + \Pi_1 + \dots + \Pi_i,$$

where

$$\Pi = -I + \Pi_1 + \dots + \Pi_k \quad (23)$$

and  $I$  denotes the identity matrix. In equation (22),  $\Pi$  is the only term that is expressed in levels.

Therefore, tests for cointegration focus on determining whether matrix  $\Pi = \beta\alpha'$  has a reduced rank, which can be at most equal to  $p - 1$ .  $\alpha'$  denotes the  $p \times r$  matrix of cointegrating vectors whereas  $\beta$  is the  $p \times r$  matrix of adjustment coefficients. The maximum likelihood estimator, which was developed by Johansen (1988), is applied to calculate the eigenvalues of  $\Pi$ . In the following, two types of tests are carried out to test for the number of cointegrating vectors or, in other words, for the number of the long-run relationship among the series. The maximum eigenvalue test, which is used to test the null hypothesis of  $r$  cointegrating vectors against the alternative of that their number is  $r + 1$ , is based on the following test statistics:

$$\lambda_{\max} = -T \ln(1 - \lambda_{r+1}), \quad (24)$$

where  $T$  is the number of observations and  $\lambda_r$  is an eigenvalue associated with the cointegrating vector  $r$ . The second cointegration test, the trace test, is based on the following:

$$\lambda_{\text{trace}} = -T \sum_{i=r+1}^k \ln(1 - \lambda_i). \quad (25)$$

The null hypothesis of the trace test is that there are at most  $r$  cointegrating vectors with the alternative hypothesis being the existence of more than  $r$  cointegrating vectors. Critical values obtained by Johansen and Juselius (1990) should be applied to assess the results of these tests. If in a system of  $p$  series  $r$  cointegrating vectors are found, the system is said to be driven by  $p - r$  common trends.

Due to its theoretical appeal, the cointegration framework has been applied in several term structure papers. Some early studies in the beginning of the 90s have focused on determining whether US term structure components are cointegrated. Hall, Anderson and Granger (1992) use treasury bill data for the period from 1970 to 1988 with one to 11 months maturity to test for the presence of cointegration among all 11 yields as well as pairwise between different yields. According to the authors, for the EH to hold there should be  $n - 1$  cointegrating vectors for a set of  $n$  series. They find that 11 interest rates have ten cointegrating vectors, i.e., the EH holds. In contrast, Zhang (1993) examines 19 interest rate series dating from 1964 to 1986 and documents the existence of 16 cointegrating vectors, or three common trends, in the US term structure.

The findings of Engsted and Tanggaard (1994), who analyze the US term structure using a data set ranging from 1952 to 1987, also indicate one common trend in the data. Dominguez and Novales (2002)

<sup>1</sup> See Granger (1983), Engle and Granger (1987).

apply cointegration analysis to estimate interest rates with one, three and six month maturity depending on one lagged forward rate. Their sample includes the US, British, Japanese, Spanish, French, Italian, Swiss and German data for the period from 1978 to 1998. With several exceptions, they found cointegration between the pairs of interest rates and conclude that forward rates can serve as unbiased predictors of future spot rates. In addition, they find that lagged forward rates can predict future spot rates better than the univariate autoregressive model. Finally, Tabak (2009) uses Brazilian swap rates for one, three, six and 12 months ranging from 1995 to 2006 to test the expectations hypothesis using term spreads. Although cointegration is present, the study rejects the pure form of the EH. Evidence for European countries using cointegration analysis is quite scarce. We intend to close this gap providing the cointegration analysis for the German term structure.

### 3. Empirical results

In this Section we present the results we obtained by estimating six models employing cointegration technique and the error correction model. We also present forecasts produced by the models for the period from November 2006 to October 2007. Fore-

casting performance is then evaluated with the help of mean absolute error (MAE), root mean square error (RMSE) and Theil's inequality coefficient.

**3.1. Cointegration analysis.** For the first model, which involves only the one-year spot rate and one lagged forward rate, we employ the Engle-Granger two-step procedure as well as Johansen approach. However, for the remaining models under consideration we employ Johansen cointegration tests as this procedure allows to identify all relevant cointegrating vectors. An important issue when using Johansen cointegration tests is the choice of the number of lags in VAR. If this number is too low, the model is specified incorrectly. From the other side, if there are too many lags, this leads to the loss of degrees of freedom. The optimal number of lags in our study is selected on the basis of various information criteria, such as Akaike (AIC), Schwarz (SIC), Final Prediction Error (FPE), sequential modified LR test statistic, and Hannan-Quinn criterion(HQ)<sup>1</sup>. Table 5 reports the results of cointegration tests for our six models<sup>2</sup>.

It was also of interest to test for cointegration in pairs, i.e., including the spot rate and each lagged forward rate. These results are shown in Table 6.

Table 5. Johansen cointegration tests, models 1 through 6, sample of 05/1995-10/2006

Model	Hypothesis	$\lambda_{trace}$	Critical value	n	$\lambda_{max}$	Critical value
1	$r=0$	31.56*	25.08	2	24.48*	20.16
	$r \geq 1$	7.08	12.76		7.08	12.76
2	$r=0$	48.61**	35.19	1	31.34**	22.30
	$r \geq 1$	17.267	20.26		12.98	15.89
3	$r=0$	50.48	54.08	2	29.00**	28.58
	$r \geq 1$	21.48	35.19		12.082	22.30
4	$r=0$	79.78**	76.97	1	38.66**	34.80
	$r \geq 1$	41.121	54.08		19.15	28.59
5	$r=0$	106.94**	103.84	1	45.69**	40.96
	$r \geq 1$	61.24	76.97		21.18	34.81
6	$r=0$	144.81**	134.68	1	47.07	47.08
	$r \geq 1$	97.74	103.85		37.00	40.96

Notes: \*, \*\* and \*\*\* denotes the rejection of the null hypothesis at the 1%, 5% and 10% level, respectively;  $n$  denotes the number of lags in VAR in levels.

Table 6. Johansen cointegration tests between  $r_1$  and  $f_i$ , sample of 05/1995-10/2006

Variables	Hypothesis	$\lambda_{trace}$	Critical value	n	$\lambda_{max}$	Critical value
$r_1, f_1$	$r=0$	31.79*	25.08 <sup>a</sup>	2	27.64*	20.16 <sup>a</sup>
$r_1, f_2$	$r=0$	9.07	17.98 <sup>b</sup>	4	5.31	13.91 <sup>b</sup>

<sup>1</sup> AIC, SIC, FPE and HQ produce the same results regarding the number of lags on VAR. In contrast, the LR test statistic indicates significantly larger number of lags. However, our results are insensitive to inclusion of more lags, i.e., cointegration is still found at the 5% level. Thus, we consider Johansen's tests to be an appropriate procedure in our case.

<sup>2</sup> All tests were conducted under no deterministic trend assumption. It is worth mentioning that the results of cointegration tests are not affected by introducing the trend assumption.

Table 6 (cont.). Johansen cointegration tests between  $r_1$  and  $f_i$ , sample of 05/1995-10/2006

Variables	Hypothesis					
		$\lambda_{trace}$	Critical value	n	$\lambda_{max}$	Critical value
$r_1, f_3$	$r = 0$	8.11	17.98 <sup>b</sup>	4	5.32	13.91 <sup>b</sup>
$r_1, f_4$	$r = 0$	8.06	17.98 <sup>b</sup>	2	5.90	13.91 <sup>b</sup>
$r_1, f_5$	$r = 0$	8.40	17.98 <sup>b</sup>	2	5.52	13.91 <sup>b</sup>
$r_1, f_6$	$r = 0$	17.56	17.98 <sup>b</sup>	3	13.85	13.91 <sup>b</sup>

Notes: \* denotes the rejection of the null hypothesis at the 1% level; n denotes the number of lags in VAR in levels; a and b are 1% and 10% critical values, respectively.

From Table 5 one can observe that cointegration for the pairs of spot and lagged forward rates only holds in the first case, for  $r_1$  and  $f_1$ <sup>1</sup>. For all other pairs both cointegration tests are not able to reject the hypothesis that variables are not cointegrated even at the ten percent level. There is some ambiguity for the last case,  $r_1$  and  $f_6$ , where both tests are (just) not able to reject the null hypothesis. The situation is completely different for the models considered in Table 5, where both tests reject the null hypothesis of no cointegration in case of models 2, 4, and 5 at the five percent significance level. For model 1 the null hypothesis is rejected even at the one percent significance level.

However, the results are not so straightforward for model 3 and 6. Whereas the maximum eigenvalue test rejects the null hypothesis of no cointegration at the five percent level, the trace statistics is not able to verify this result. The situation is reversed for model 6. The trace test indicates cointegration at the five percent level while the maximum eigenvalue test is (just) not able to reject the null. For all models the number of cointegrating vectors is found to be one<sup>2</sup>. Thus, we have found that there is a long-run relationship among spot rates and lagged forward rates. In this case it could be possible to use this information for constructing forecasts. Table 7 shows the normalized cointegrating coefficients.

Table 7. Parameter estimates for models 1 to 6, sample of 05/1995-10/2006

Model	Parameter estimates						
	Constant	$\beta$					
1	-0.015 (0.00979) [-1.55]	1.2316* (0.2241) [5.49]					
2	-0.0109 (0.01393) [0.78]	1.662* (0.261) [6.36]	-0.476*** (0.265) [-1.79]				
3	-0.007 (0.014) [-0.05]	1.645* (0.268) [6.12]	-0.554 (1.004) [-0.53]	0.0134 (1.008) [-0.013]			
4	-0.0009 (0.0027) [-0.034]	3.050* (0.459) [6.64]	-3.353*** (1.829) [-1.83]	2.95* (1.817) [1.62]	-1.338** (0.486) [2.76]		
5	-0.0173 (0.031) [-0.55]	-3.99* (0.558) [-7.16]	2.25 (1.978) [1.13]	-2.20 (1.958) [-1.12]	1.204*** (0.607) [1.98]	2.099* (0.65) [3.22]	
6	-0.0211 (0.012) [-1.79]	-1.441* (0.186) [-7.76]	-0.024 (0.633) [-0.04]	-0.189 (0.618) [-0.30]	0.393** (0.194) [2.03]	0.84* (0.217) [3.686]	0.579* (0.203) [2.85]

Notes: Numbers in parentheses stand for the maximum likelihood standard errors; numbers in square brackets are t-statistics.

The results from maximum likelihood estimation of parameters, as opposed to the conclusions drawn from the cointegration tests, are not so promising.

One can infer from Table 7 that only  $f_1$ , i.e., the forward rate determined one period before, is highly significant in all six models and therefore exhibits explanatory power with respect to the future one-year spot rate. However, it has a positive sign only for models 1 through 4. In models 5 and 6 the wrong di-

<sup>1</sup> When the Engle-Granger two-step procedure is applied to check if  $r_1$  and  $f_i$  are cointegrated, the KPSS test is not able to reject the null hypothesis of a stationary process. DF-GLS confirms this result, rejecting the null of a unit root process at the 5% level. In contrast, the out-come of the ADF and the PP-test is a non-stationary error term. As Johansen approach indicates cointegration at the 1% level, we consider these results to be an evidence of poor performance of the ADF and the PP test.

<sup>2</sup> We also performed cointegration tests (not reported in this paper) using the full data set available for 1978-2006, and could confirm cointegration also in this sample. The number of cointegrating vectors is, however, more than just one for several models.

rection is predicted by the lagged forward rate. Forward rates which prevailed two and three periods before,  $f_2$  and  $f_3$ , are insignificant in most of the considered models and therefore cannot contribute to the prediction of the future spot rate. Up to model 5,  $f_2$  also has a negative sign. Although  $f_3$  has a positive sign in models 3 and 4, it becomes negative starting from model 5.

Whereas forward rates seem to have no forecasting ability with respect to the future spot interest rate, forward rates lying farther in the past might be more useful in explaining the spot rate. In models 4, 5 and 6,  $f_4$  is significant. In models 5 and 6,  $f_5$  is significant at the one percent level whereas  $f_6$  also seems to have some predictive power in model 6 at the one percent level. It is worth noting that both  $f_5$  and  $f_6$  have positive signs. To summarize, for models 4, 5 and 6 the first and the last forward rates are significantly different from zero at least at the five percent significance level. Thus, we found evidence that forward rates one period before as well as forward rates that are lying five and six periods before contain some explanatory power regarding the one-year spot rate, although sign reversion starting from model 5 is puzzling. Forward rates lying in the “middle” do not seem to be a useful tool in forecasting spot rates. While Table 7 represents the parameter estimates for the long-run relationship

among the series, it is also interesting to investigate the short-run dynamics which is captured through the error correction model.

**3.2. Error correction model.** As cointegration was found, according to the Granger representation theorem, an error correction model is a valid representation of the data. We set up an error correction model for each of the six models. The results are reflected in Table 8, where both the estimates of cointegrating vectors and adjustment coefficients are presented. The latter are of a crucial interest as their significance indicates the validity of the error correction representation of the data. As Table 8 suggests, with the exception of the first model where the significance is at the five percent level, the adjustment parameters are significant at the one percent level. Thus, the conclusion that the respective series are cointegrated is reinforced by the significant adjustment parameters. This means that for these models the spot rate responds to the deviations from the long-run value, i.e., the ECM works. However, the adjustment coefficient  $\delta_1$  has a wrong sign in models 1 through 4. We would expect it to be negative. In case the value of  $r_1$  is above its long-run value, the change in  $r_1$  should be negative to compensate for the disequilibrium in the previous period. We, however, observe negative signs only in case of models 5 and 6. Thus, only for these two models the ECM makes sense.

Table 8. Error correction model, sample of 05/1995-10/2006

$$\Delta r_t = \delta_0 + \delta_1 \left( r_{t-1} - \sum_{i=1}^I \hat{\beta}_i f_{i,t-i} - \hat{\beta}_0 \right) + \varepsilon_t$$

Model	1	2	3	4	5	6
$f_1$	1.2316* (0.2241) [5.49]	1.662* (0.261) [6.36]	1.645* (0.268) [6.12]	3.050* (0.459) [6.64]	-3.99* (0.558) [-7.16]	-1.441* (0.186) [-7.76]
$f_2$		-0.476*** (0.265) [-1.79]	-0.554 (1.004) [-0.53]	-3.353*** (1.829) [-1.83]	2.25 (1.978) [1.13]	-0.024 (0.633) [-0.04]
$f_3$			0.0134 (1.008) [-0.013]	2.95* (1.817) [1.62]	-2.20 (1.958) [-1.12]	-0.189 (0.618) [-0.30]
$f_4$				-1.338** (0.486) [2.76]	1.204*** (0.607) [1.98]	0.393** (0.194) [2.03]
$f_5$					2.099* (0.65) [3.22]	0.84* (0.217) [3.686]
$f_6$						0.579* (0.203) [2.85]
$\beta_0$	-0.015 (0.0098) [-1.55]	-0.01093 (0.01393) [0.78]	-0.007 (0.014) [-0.05]	-0.0009 (0.027) [-0.034]	-0.0173 (0.031) [-0.55]	-0.021 (0.0118) [-1.79]
$\delta_1$	0.0291** (0.0117) [2.49]	0.034* (0.0089) [3.874]	0.026* (0.01) [2.62]	0.0219* (0.0049) [4.51]	-0.023* (0.004) [-5.85]	-0.079* (0.01) [-6.43]

Table 8 (cont.). Error correction model, sample of 05/1995-10/2006

Model	1	2	3	4	5	6
R <sup>2</sup> (adj.)	0.111	0.098	0.118	0.129	0.20	0.23
F-statistic	9.39	14.89	4.43	20.08	33.98	41.11

Notes: \*, \*\* and \*\*\* denote the rejection of the null hypothesis at the 1%, 5% and 10% level, respectively; numbers in parentheses stand for the maximum likelihood standard errors; numbers in square brackets are t-statistics.

Finally, we consider the adjusted coefficients of determination in order to examine goodness of fit of our models. The results are also not in favor of the forecast ability of forward rates. The adjusted R<sup>2</sup> of the first model only slightly exceeds 11 percent, for the second model this value is even lower, 9.8 percent. Then, starting from model 3 the adjusted R<sup>2</sup> increases gradually achieving its highest level at 23 percent for model 6. The most significant increase by 7.1 percent in the coefficient of determination occurs when we move from model 4 to model 5, i.e., the inclusion of  $f_5$  considerably improves the goodness of fit of the model.

**3.3. Predictions.** Although the results regarding the significance of coefficients and goodness of fit are

not very promising, the variables in all models are cointegrated and the error correction representation is valid for some of them. It is therefore of crucial interest, whether the fact that cointegration is present can help to improve forecasts. We use the estimated parameters for the sample from 1995 to 2006 to make forecasts for the period November 2006 to October 2007. To assess forecasting performance of our models, we compare the forecasts from the cointegration equations with the naive model which uses past period value of  $r_1$  to make a forecast. As a performance measure, mean absolute error, root mean square error and Theil's inequality coefficient  $U$  is employed:

$$MAE = \frac{1}{N} \sum_{i=1}^N |r_i - \hat{r}_i|, RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (r_i - \hat{r}_i)^2}, U = \frac{\sqrt{\frac{1}{N} \sum_{i=1}^N (r_i - \hat{r}_i)^2}}{\sqrt{\frac{1}{N} \sum_{i=1}^N r_i^2 + \frac{1}{N} \sum_{i=1}^N \hat{r}_i^2}}, \quad (26)$$

where  $r_i$  and  $\hat{r}_i$  denote the true and the forecasted values, respectively. The forecast accuracy measures are specified in Table 9. Theil's inequality coefficient lies between zero and one. For all measures, a smaller value is desirable.

Table 9. Forecasting performance, models 1 to 6 versus naive model

Model	MAE	RMSE	U
Naive	0,0104	0,0105	0,0555
$r_1, \hat{r}_1$	0,0258	0,0263	0,1851
$r_1, \hat{r}_1, \hat{r}_2$	0,0280	0,0286	0,2114
$r_1, \hat{r}_1, \hat{r}_2, \hat{r}_3$	0,0268	0,0275	0,1972
$r_1, \hat{r}_1, \hat{r}_2, \hat{r}_3, \hat{r}_4$	0,0343	0,0356	0,2959
$r_1, \hat{r}_1, \hat{r}_2, \hat{r}_3, \hat{r}_4, \hat{r}_5$	0,0113	0,0475	0,0584
$r_1, \hat{r}_1, \hat{r}_2, \hat{r}_3, \hat{r}_4, \hat{r}_5, \hat{r}_6$	0,0061	0,0073	0,0368

Out of six estimated models, only model 6, which involves all six lagged forward rates, was able to beat the naive model according to all forecast accuracy measures. Model 5 is the second-best model according to the MAE and Theil's coefficient but not to the RMSE which identifies model 1 as the second-best model. A model providing the worst forecast is model 4 according to the MAE and Theil's coefficient and model 5 according to RMSE. Thus, past forward rates exhibit rather poor predictive power with respect to the future one-year spot rate.

Models including one to four lagged forward rates have no predictive power at all whereas for the model including six forward rates there seem to be some forecasting ability as it outperforms a naive model.

## Conclusion

We have investigated the predictive power of forward interest rates lagged up to six years with respect to the future one-year spot rate using a recent data set from 1995 to 2007. In our study the cointegration framework was applied as all time series under consideration proved to be  $I(1)$ . Maximum eigenvalue and trace cointegration tests have indicated the presence of a long-run relationship and a single cointegrating vector in the considered series in all six models. This result is in line with several previous studies which document the presence of cointegration in the term structure of interest rates.

Despite the fact that the one-year spot interest rate and lagged forward rates form a cointegration relationship, we do not find reliable evidence that forward rates can be used as predictors of the future spot rates. In the majority of the considered models only the first forward rate is significantly different from zero. The six-year model seems to fit the data the best in this respect containing the largest number of significant forward rates. In general, only the

one-year and the five-year and six-year forward rates seem to fit the data to some certain extent. An error correction representation estimated for each model has a significant adjustment coefficient but the sign of this coefficient is negative only for the five-year and six-year models. In addition, the ECM has a poor fit, with the coefficient of determination ranging from ten percent for the two-year model to the maximum of 23 percent for the six-year model.

Finally, we used maximum likelihood estimates of the cointegration equation obtained with the sample

from May 1995 to October 2006 to construct a forecast for the next 12 month. Several forecast accuracy measures indicate that the one-year to five-year models are not worth the effort of estimating them as using the last period's value of one-year spot rates yields lower forecast errors. Only the six-year model performs better than the naive model. This reinforces our conclusion derived from the cointegration and error correction analysis. We found that forward rates contain very poor predictive ability and generally cannot serve as predictors of future spot rates.

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