

“Interbank Borrowing and Two-Tier Banking”

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Interbank borrowing and two-tier banking

Abstract

In this paper, an interbank market with a large number of local banks is modelled. Because of costly interbank borrowing, individual local banks choose to over-invest in liquid reserves, and there is an aggregate surplus of such reserves across the banking system. This liquidity surplus is utilized by second-tier, correspondent banks which invest it in more productive, less liquid assets. The two-tier banking system, however, is more prone to panic runs because of the increased interbank dependence and the reduced aggregate liquidity. Government intervention may therefore be more important to maintain stability of the two-tier system.

Keywords: interbank borrowing, two-tier banking system, liquidity surplus.

JEL Classification: G21.

Introduction

Historically, United States banks were restricted in one form or another form of branching across states as well as within them¹. Such branching restrictions created a banking system that consisted of a large number of local banks (first-tier banks) co-existing with a small number of national banks (second-tier, correspondent banks). If there were lessons to be learned from this experience, we must understand how such a two-tier banking system might affect credit allocation by individual banks and the financial stability of this banking system. This latter issue is particularly important since the U.S. banking system was subject to periodic banking panics prior to the enactment of the Federal Deposit Insurance Act in the 1930s². More broadly, the analysis here could also have implications for the global banking system where individual countries erect barriers to entry by foreign banks into their domestic markets. Such barriers, however, might give rise to a global liquidity surplus which might motivate the rise of global correspondent banks. To examine these important issues, I model an economy that initially consists of a large number of independent local (unit) banks. As long as there is a well-functioning interbank market having little deadweight loss, unit banking and, for that matter, branching restrictions, would have no impact on the allocation of credit by individual banks. This is also the case even if there is asymmetric information about individual banks' investment opportunities and their liquidity needs. However, bank credit allocations can be substantially affected if interbank borrowing by unit banks is to incur significant costs, for

example, as a result of monitoring by interbank lenders. In this environment, banks must individually increase their liquid reserves and the unit banking system as a whole will accumulate a surplus of such reserves. I show that this reserve surplus can give rise to second-tier banks that function as correspondent banks of unit banks by utilizing the surplus reserves. Because of the increased interbank dependence and the reduced aggregate liquidity reserves, the two-tier banking system is shown to be more prone to banking panics. A large number of papers have studied bank runs and the stability of banking systems, including Bryant (1980), Diamond and Dybvig (1983), Jacklin and Bhattacharya (1988), and Smith (1991). While this literature provided many important insights, relatively few have focused specifically on the role of interbank markets³. Bhattacharya and Gale (1987) examine interbank risk sharing and show that a competitive interbank market cannot achieve the first-best allocation because of individual banks' incentive to reduce liquidity reserves. A central bank that provides restricted interbank borrowing is shown to be able to improve welfare by imposing a regulatory reserve requirement that mitigates the under-reserve problem. In contrast, the analysis here is independent of risk sharing consideration. It is the need for interbank borrowing arising from uncertain loan demands that causes individual banks to over-invest in liquid reserves.

The rest of this paper is organized as follows. The model is set up in Section 1. In Section 2, interbank borrowing costs are shown to cause unit banks to over-invest in liquid reserves and, as a result, the system of unit banks will accumulate a liquidity surplus. In Section 3, a two-tier bank is shown to mitigate this liquidity surplus problem. Section 4 discusses the model's policy implications. The last section concludes. All proofs are contained in Appendix A.

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¹ Interstate branching restrictions were internally dismantled in the United States with the passage of the Interstate Banking and Branching Efficiency Act in 1994.

² See Friedman and Schwartz (1963) for a comprehensive survey of the U.S. banking history.

³ Exceptions are, for example, Bhattacharya and Gale (1987), Chari (1989), Smith (1991), and Allen and Gale (2000).

1. The model setup

Consider an economy with three dates, $t = 0; 1; 2$, or two time periods, between $t = 0$ and 1 and between $t = 1$ and 2. There are a large number of ex ante identical local banks (unit banks). A unit bank is small in size and operates in a local community. At $t = 0$, the bank receives aggregate deposits in the amount of one unit of the economy's sole consumption good and nothing at any other date. The economy is universally risk neutral, and since risk sharing is not the focus of this analysis, all individual deposits will mature at $t = 2$. Consistent with a competitive banking system, I also assume that a bank maximizes the welfare of its depositors, as it must provide them with best possible deals.

There are two investment technologies that are publicly available in the economy. The first is a liquid technology that will pay off one unit for every unit invested in it for one period. The second is an illiquid technology that will pay off $r_q > 1$ units for every unit invested in it for two periods. The illiquid investment will pay off only $r_u \in (0; 1)$ if it is interrupted in one period.

A bank also has access to a technology that is only privately available in its own community, possibly as a result of the bank's local know how. I introduce this private technology to capture possible opportunity costs associated with individual banks' liquidity constraint which may cause them to abandon good investments. Specifically, the private technology's payoff is as follows. For every unit a bank invests in its own private technology at $t = 0$, a proportion $\theta \in (0; 1)$ of it will turn out to be good at $t = 1$, and the rest, $1 - \theta$, will be bad.

The payoff to the good portion will depend on whether the bank upgrades this investment at $t = 1$. Upgrading one unit of the good investment will require additional $\phi > 0$ units of investment at $t = 1$, and the per unit payoff of the upgraded investment will be $r_p > r_q$ at $t = 2$. Thus, given the upgrading, the payoff at $t = 2$ to the $1 + \phi$ total good investment will be $(1 + \phi)r_p$. If any part of the good investment is not upgraded at $t = 1$, this part will also turn bad. The payoff to the bad investment will be zero.

For an individual bank, the proportion θ is a random variable. However, the aggregate realization of θ by all n banks is non-stochastic. In particular, if θ_i is bank i 's realized θ , then $\sum_{i=1}^n \theta_i = n\bar{\theta}$, where $\bar{\theta}$ is the mean of random variable θ . Although the assumption of an aggregate certainty seems restrictive, this simplifying assumption does not trivialize the problem. Indeed, in spite of the

aggregate certainty, the results show that individual banks may still choose to over-invest in the liquidity technology. In the analysis, I also assume that information about a bank's investment portfolio and its realized θ is only privately known by the bank. There is no market for any bank's investment claims at either $t = 0$ or $t = 1$.

Consider now the choice problem of a bank, say, bank i , $i \in \{1, 2, \dots, n\}$. Let $S_i \geq 0$, $Q_i \geq 0$, and $P_i \geq 0$ denote, respectively, the bank's investment at $t = 0$ in the liquid, the illiquid, and the private technology. Upon learning its realized θ_i at $t = 1$, bank i will choose a portion, $\alpha_i \in [0, \theta_i]$, of its good investment to upgrade and a portion $U_i \in [0, Q_i]$ of its illiquid investment to liquidate. In general, the early liquidation of the illiquid investment may be desirable because of possibly costly interbank borrowing. Accordingly, bank i can be a net borrower or a net lender in the interbank market. Let $B_i \in (-\infty, \infty)$ denote the bank's net borrowing from the interbank market at $t = 1$, and let γ be the required payoff at $t = 2$ per unit borrowing in the interbank market. Clearly, interbank loans must in equilibrium have $\gamma \geq 1$. Thus, the required net payment of interbank loans by bank i 's is B_i at $t = 2$, which is negative if the bank is a net lender. With a net borrowing amount of B_i , bank i may incur additional costs at $t = 2$, which is denoted by $m(B_i)$. I will interpret such costs and specify the cost function in the next section.

As usual, I first formulate bank i 's choice problem at $t = 1$, taking as given the bank's initial investment choices. Given γ , P_i , Q_i , S_i , and the bank's realized θ_i , bank i will choose α_i , U_i , and B_i to solve the following, problem (2.1), at $t = 1$:

$$\max \alpha_i(1 + \phi)P_i r_p + (Q_i - U_i)r_q - B_i\gamma - m(B_i), \quad (1)$$

$$\text{s.t. } \alpha_i\phi P_i \leq S_i + U_i r_u + B_i, \quad (2)$$

$$0 \leq \alpha_i \leq \theta_i, \quad (3)$$

$$0 \leq U_i \leq Q_i. \quad (4)$$

In problem (2.1), expression (1) is bank i 's interim payoff function, condition (2) is its interim resource constraint, and conditions (3) and (4) restrict α_i and U_i within the feasible regions.

At $t = 0$, given the optimal choices of α_i , U_i , and B_i from problem (2.1), bank i chooses P_i , Q_i , and S_i to solve the following problem (2.2):

$$\max E[\alpha_i(1 + \phi)P_i r_p + (Q_i - U_i)r_q - B_i\gamma - m(B_i)], \quad (5)$$

$$\text{s.t. } P_i + Q_i + S_i \leq 1, \quad (6)$$

$$P_i, Q_i, S_i \geq 0. \quad (7)$$

Expression (5) is bank i 's expected payoff function, condition (6) is its initial resource constraint, and condition (7) is self-explanatory.

Definition 1. An (rational expectations) interbank market equilibrium is a collection of $\gamma \geq 0$, $P_i \geq 0$, $Q_i \geq 0$, $S_i \geq 0$, $\alpha_i \in [0, \theta_i]$, $U_i \in [0, Q_i]$ and $B_i \in (-\infty, \infty)$, for all $i \in \{1, 2, \dots, n\}$, such that

1) given γ , P_i , Q_i , S_i , and θ_i , bank i chooses α_i , U_i , and B_i to solve problem (2.1);

2) given γ and the optimal α_i , U_i , and B_i derived from problem (2.1), bank i chooses P_i , Q_i , and S_i to solve problem (2.2); and

3) the interbank market clears at $t = 1$, or

$$\sum_{i=1}^n B_i = 0.$$

The first case of interest is a benchmark with costless interbank borrowing, i.e., $m(B_i) = 0$. In this case, the following result obtains.

Proposition 1. Fix $m(B_i) = 0$. For r_p sufficiently large, the unique interbank market equilibrium is, for all $i \in \{1, 2, \dots, n\}$,

$$\gamma = \bar{\theta}(1 + \phi)r_p / (1 + \bar{\theta}\phi),$$

$$P_i = 1 / (1 + \bar{\theta}\phi),$$

$$Q_i = 0,$$

$$S_i = \bar{\theta}\phi / (1 + \bar{\theta}\phi),$$

$$\alpha_i = \theta_i,$$

$$U_i = Q_i,$$

$$B_i = \theta_i\phi P_i - S_i.$$

Proposition 1 shows that with frictionless interbank borrowing, individual banks invest only in the liquid technology and their own private technology, and there is no investment in the publicly available illiquid technology. This is because the private technology is more productive than the illiquid one, while the liquid technology provides the necessary liquid funds for the interim upgrade of the private technology that turns out to be good.

Then, given its realized θ_i in the interim, bank i will choose to upgrade all the good investment, facilitated by either borrowing or lending in the interbank market, depending on its net liquidity position. In equilibrium,

$$\sum_{i=1}^n S_i = \sum_{i=1}^n \theta_i\phi P_i = n\bar{\theta}\phi P_i.$$

That is, each bank's liquidity investment equals its expected liquidity needs, i.e., $S_i = \bar{\theta}\phi P_i$, and there is no aggregate liquidity surplus. Indeed, upon realizing θ_i , bank i 's interim liquidity needs are $\theta_i\phi P_i$. If $\theta_i\phi P_i > S_i$, i.e., $\theta_i > \bar{\theta}$, the bank is a net borrower of interbank funds, but if $\theta_i < \bar{\theta}$, it is a net lender.

The equilibrium investments characterized in Proposition 1 are identical to those of the first best. The first best investments are achieved when these choices maximize the aggregate payoff of the entire banking system, subject to the aggregate resource constraints at $t = 0$ and $t = 1$. Since all banks are identical and there is no aggregate uncertainty, the first-best maximization problem is identical to a planner maximizing the expected payoff of a representative bank, subject to the deterministic aggregate constraints on its initial and interim investments. In accordance with the previous notation, let S , Q , and P denote the representative bank's corresponding $t = 0$ investments in the liquid, illiquid, and private technologies, and let α denote the portion of the good investment the bank chooses to upgrade at $t = 1$. With no aggregate uncertainty, the bank should not have to liquidate any investment prematurely. Thus, the planner simply chooses α , P , Q , and S to solve the following first-best problem (2.3):

$$\max \alpha(1 + \phi)Pr_p + Qr_q, \tag{8}$$

$$\text{s.t. } P + Q + S \leq 1, \tag{9}$$

$$\alpha\phi P \leq S, \tag{10}$$

$$0 \leq \alpha \leq \bar{\theta}, \tag{11}$$

$$P, Q, S \geq 0. \tag{12}$$

Problem (2.3) integrates problems (2.1) and (2.2). The objective function (8) is similar to the earlier (1) and (5), except that $U_i = 0$ and that the planner is not concerned with interbank interest payments. Conditions (9) and (10) are the planner's resource constraints at $t = 0$ and $t = 1$, respectively. Conditions (11) and (12) restrict the choice variables within the feasible regions.

Proposition 2. The first-best investments α , P , Q , and S are identical to the equilibrium investments α_i , P_i , Q_i , and S_i in Proposition 1.

Proposition 2 shows that even though individual banks' investments and liquidity needs are only privately known, the interbank market equilibrium will enable the first best. That is, with no costly interbank borrowing, bank investments would be unaffected by the unit banking structure and, for that matter, any branching restrictions would not be consequential. However, as will be seen shortly, if interbank borrowing is costly, the interbank market equilibrium will be substantially different, and so will be the interbank market structure.

2. Costly interbank borrowing

Suppose now there are costs associated with interbank borrowing. Such costs may be attributed

to, for example, information production and monitoring costs by interbank lenders. A standard moral hazard story would have interbank lenders carry out costly monitoring of borrowers in order to prevent the misuse of interbank loans¹. Unless stated otherwise, the notations used in this section will be identical to those used earlier. In particular, if the amount of interbank borrowing is B_i by bank $i \in \{1, 2, \dots, n\}$, its borrowing cost is now specified to be

$$m(B_i) = \begin{cases} kB_i^2/2 & \text{if } B_i \geq 0, \\ 0 & \text{if } B_i < 0. \end{cases} \quad (13)$$

Parameter $k > 0$ measures the costliness of this borrowing. Unlike the interest payment on an interbank loan, which has an expected net cost of zero, the borrowing cost $m(B_i)$ is asymmetric and is a deadweight loss to the banking system. While the borrower incurs this cost, the lender does not benefit from it². For tractability, I also specify the following probability distribution of θ_i : for all $i \in \{1, 2, \dots, n\}$,

$$\theta_i = \begin{cases} \theta_h & \text{with probability } 1/2, \\ 0 & \text{with probability } 1/2, \end{cases} \quad (14)$$

with $\theta_h \in (0, 1)$ and $\bar{\theta} = \theta_h/2$. To be consistent with deterministic aggregate liquidity needs, I assume that n is an even number and a half of all banks, i.e., $n/2$ banks, will realize θ_h .

Given $m(B_i)$ and as specified in equations (13) and (14), the solutions to problems (2.1) and (2.2) can be derived. To facilitate exposition, a critical value of parameter k is defined below:

$$k^*(\gamma) \equiv \frac{(2 + \theta_h\phi)(\theta_h(1 + \phi)r_p - (2 + \theta_h\phi)\gamma)}{\theta_h\phi(1 + \theta_h\phi)}.$$

Proposition 3. Fix $\gamma > 0$. Then, for r_p sufficiently large and $k > k^*(\gamma)$, the solutions to problems (2.1) and (2.2) have

$$P_i = \frac{1}{1 + \theta_h\phi} + \frac{\theta_h(1 + \phi)r_p - (2 + \theta_h\phi)\gamma}{k(1 + \theta_h\phi)^2},$$

$$Q_i = 0,$$

$$S_i = \frac{\theta_h\phi}{1 + \theta_h\phi} - \frac{\theta_h(1 + \phi)r_p - (2 + \theta_h\phi)\gamma}{k(1 + \theta_h\phi)^2},$$

$$\alpha_i = \theta_i,$$

$$U_i = \begin{cases} 0 & \text{if } \theta_i = 0 \text{ and } r_u\gamma \leq r_q, \\ Q_i & \text{otherwise,} \end{cases}$$

$$B_i = \theta_i\phi P_i - S_i = \frac{\theta_h(1 + \phi)r_p - (2 + \theta_h\phi)\gamma}{k(1 + \theta_h\phi)}.$$

¹ Such costs may also be interpreted as non-pecuniary disutility imposed on borrowers because of consistent lender monitoring.

² The qualitative results would remain unchanged with other cost functions as long as they admitted a costly spread between borrowing and lending and $m(\cdot)$ were convex.

Proposition 3 shows that individual banks will again choose only to invest in the private and the liquid technology. The restriction $k > k^*(\gamma)$ is important here; it ensures that the interbank market equilibrium will result in over-investment in the liquid technology. The interbank equilibrium is characterized next.

Proposition 4. For r_p sufficiently large and $k > k^*(1)$, the unique interbank market equilibrium is the collection of $\gamma = 1$ and the $P_i, Q_i, S_i, \alpha_i, U_i$, and B_i , for all $i \in \{1, 2, \dots, n\}$, as given in Proposition 3 with $\gamma = 1$.

Proposition 4 shows that with costly interbank borrowing, the equilibrium investments by banks are significantly different from the first best. More importantly, the interbank market equilibrium now leads to over-investment in the liquid technology by individual banks and an aggregate liquidity surplus in the entire banking system. To see the later, note that the difference between the aggregate investment in the liquid technology and the interim liquidity needs for investment upgrading needs is

$$\sum_{i=1}^n S_i - \sum_{i=1}^n \theta_i\phi P_i = n \left(S_i - \frac{1}{2} \theta_h\phi P_i \right) = \frac{n}{2(1 + \theta_h\phi)}.$$

$$\left(\theta_h\phi - \frac{(2 + \theta_h\phi)(\theta_h(1 + \phi)r_p - (2 + \theta_h\phi))}{k(1 + \theta_h\phi)} \right) > 0,$$

where the last inequality follows from $k > k^*(1)$.

Intuitively, as the costliness, k , of interbank borrowing increases, the cost to the borrowing bank increases, but there is no corresponding gain to the lending bank. This problem is made worse because a bank's interim borrowing needs increase with its initial investment P_i in its own private technology. Thus, for a large k , individual banks are better off increasing their liquid investments and, as a consequence, the banking system accumulates surplus liquid funds. The equilibrium liquidity surplus, $n(S_i - \theta_h\phi P_i/2)$, is strictly increasing in k since P_i is decreasing in k while S_i is increasing in it. Interestingly, the interbank equilibrium is unique because the interbank loan return must be $\gamma = 1$. If $\gamma < 1$, no one would want to lend, but if $\gamma > 1$, the liquidity surplus would imply an excess supply of liquid funds.

3. Two-tier banking

Costly interbank borrowing has been to result in a surplus of liquid reserves by the unit banking system. In this section, I examine the emergence of second-tier banks that utilize the surplus funds. That is, I now examine the role of a unit-banks' bank – a

correspondent bank¹. In accordance with the earlier assumptions of the technologies, a second-tier (correspondent) bank has access only to the economy's publicly available liquid and illiquid technologies. It has no access to a local community's private technology presumably because only the unit bank performs special services in the community it operates and these services cannot be easily transferred². Since the second-tier bank invests only in the public technologies, the suppliers of this bank's funds need not engage in costly information production and monitoring. Thus, there are no borrowing costs associated with the second-tier bank's borrowing from unit banks³.

As before, let S_i , Q_i , and P_i , $i \in \{1, 2, \dots, n\}$, denote, respectively, unit bank i 's investments in the liquid, the illiquid, and the private technology at $t = 0$. Unlike in Section 2, a unit bank can now deposit its liquid funds in the second-tier bank rather than investing them in the liquid technology directly. Thus, S_i now refers to the amount of unit bank i 's interbank deposits at $t = 0$. Since a unit bank's interim liquidity needs are only privately known, interbank deposits are in the form of demand deposits (as in Diamond and Dybvig, 1983). If unit bank i realizes $\theta_i = \theta_i$ at $t = 1$, it will first withdraw its interbank deposit S_i to upgrade its good investment because, unlike interbank borrowing, deposit withdrawals incur no borrowing costs. If the withdrawal is insufficient to meet the liquidity needs, bank i will then borrow additional funds from the second-tier bank. Let α_i now be the portion of the good investment bank i chooses to upgrade, and let B_i be its net interbank borrowing at $t = 1$, with the same interbank borrowing cost $m(B_i)$. Also, let $\gamma \geq 0$ be the required return on interbank loans extended by the second-tier bank. As before, all payments are made at $t = 2$, except that interbank deposits can be withdrawn at the face value at $t = 1$. For unit bank i , $i \in \{1, 2, \dots, n\}$, its investment choice problems are identical to problems (2.1) and (2.2). That is, given γ , P_i , Q_i , S_i , and its realized θ_i , unit

bank i chooses α_i , U_i , and B_i at $t = 1$ to solve the following choice problem (4.1):

$$\begin{aligned} \max \quad & \alpha_i(1 + \phi)P_i r_p + (Q_i - U_i)r_q - B_i\gamma - m(B_i), \\ \text{s.t.} \quad & \alpha_i\phi P_i \leq S_i + U_i r_u + B_i, \\ & 0 \leq \alpha_i \leq \theta_i, \\ & 0 \leq U_i \leq Q_i. \end{aligned}$$

Given the optimal α_i , U_i , and B_i from problem (4.1), bank i chooses P_i , Q_i , and S_i to solve the following choice problem (4.2):

$$\begin{aligned} \max \quad & E[\alpha_i(1 + \phi)P_i r_p + (Q_i - U_i)r_q - B_i\gamma - m(B_i)], \\ \text{s.t.} \quad & P_i + Q_i + S_i \leq 1, \\ & P_i, Q_i, S_i \geq 0. \end{aligned}$$

Consider now the choice problem of the second-tier bank. Let $Q_c \geq 0$ denote the second-tier bank's investment in the illiquid technology at $t = 0$, and $S_c \geq 0$ be its investment in the liquid technology. Consistent with a competitive two-tier banking system, the second-tier bank maximizes the aggregate payoff of the banking system and that all gains from two-tier banking are transferred to unit banks at $t = 2$. Then the second-tier bank chooses γ , Q_c , and S_c to solve the following problem (4.3):

$$\max \sum_{i=1}^n (\theta_i(1 + \phi)P_i r_p + (Q_i - U_i)r_q - m(B_i)) + Q_c r_q, \quad (15)$$

$$\text{s.t.} \quad Q_c + S_c \leq \sum_{i=1}^n S_i, \quad (16)$$

$$\sum_{i=1}^n \alpha_i \phi P_i \leq S_c, \quad (17)$$

$$S_c, Q_c, \gamma \geq 0. \quad (18)$$

In problem (4.3), expression (15) describes the aggregate welfare of the two-tier banking system. Conditions (16) and (17) are the second-tier bank's initial resource and interim liquidity constraints, respectively. Condition (18) restricts the choice variables within the feasible regions.

Definition 2. A two-tier banking equilibrium is a collection of $\gamma \geq 0$, $Q_c \geq 0$, $S_c \geq 0$, $P_i \geq 0$, $Q_i \geq 0$, $S_i \geq 0$, $\alpha_i \in [0, \theta_i]$, $U_i \in [0, Q_i]$, and $B_i \in (-\infty, \infty)$, for all $i \in \{1, 2, \dots, n\}$, such that

- 1) given γ , P_i , Q_i , S_i , and θ_i , unit bank i chooses α_i , U_i , and B_i to solve problem (4.1);
- 2) given γ and the optimal α_i , U_i , and B_i from problem (4.1), unit bank i chooses P_i , Q_i , and S_i to solve problem (4.2); and
- 3) the second-tier bank chooses γ , Q_c , and S_c to solve problem (4.3).

¹ Since the second-tier bank does not branch across communities, forming this bank does not breach branching restrictions.

² In this simple model, if the second-tier bank had access to the private technology, the optimal mechanism would have unit banks specialize in collecting deposits in their own communities and the second-tier bank specialize in investment. This is not empirically observed, however. In a richer model with scale-dependent technologies, the complete separation of deposit collection and investment would not be optimal.

³ One reason why unit banking may lead to higher borrowing costs is that there is a greater difficulty involved in generating information regarding independent, small banks. In fact, the results of my model would be qualitatively unchanged even with some borrowing costs by the second-tier bank, as long as these costs were less than those incurred by unit banks. In the last section, I will discuss plausible reasons for the differential borrowing costs between the two types of banks.

Since problems (4.1) and (4.2) are identical to the earlier problems (2.1) and (2.2), the result of Proposition 3 is applicable here. The two-tier banking equilibrium is now characterized.

Proposition 5. For r_p sufficiently large and $k > k^*(r_q)$, the unique two-tier banking equilibrium is, for all $i \in \{1, 2, \dots, n\}$,

$$\gamma = r_q,$$

$$P_i = \frac{1}{1 + \theta_h \phi} + \frac{\theta_h(1 + \phi)r_p - (2 + \theta_h \phi)r_q}{k(1 + \theta_h \phi)^2},$$

$$Q_i = 0,$$

$$S_i = \frac{\theta_h \phi}{1 + \theta_h \phi} - \frac{\theta_h(1 + \phi)r_p - (2 + \theta_h \phi)r_q}{k(1 + \theta_h \phi)^2},$$

$$\alpha_i = \theta_i,$$

$$U_i = \begin{cases} Q_i & \text{if } \theta_i = \theta_h, \\ 0 & \text{if } \theta_i = 0, \end{cases}$$

$$B_i = \theta_i \phi P_i - S_i,$$

$$Q_c = n \left(S_i - \frac{1}{2} \theta_h \phi P_i \right),$$

$$S_c = \frac{n}{2} \theta_h \phi P_i.$$

It is interesting to note that the two-tier banking equilibrium has $\gamma = r_q$. This is because the opportunity cost for the second-tier bank is the illiquid investment, not the liquid one. As a result, each unit bank's investment under the two-tier banking system is different from that under the unit banking system. Importantly, the two-tier banking system improves the aggregate welfare because the unit banking equilibrium is feasible under two-tier banking but is not chosen. Intuitively, the second-tier bank improves economic efficiency by utilizing the liquidity surplus accumulated by unit banks for more productive, less liquid investment. Indeed, the liquidity surplus, $n(S_i - \theta_h \phi P_i / 2)$, is completely utilized by the second-tier bank for its investment in the illiquid technology with $Q_c = n(S_i - \theta_h \phi P_i / 2)$, and the two-tier banking system, as a whole, no longer exhibits a liquidity surplus. Although the required return on interbank loans is $\gamma = r_q > 1$, individual unit banks are better off using the second-tier bank. This is because competitive two-tier banking ensures that unit banks will receive all efficiency gains.

4. Stability of two-tier banking

While the two-tier banking system clearly improves economic efficiency, it is more vulnerable to banking panics because of the reduced aggregate liquidity reserves. Indeed, the second-tier bank will

have to liquidate its illiquid investment prematurely if there are runs on interbank deposits. If some unit banks with no liquidity needs believe that others will withdraw their interbank deposits early, it is rational for these unit banks to do the same¹. To see this, note that the maximal amount of interbank funds available to the second-tier bank at $t = 1$ is only

$$S_c + Q_c r_u = \frac{n}{2} \theta_h \phi P_i + n \left(S_i - \frac{1}{2} \theta_h \phi P_i \right) r_u < n S_i, \quad (19)$$

where $Q_c r_u$ is the early liquidation value of Q_c and the last inequality follows from $r_u < 1$. Thus, if r_u is small and a large number of unit banks choose to withdraw in the interim, there will not be enough funds available at $t = 2$. In the case of such runs on interbank deposits, a unit bank that has not withdrawn its deposits at $t = 1$ will receive nothing at $t = 2$. Therefore, anticipating such runs, all unit banks will want to make early withdrawals, thereby ensuring the banking panic.

Two factors make such panic runs very costly for the second-tier banking system. First, the second-tier bank will have to liquidate its illiquid investment prematurely. This cost can be substantial if r_u is small. Second, there can be substantial costs to its respondent unit banks that have the good investment. Condition (19) implies that during banking panics, many unit banks will be unable to withdraw their deposits before the second-tier bank runs out of funds. If these unit banks are the ones with the good investment, they must either increase their interbank borrowing from other unit banks, or abandon some of the good investment. There are additional costs in either instance. For example, with additional borrowing, the cost to unit banks is increased by

$$\frac{1}{2} k \left((\theta_h \phi P_i)^2 - (\theta_h \phi P_i - S_i)^2 \right) > 0.$$

One way to improve the stability of the two-tier banking system is to create a lender of last resort – a central bank. During banking panics, the lender of last resort can sell government-back securities to recycle liquid funds to the banking system, thereby preventing costly asset liquidation². It is interesting to see that a government deposit insurance scheme that provides coverage only to individual depositors, as is usually the case, may not prevent panic runs in the interbank market. Therefore, other forms of regulation, such as the creation of a central bank, may be necessary in an inter-connected banking system. In this regard, a globally linked banking

¹ For related studies, see Bryant (1980), Diamond and Dybvig (1983), Postlewaite and Vives (1987), Jacklin and Bhattacharya (1988), and Qi (1994).

² In monetary economy, the central bank can also supply emergency liquidity by printing fiat money, effectively creating an inflation tax.

system may also require a greater coordination among central banks in different countries to ensure stability.

Conclusion

This paper model an interbank market. It argues that branching restrictions would have no impact on bank credit allocation as long as interbank borrowing is not costly. However, with costly interbank borrowing, such restrictions would cause individual local banks to over-invest in liquid reserves and, as a result, such a banking system would have an aggregate liquidity surplus. A second-tier, correspondent bank is shown to mitigate this liquidity surplus problem by serving as the local banks' bank. Because of the reduced aggregate liquidity reserves and the increased interbank dependence, the two-tier banking system is more vulnerable to banking panics. A government sponsored lender of last resort – a central bank – may be necessary to maintain financial stability.

The implications of the model seem to be consistent with casual observations of U.S. interbank markets, i.e., the Fed funds markets. The U.S. banking system consists of a large number of community-based local banks that co-exist with a much smaller number of regional or national correspondent banks. The interbank flows of funds are facilitated through the overnight Fed funds markets¹. Consistent with the story of a liquidity surplus among local banks, empirical evidence for the most part has documented that small banks are the net suppliers of liquid funds in the overnight interbank markets and large banks are the net users of such funds. However, there is little liquidity surplus in the entire banking system that includes the correspondent banks². The fact that Fed funds are the surplus over regulatory reserve requirements does not invalidate

this interpretation because the aggregate reserves by small banks persistently exceed the statutory minimum. The model seems also to be consistent with the observation that there is little interbank borrowing among small, local banks even though they do sometimes borrow from their own correspondent banks. The model suggests that interbank deposits are in the form of demand debt because of the private liquidity needs by individual local banks. Evidently, Fed fund borrowing is generally in the form of unsecured, callable, and automatically rolling-over overnight loans.

At this point, I should discuss reasons for why the second-tier bank's interbank borrowing may not be as costly as similar borrowing among local banks. In addition to its better diversification, a correspondent bank, given its size, is likely to be more closely monitored by other stakeholders, such as large debtholders and governments. There are also greater concerns for reputation by such banks. The perception that large banks are “too-big-to-fail” may be beneficial as well. Interesting extensions of the present model would be to explicitly incorporate these factors.

The model could also be extended in other directions. One could build a more general model with scale-dependent technologies. The interbank funds market could be examined in the context of interbank relationship building. For example, the correspondent relationships might be viewed as efforts by banks to develop long-term relationships that would reduce further costs associated with interbank borrowing. Finally, the model could be modified to examine policy issues relevant in the interbank market, such as the regulatory reserve requirements as well as the central bank's control of the interbank fund rate.

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¹ In less than three decades, the Fed funds trading volume has grown from a daily average of about 10 billion units in 1960's to about 130 billion dollars in mid 1980's (See Goodfriend and Whelpley (1986) and Choi (1989)). The rapid growth has been attributed to sharp increases of inflation and nominal interest rates in 1970 and early 1980's. Small banks have found it increasingly costly to keep excess reserves unused. At the same time, other non-bank institutions have found it increasingly important to channel their liquid funds to the Fed funds market. This latter development has developed into important new sources of funds (See Stigum (1990) for a comprehensive survey of the Fed funds market).

² See Federal Reserve Bulletin published regularly by the Federal Reserve Board

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Appendix A.

Proof of Proposition 1:

First, note that $P_i, S_i > 0$, and $\gamma \geq 1$ for r_p sufficiently large. Consider now bank i 's problem (2.1), given $m(B_i) = 0$. The Lagrangian is

$$L = \alpha_i(1 + \phi)P_i r_p + (Q_i - U_i)r_q - B_i\gamma + \lambda_1(S_i + U_i r_u + B_i - \alpha_i \phi P_i) \\ + \lambda_2(\theta_i - \alpha_i) + \lambda_3 \alpha_i + \lambda_4(Q_i - U_i) + \lambda_5 U_i,$$

where λ_j 's, hereafter, will denote the corresponding Lagrangian multipliers. Then, given P_i, Q_i, S_i , and θ_i , for r_p sufficiently large, the solution has $\alpha_i = \theta_i, U_i = Q_i$, and $B_i = \theta_i \phi P_i - S_i - Q_i r_u$, with $\lambda_1 = \gamma > 0, \lambda_3 = \lambda_5 = 0$,

$$\lambda_2 = ((1 + \phi)r_p - \gamma\phi)P_i = \frac{(1 + \phi)P_i r_p}{1 + \theta\phi} > 0, \text{ and}$$

$$\lambda_4 = \gamma r_u - r_q = \frac{\bar{\theta}(1 + \phi)r_u r_p}{1 + \theta\phi} - r_q > 0.$$

To solve bank i 's problem (2.2), I first substitute the optimal α_i, U_i , and B_i into (5). Then, the Lagrangian of problem (2.2) is

$$L = \bar{\theta}(1 + \phi)P_i r_p - (\bar{\theta}\phi P_i - S_i - Q_i r_u)\gamma \\ + \lambda_1(1 - P_i - Q_i - S_i) + \lambda_2 P_i + \lambda_3 Q_i + \lambda_4 S_i.$$

Then, for r_p sufficiently large, the solution has $P_i + S_i = 1, Q_i = 0$, and $\gamma = \bar{\theta}(1 + \phi)r_p / (1 + \bar{\theta}\phi)$ with $\lambda_1 = \gamma > 0, \lambda_2 = \lambda_4 = 0$, and $\lambda_3 = (1 - r_u)\gamma > 0$. Finally, market clearing implies

$$\sum_{i=1}^n B_i = \sum_{i=1}^n (\theta_i \phi P_i - S_i) = n(\bar{\theta}_i \phi P_i - S_i) = 0.$$

By solving the market clearing condition and $P_i + S_i = 1$ simultaneously, it follows that $P_i = (1 + \bar{\theta}\phi)^{-1}$ and $S_i = \bar{\theta}\phi / (1 + \bar{\theta}\phi)$. Therefore, for r_p sufficiently large, the collection of $\gamma, P_i, Q_i, S_i, \alpha_i, U_i$, and B_i is the unique interbank market equilibrium. Q.E.D.

Appendix B.

Proof of Proposition 2:

The Lagrangian of the first-best problem (2.3) is

$$L = \alpha(1 + \phi)Pr_p + Qr_q + \lambda_1(1 - P - Q - S) + \lambda_2(S - \alpha\phi P) \\ + \lambda_3(\bar{\theta} - \alpha) + \lambda_4 \alpha + \lambda_5 P + \lambda_6 Q + \lambda_7 S.$$

Then, for r_p sufficiently large, the solution has $P = (1 + \bar{\theta}\phi)^{-1}, Q = 0, S = \bar{\theta}\phi / (1 + \bar{\theta}\phi)$, and $\alpha = \bar{\theta}$, with $\lambda_1 = \lambda_2 = \bar{\theta}(1 + \phi)r_p / (1 + \bar{\theta}\phi) > 0, \lambda_3 = \lambda_1 P / \bar{\theta} > 0, \lambda_4 = \lambda_5 = \lambda_7 = 0$, and $\lambda_6 = \lambda_1 - r_q > 0$. Therefore, α, P, Q , and S are the same as the corresponding investments in Proposition 1. Q.E.D.

Appendix C.

Proof of Proposition 3:

First, note that for $k > k^*(\gamma)$,

$$P_i \in \left(\frac{1}{1 + \theta_h \phi}, \frac{2}{2 + \theta_h \phi} \right), \text{ and } S_i \in \left(\frac{\theta_h \phi}{2 + \theta_h \phi}, \frac{\theta_h \phi}{1 + \theta_h \phi} \right).$$

Consider now bank i 's problem (2.1), given $m(B_i)$ and θ_i as in (13) and (14). Then, the Lagrangian

$$\text{is } L = \alpha_i(1 + \phi)P_i r_p + (Q_i - U_i)r_q - B_i\gamma - m(B_i) + \lambda_1(S_i + U_i r_u + B_i - \alpha_i \phi P_i) + \lambda_2(\theta_i - \alpha_i) + \lambda_3 \alpha_i + \lambda_4(Q_i - U_i) + \lambda_5 U_i.$$

If $\theta_i = \theta_h$, bank i is a net borrower and $B_i \geq 0$; then, $m(B_i) = kB_i^2/2$. Given γ , P_i , Q_i , and S_i , for r_p sufficiently large, the solution has $\alpha_i = \theta_h$, $U_i = Q_i$, and $B_i = \theta_h \phi P_i - S_i - Q_i r_u$, with

$$\lambda_1 = \gamma + kB_i = \frac{\theta_h(1 + \phi)r_p - \gamma}{1 + \theta_h \phi} > 0,$$

$$\lambda_2 = ((1 + \phi)r_p - \lambda_1 \phi)P_i = \frac{((1 + \phi)r_p + \phi\gamma)P_i}{1 + \theta_h \phi} > 0,$$

$\lambda_3 = \lambda_5 = 0$, and $\lambda_4 = (\gamma + kB_i)r_u - r_q > 0$. If $\theta_i = 0$, bank i is now a net lender and $B_i < 0$; then, $m(B_i) = 0$. Again, given γ , P_i , Q_i , and S_i , for r_p sufficiently large, the solution has $\alpha_i = 0$,

$$U_i = \begin{cases} Q_i & \text{if } r_u > r_q, \\ 0 & \text{if } r_u \leq r_q, \end{cases}$$

And $B_i = -S_i - U_i r_u$, with $\lambda_1 = \gamma > 0$, $\lambda_2 = ((1 + \phi)r_p - \gamma\phi)P_i > 0$, $\lambda_3 = 0$,

$$\lambda_4 = \begin{cases} r_u - r_q > 0 & \text{if } r_u > r_q, \\ 0 & \text{if } r_u \leq r_q, \end{cases}$$

$$\lambda_5 = \begin{cases} 0 & \text{if } r_u > r_q, \\ r_q - r_u > 0 & \text{if } r_u \leq r_q. \end{cases}$$

Now, I proceed to solve bank i 's problem (2.2). As it turns out, the optimal P_i , Q_i , and S_i are identical in either the case of $r_u > r_q$, or $r_u \leq r_q$. To avoid repetition, I only present the proof for the case of $r_u \leq r_q$. By substituting $m(B_i)$, θ_i , and the optimal α_i , U_i , and B_i into (5), the Lagrangian of problem (2.2) is

$$L_0 = \frac{1}{2} \left(\theta_h(1 + \phi)P_i r_p - (\theta_h \phi P_i - S_i - Q_i r_u)\gamma - \frac{1}{2}k(\theta_h \phi P_i - S_i - Q_i r_u)^2 \right) + \frac{1}{2}(Q_i r_q + S_i \gamma) + \lambda_1(1 - P_i - Q_i - S_i) + \lambda_2 P_i + \lambda_3 Q_i + \lambda_4 S_i.$$

Then, given $\gamma > 0$, for r_p sufficiently large and $k > k^*(\gamma)$, the solution has P_i , Q_i , and S_i as in the proposition, with

$$\lambda_1 = \gamma + k(\theta_h \phi P_i - S_i)/2 > 0, \lambda_2 = \lambda_4 = 0, \text{ and } \lambda_3 = \frac{1}{2}k(1 - r_u)(\theta_h \phi P_i - S_i) + \gamma - \frac{1}{2}(r_u + r_q) > 0.$$

Therefore, the solutions to both problems are as given. Q.E.D.

Appendix D.

Proof of Proposition 4:

Note that given $\gamma > 0$, bank i 's optimal choices P_i , Q_i , S_i , α_i , U_i , and B_i are as in Proposition 3. What remains is to show that $\gamma = 1$ is the unique interbank equilibrium loan return. In particular, if $\gamma < 1$, no bank would want to lend funds at $t = 1$ because it is better off investing in the liquid technology. On the other hand, if $\gamma > 1$, all banks with $\theta = 0$ would want to lend all of their liquid funds. In this case, however, for $k > k^*(\gamma)$, the market would not clear since

$$\begin{aligned} \sum_{i=1}^n B_i &= \sum_{i=1}^n \left(\frac{1}{2} (\theta_h \phi P_i - S_i) - \frac{1}{2} S_i \right) \\ &= \frac{n}{2} \left(\frac{(2 + \theta_h \phi) (\theta_h (1 + \phi) r_p - (2 + \theta_h \phi) \gamma)}{k(1 + \theta_h \phi)^2} - \frac{\theta_h \phi}{1 + \theta_h \phi} \right) < 0; \end{aligned}$$

that is, there would be an excess supply of funds at $t = 1$. Therefore, the market equilibrium must have $\gamma = 1$. Q.E.D.

Appendix E.

Proof of Proposition 5:

Note that bank i 's problems (4.1) and (4.2) are identical to problems (2.1) and (2.2). Thus, given $\gamma > 0$, for r_p sufficiently large and $k > k^*(\gamma)$, the solutions to problems (4.1) and (4.2) are identical to those in Proposition 3. What remains is to solve the second-tier bank's problem (4.3). In particular, by substituting the optimal P_i , Q_i , S_i , α_i , U_i , and B_i into (15) through (17), the Lagrangian of problem (4.3) is

$$\begin{aligned} L &= \frac{n}{2} \theta_h (1 + \phi) P_i r_p - \frac{n}{4} k (\theta_h \phi P_i - S_i)^2 + Q_c r_q + \lambda_1 (n S_i - Q_c - S_c) \\ &+ \lambda_2 \left(S_c - \frac{n}{2} \theta_h \phi P_i \right) + \lambda_3 S_c + \lambda_4 Q_c + \lambda_5 \gamma. \end{aligned}$$

Then, the solution to problem (4.3) has $S_c = n \theta_h \phi P_i / 2$, $Q_c = n(S_i - \theta_h \phi P_i / 2)$, and $\gamma = r_q$, with $\lambda_1 = \lambda_2 = r_q$, and $\lambda_3 = \lambda_4 = \lambda_5 = 0$. Therefore, by substituting $\gamma = r_q$ in the results of Proposition 3, the two-tier banking equilibrium is as established. Q.E.D.