“Forecasting based on spectral time series analysis: prediction of the Aurubis stock price”

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FORECASTING BASED ON SPECTRAL TIME SERIES ANALYSIS: PREDICTION OF THE AURUBIS STOCK PRICE

Abstract

The attempt to predict stock price movements has occupied investors ever since. Reliable forecasts are a basis for investment management, and improved forecasting results lead to enhanced portfolio performance and sound risk management. While forecasting using the Wiener process has received great attention in the literature, spectral time series analysis has been disregarded in this respect. The paper's main objective is to evaluate whether spectral time series analysis can produce reliable forecasts of the Aurubis stock price. Aurubis poses a suitable candidate for an investor's portfolio due to its sound economic and financial situation and the steady dividend policy. Additionally, reliable management contributes to making Aurubis an investment opportunity. To judge if the achieved forecast results can be considered satisfactory, they are compared against the simulation results of a Wiener process. After de-trending the time series using an Augmented Dickey-Fuller test, the residuals were compartmentalized into sine and cosine functions. The frequencies, amplitude, and phase were obtained using the Fast Fourier transform. The mean absolute percentage error measured the accuracy of the stock price prediction, and the results showed that the spectral analysis was able to deliver superior results when comparing the simulation using a Wiener process. Hence, spectral time series can enhance stock price forecasts and consequently improve risk management.

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INTRODUCTION

Stock return forecasting has been a widely discussed task among technical analysts (Shalini et al., 2019; Picasso et al., 2019; Reschenhofer et al., 2019) and investors making discussions based on fundamental analysis principles (Xidonas et al., 2009). Some authors suggest combining these two approaches to get an optimal output of a selected portfolio (Fernandez et al., 2019; Hasuike & Mehlawat, 2018; Meng & Chen, 2018; Eiamkanitchat et al., 2016). Forecasting of a stock price should be based on appropriate methods, which, as a result, can contribute to a higher return of portfolio (Yu, 2011; Östermark, 1991). Various methods and approaches exist for picking the stocks, creating the portfolio, and forecasting future returns. The authors previously discussed the economic value added as a possible stock selection method (Maitah et al., 2015). In this paper, the Wiener process and the spectral time series analysis as methods for deriving stock price forecasts are employed using the stock price data from the Aurubis stock price. Consequently, it is determined which method produced more reliable forecasts.

The Black-Scholes model assumes that the underlying follows a Geometric Brownian motion and has received great attention regard-

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ing the simulation of stock returns after its introduction in the 1970s by Myron Scholes, Fischer Black, and Robert Merton. It became popular among traders internationally, making it one of the most used models to simulate the return on assets to financial price options (Hull, 2009).

Despite the popularity of the model, the assumption that the stock price follows a GBM is not supported by empirical studies. The Black-Scholes model assumes a smooth movement of asset prices (no jumps), and it would describe stock behavior perfectly in the case of an ideal situation without the information of the outside world. In reality, markets tend to jump based on good and/or bad news entering the market regarding the entire economy or an individual company. Further, the Black-Scholes model assumes a normal distribution of returns. Empirical studies have shown that the return distribution of asset returns is skewed to the left while having a higher peak and heavier tails (leptokurtic feature) than assumed by the normal distribution of the Black-Scholes model (Yi, 2010).

In order to produce better forecasting results, spectral time series analysis poses a viable alternative. Spectral time series analysis has been used in literature to analyze derivatives and the behavior of time series (Oest, 2002). It has also been used to forecast the growth rate and returns (Keim et al., 2006) and examine asset volatility and stock return series (Tsay, 2005). It was applied within various areas, including business, science, finance, communication, and entertainment. Spectral time series has proven to be a resilient tool for various types of analyses, as shown by Fumi et al. (2013), Hassani (2007), and Marques et al. (2006), among others.

1. LITERATURE REVIEW

While the Wiener process is widely discussed in the literature on stock price forecasting (Abidin & Jaffar, 2017; Azizah et al., 2018; Reddy & Clinton, 2016), spectral time series analysis has not received great attention regarding forecasting stock price returns. Various studies have been conducted, which demonstrated the importance of spectral time series analysis and that it can deliver superior results, including those of Fumi et al. (2013) and Hassani (2007). Even though the studies have demonstrated the superiority of spectral time series over other forecasting methods, the studies have not been used to forecast the stock price. Forecasting stock price returns can be considered important since it helps investors develop an effective market trading strategy. Further, they can anticipate investment losses and provide optimal benefits for investors (Azizah et al., 2020).

Spectral time series describes given phenomena in time and in many cases and "*may take a chaotic process*" (Grzesica & Wiecek, 2016, p. 254). Financial time series analyses had attracted substantial attention when the Nobel Prize was awarded to Engle and Granger (Tsay, 2005). Moreover, spectral time series analysis is used for the assessment of derivatives, for the investigation of the stochastic behavior of time series, and for the optimization of portfolios. It is also utilized to determine the correlation between different products and the prognoses of crises (Oest, 2002). Risk and return can be considered the two most important asset price characteristics from a stakeholder's perspective. Since analysts and investors are interested in future growth rates regarding the returns, spectral time series analysis poses a suitable tool for this type of analysis (Keim et al., 2006). Jenkins and Priestley (1957) pointed to the problem that "*the raw data consist of time-series which may be essentially discrete in time*". Due to this fact, a time series can be presented in the form of a trace. According to Grzesica and Wiecek (2016), time series analysis helps detect the nature of a time series and allows predicting future values. Financial time series analysis relates the theory and practice of asset valuation over time and can be described as an empirical discipline. Nonetheless, it is different from other time series analysis forms since financial theory and empirical time series are related to uncertainty. This uncertainty is reflected by asset volatility and stock return series (Tsay, 2005). Thus, statistical theory plays an important role in financial time series and the stock market analysis (Tsay, 2005). Time-related data sets and consequently the analysis of these data sets appear in many domains like busi-

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ness and science, including finance (stock market data, credit card transactional data), communication (telephone data, signal processing data, network monitoring) or entertainment, including music and videos (Keim et al., 2006). Fumi et al. (2013) showed that Fourier analysis could forecast the demand for a fashion company. When applying the Fourier transform, the mean average percentage error regarding forecasting can be reduced by approximately 30% compared to other forecasting techniques, including exponential smoothing and moving averages. Also, Hassani (2007) forecasted the number of accidental deaths in the USA using several forecasting techniques. Spectral time series analysis outperformed the other techniques as measured by mean absolute percentage error. Marques et al. (2006) proved the suitability of spectral analysis as a forecasting tool based on their prediction of a hydrological time series. To sum up, there are three main categories which spectral time series analysis focuses on: the analysis of financial data (e.g., price changes, volatility, and correlations, crisis analysis), models describing financial data (e.g., stable distributions, agent models, turbulences), and the handling of financial products including the assessment of derivatives, portfolio optimization and risk management (Oest, 2002). In other words, spectral time series analysis can be specifically used to model financial returns, price volatilities, autocorrelation, and accuracy estimates. Further, it can be utilized to test random walk hypotheses, trend forecasting, and valuing options. Time series analysis also poses evidence against the efficiency of futures markets (Taylor, 2007). The advantages of spectral time series analysis include its easy implementation because the underlying behavior is interpreted based on the simplest statistical behavior (Scargle, 1982). Besides, the technique has been developed to detect trends and identify to detect and smooth seasonality. All factors contributing and influencing the time series are identified (Azar et al., 2001). One of the major disadvantages of time series analysis is that at least second-order stationarity is required to derive meaningful models. When analyzing the periodogram in more detail, the focus is drawn to microscopic details that might lead to biased conclusions (Birr et al., 2016).

Since its stock price reflects a company's well-being, the Aurubis stock can be considered a proxy for companies within the copper industry in the EU. With millions of jobs at risk, which are concerned with the welfare of the copper branch, the analysis and prediction of the stock price are supposed to deliver clarity regarding the industry's status quo. Aurubis AG, with its headquarters in Hamburg, is a leading copper multinational and the biggest recycler of copper globally. Its product assortment comprises standard and special products made from copper and copper alloying (Aurubis AG, 2020). In the European Union, the copper industry consists of around 500 companies that generate a profit of around EUR 45 billion every year and employ 50,000 people. When considering the downstream industries of copper, including utility companies, car manufacturers, producers of electronics, and many more, several million people are employed. In 2016, around 12% of global copper production was generated within the EU (Deutsches Kupferinstitut Copper Alliance, 2020). From a macro perspective, the ongoing trade war between the US and China has had a major impact on copper. It revealed a high correlation with the trade war, while no other metal is closely tied to the controversy. As a result, global manufacturing has experienced a slowdown in the form of an industrial recession in Japan and Germany.

2. AIMS

The paper's main objective is to determine if the spectral time series analysis delivers satisfactory results when forecasting stock prices. The aim of performing a spectral time series analysis poses the spectral decomposition which draws attention to the cyclical process. In order to determine whether the results can be regarded satisfactory, the obtained forecasts are compared against simulated prices under a Wiener Process. From the predicted stock prices, it is derived whether spectral time series analysis is a suitable tool for forecasting stock returns.

3. METHODOLOGY

In order to determine whether the forecasted results under the spectral time series analysis are satisfactory, the results obtained by the spectral time series analysis are compared against the stock price simulation under a Wiener process. Well-grounded investment performance predictions are of special importance to an investor's risk management as they help reduce uncertainty and provide clear expectations regarding the expected returns. Various studies, including Abidin and Jaffar (2017), Azizah et al. (2018), and Reddy and Clinton (2016), have demonstrated the importance of the Wiener process regarding producing stock price forecasts. Reliable forecasts pose the fundament for an investment decision, and superior simulation results lead to better investment decisions and enhanced portfolio performance. Fumi et al. (2013) and Grzesica and Wiecek (2016), among others, have shown that spectral analysis provided superior forecasts and hence, might pose a viable alternative to the traditional Wiener process. If the results achieved within this case study are satisfactory, spectral time series analysis might be implemented by investors as a tool used in order to reduce the risk of an investment.

If spectral analysis delivers satisfactory results regarding the forecasted prices within this study, the current economic condition of copper commerce can be derived. The data used in the study were secondary; all daily closing prices have been retrieved from Bloomberg. For the Aurubis stock price analysis, the calibration dataset includes the daily closing prices from 25 August 2014 to 10 September 2018 (1,024 observations). The validation dataset was chosen to comprise the daily closing prices from 11 September 2018 to 28 March 2019 (137 observations). Even though non-probability sampling was applied in the paper, the longitudinal study design allows elaborating the forecast quality during several points in time. The forecast quality is assessed using end-of-day prices.

In order to be able to model and forecast a time series, a general step by step approach was introduced by Brockwell and Davis (2016) and applied in the paper:

As a first step the given time series was plotted in order to judge whether they show severe changes in behavior or outliers.

To obtain a stationary series, the trend and other recurring components need to be removed using the techniques like differencing and averaging.

After the stationarity requirement was met, a model needs to be chosen which fits the residuals whereby the quality of the fit is determined using sample statistics (parametric approach) or model-free approaches (non-parametric approach such as time series analysis as proposed by Birr et al. (2016).

Consequently, the residuals can be inverted in order to obtain a forecast of the original time series. According to Grzesica and Wiecek (2016), spectral time series analysis is performed as aforementioned, whereby the main goal of performing spectral decomposition is to draw attention to the cyclical process. The decomposition is possible by splitting the stationary residuals of a given dataset into trigonometric functions like the sine and cosine. In the context of spectral time series analysis, those functions are often denoted as harmonics.

To be able to apply (spectral) time series analysis on a given dataset, the stationarity requisition should be fulfilled to set up statistically significant models (Warner, 1998). A time series can be considered stationary if it meets the following requirements (Sun et al., 2018):

Constant mean, given by: $\mu_x(t) = E(X_t)$ where $\mu_x(t)$ is not dependent on t (Brockwell & Davis, 2016).

Autocorrelation does not depend on itself, but only on the relative position within the time series, given by:

$$\gamma_x(r,s) = \operatorname{cov}(X_r, X_s) = \\ = E\left[\left(X_r - \mu_x(r)\right) - X_s - \mu_x(s)\right], \quad (1)$$

where $\{X_t\}$ denotes a time series $E\left(X_t^2\right) < \infty$ and $\gamma_x(t+h,t)$ is not dependent on t for each h (Brockwell & Davis, 2016).

If both criteria are fulfilled, the time series can be considered weakly stationary. If stationarity is not given for a specific dataset, the condition can be fulfilled by deducting the trend and seasonal component from the original time series. To determine whether a time series fulfills the stationarity requirement, which enables further modeling, a common practice in econometrics is to perform an augmented Dickey-Fuller (or ADF) test. The

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ADF is a unit-root test performed by running the following regression:

$$\Delta x_t = \mu + \gamma t + \alpha x_{t-1} + \sum_{j=1}^{k-1} \beta_j \Delta x_{t-j} + u_t, \quad (2)$$

where x_t denotes a time series, Δ denotes the difference operator and u_t denote the residuals (Chung & Kon, 1995).

As proposed by Brockwell and Davis (2016), the time series can be represented as the sum of a seasonal component, a trend component, and the residuals. To obtain a stationary time series, various techniques can be applied. A moving average filter is an optimal tool to reduce the random noise for a given dataset, determining the trend component (Azami et al., 2012). Moving average filters are also known as smoothers and was described by Chan (2011):

$$Sm(X_t) = \sum_{r=-q}^{8} a_r X_{t+r}. \quad (3)$$

The seasonal component will be determined by averaging for each unit of time across all periods (Moncrieff et al., 2004). The error time is the remainder after the seasonal, and the trend component was deducted from the original dataset. After achieving stationarity by representing the data of a trend, seasonal and residual component, the time series can be further processed. The first spectral representation theorem states that the connection between the covariance function of a stationary process and the spectral distribution function is given by a Fourier transform (Chan, 2011). The spectral representation of the autocovariance function of a stationary process

$$\{Y_t\} \text{ is given if } \gamma(k) = \int_{-\pi}^{\pi} e^{ik\lambda} dF(\lambda), \quad (4)$$

where F has the properties $[-\pi, \pi]$ with $F[-\pi] = 0.$

The second spectral theorem links the stationary process $\{Y_t\}$ to another stationary process with independent increments $\{Z(\lambda)\}$ via the spectral domain. In other words:

$$Y_t = \int_{-\pi}^{\pi} e^{it\lambda} dZ(\lambda). \quad (5)$$

According to the third theorem, the spectral density function is connected to the autocovariance function. The ACF of a stationary process Y_t is given by $\gamma(\cdot)$ if it is even, and the function is given as follows (Chan, 2011):

$$f(\lambda) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma(k) e^{-ik\lambda} \geq 0. \quad (6)$$

The first spectral representation theorem (Brockwell & Davis, 2016) showed that the expression of a time series using Fourier coefficients is extremely important in the field of signal processing and structural design. Fourier components represent sinusoidal waves of different frequencies. Using the frequency domain approach, according to Montgomery, Jennings, and Kuhlahci (2015), the Fourier representation is denoted as:

$$y_t = \sum_{k=1}^{n} a_k \sin\left(2\pi t \frac{k}{n}\right) + \sum_{k=1}^{n} b_k \cos\left(2\pi t \frac{k}{n}\right), \quad (7)$$

where a and b are considered Fourier coefficients and expressed as:

$$a_t = \frac{2}{n} \sum_{k=1}^{n} \cos\left(2\pi t \frac{k}{n}\right), \quad (8)$$

and

$$b_t = \frac{2}{n} \sum_{k=1}^{n} \sin\left(2\pi t \frac{k}{n}\right). \quad (9)$$

The spectral density function is given by the Fourier transform of the autocorrelation function. For some cases, the spectral density provides a simpler interpretation compared to the autocorrelation function. This is because spectrum estimates at neighboring sample autocorrelations are widely independent, while the autocorrelation function reveals dependencies (Montgomery et al., 2015). In order to obtain the spectral density, the Fourier coefficients are a and b in the form of

$$P\left(\frac{k}{n}\right) = \frac{n}{2}\left(a_k^2 + b_k^2\right) \quad (10)$$

and smoothed consequently. Further, the Fourier coefficients reveal the following relationship using the identity

$$e^{i\theta} = \cos\theta + i\sin\theta. \quad (11)$$

By exploiting the relationship mentioned above, the magnitude (C) is given by $\sqrt{a^2 + b^2}$. The angle (also referred to as phase or ϕ) is rewritten as

$\tan^{-1}(b/a)$. The period λ (i.e., the time it takes a wave to go through a whole cycle) is connected with the frequency ω by $\lambda = 2\pi/\omega$. Generally, the frequency ω denotes the angular speed measures in radians per unit of time. ϕ represents the angular amount by which the sine wave is shifted. The time shift is, therefore, given by ϕ/ω (Cochrane, 1997).

Following the simulation approach as proposed by Fumi et al. (2013), to obtain interpretable results, all $f_{N/2}$ Fourier coefficients except for f_0 need to be sorted in decreasing order regarding amplitude and consequently, the inverse Fourier transform needs to be applied $N/2$ times progressively. The component's amplitudes n_t can then be calculated as the absolute value of the complex number $A_n = a_n + b_n$, where *a* is the real part and *b* is the imaginary part. According to Fumi et al. (2013), the forecasts can then be obtained using the formula

$$y(t) = amplitude_0 + amplitude_1 \times \times \cos(2\pi \cdot frequency \cdot t - phase). \quad (12)$$

The Wiener process can be considered one of the most important stochastic processes in theory and application, and it poses a particular type of Markov process with a mean change of 0 and a variance rate of 1, with its main properties being given by $W_0 = 0$, the trajectories are continuous functions of $t \in [0, \infty]$, the expectation $EW_t = 0$, the correlation function is given by

$$E(W_t W_s) = t \wedge s, \quad (a \wedge b = \min(a,b)), \text{ for any } t_1, t_n \quad (13)$$

the random vector $\left(W_{t_1}, W_{t_n}\right)$ is Gaussian. For any s, t it holds that

$$Ew_t^2 = t \text{ as well as} \quad (14)$$

$$E[W_t - W_s] = 0 \text{ and} \quad (15)$$

$$E[W_t - W_s]^2 = |t - s|. \quad (16)$$

Increments of the Wiener process on non-overlapping intervals are independent, i.e., for $(S_1, T_1) \cap (S_2, T_2) = \phi$ the random variables $Wt_2 - Ws_2$, $Wt_1 - Ws_1$ are independent, with the paths of a Wiener process not being differentiable functions and the martingale property (Hull, 2009).

$$W_0^s = \{W_u, 0 \le u \le s\}, \quad (17)$$

$$E\left(\frac{W_t}{W_0^s}\right) = W_s, \quad (18)$$

$$E\left\{(W_t - W_s)^2 \middle\| W_0^s\right\} = t - s. \quad (19)$$

A variable z is said to follow the Brownian motion if it satisfies the following conditions (Hull, 2009). The change Δz during a small period Δt is

$$\Delta z = \in \sqrt{\Delta t}, \quad (20)$$

where $\in$ follows a standard normal distribution $\phi(0,1)$. Additionally, for every 2 different short intervals of time Δt have independent values of Δz and it peruses from the first condition 1 that Δz has a normal distribution itself, with mean of $\Delta z = 0$, standard deviation of $\Delta z = \sqrt{\Delta t}$ and variance of $\Delta z = \Delta t$.

Considering a long period of time T the changes of the value of z can be indicated as $z(T) - z(0)$ and it can be esteemed as the sum of changes in z in N small time intervals of length Δt where: $N = T/\Delta t$ and $z(T) - z(0) = \sum_{i=1}^{N} \in_i \sqrt{\Delta t}$, where $\in_i$ $(i = 1, 2, \ldots N)$ are distributed $\phi(0.1)$, knowing from condition 2 that $\in_i$ are independent of each other. From the last equation of $z(T) - z(0) = \sum_{i=1}^{N} \in_i \sqrt{\Delta t}$, it has to hold that $z(T) - z(0) = 0$, and the variance of $z(T) - z(0) = N\Delta t = T$ and the standard deviation is given by $z(T) - z(0) = \sqrt{T}$.

Finally, to determine the quality of the forecast results derived by both the Wiener process and the spectral time series analysis, Khair et al. (2017) proposed using the mean absolute percentage error to evaluate the accuracy of a prediction. The MAPE is given by the following formula:

$$MAPE = \frac{\sum \frac{\left|z_1 - zt'\right|}{z_1}}{n} \cdot 100. \quad (21)$$

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4. RESULTS

To set up a precise model that allows meaningful forecasts, the stationarity requirement needs to be met. To test for stationarity, the ADF test has been performed. When performing the ADF test on the raw, unprocessed dataset of the Aurubis stock price, the test statistic revealed a value of 0.843. Since the *Z*-scores of all three confidence intervals (99%, 95%, and 90%) were smaller than the test statistic's value, it can be concluded that the raw dataset has a unit root. A unit root implies that the process is non-stationary and needs further adjustment for modeling. By having a closer look at the Aurubis stock price plot, it is visible that the price oscillates around a persistent mean for long periods. This observation implies that the time series can be decomposed using the additive model approach (Brockwell & Davis, 2016). Representing the time series as trend, seasonal and residual components draw the following picture (see Figure 1).

After the decomposition of the raw dataset of the Aurubis stock price into trend, seasonal and random components, the residuals can only be used for further calculations if they fulfill the stationarity requirement. Since no meaningful statistical models can be obtained using a non-stationary time series (Warner, 1998), another Augmented Dickey-Fuller test is inalienable. On applying the ADF test on the residuals, the value of the test-statistic has improved significantly to around –6.0. This implies that there is no unit root, and hence the time series can be considered stationary, and no further modeling regarding the stationarity of the time series is needed. To obtain the spectral density, the Fourier coefficients *a* and *b* in the form of

$$P\left(\frac{k}{n}\right)=\frac{n}{2}\left(a_k^2+b_k^2\right) \qquad (22)$$

are determined and smoothed. The unsmoothed periodogram reveals the picture presented in Figure 2.

When applying the Fast Fourier transform, complex numbers are returned. From those, the approach mentioned above helps extract the information required to produce forecasts. It is possible to derive the amplitude, frequency, and phase that form the stock price pattern by transforming the complex numbers. Estimating the Fourier coefficients using the frequency domain approach as

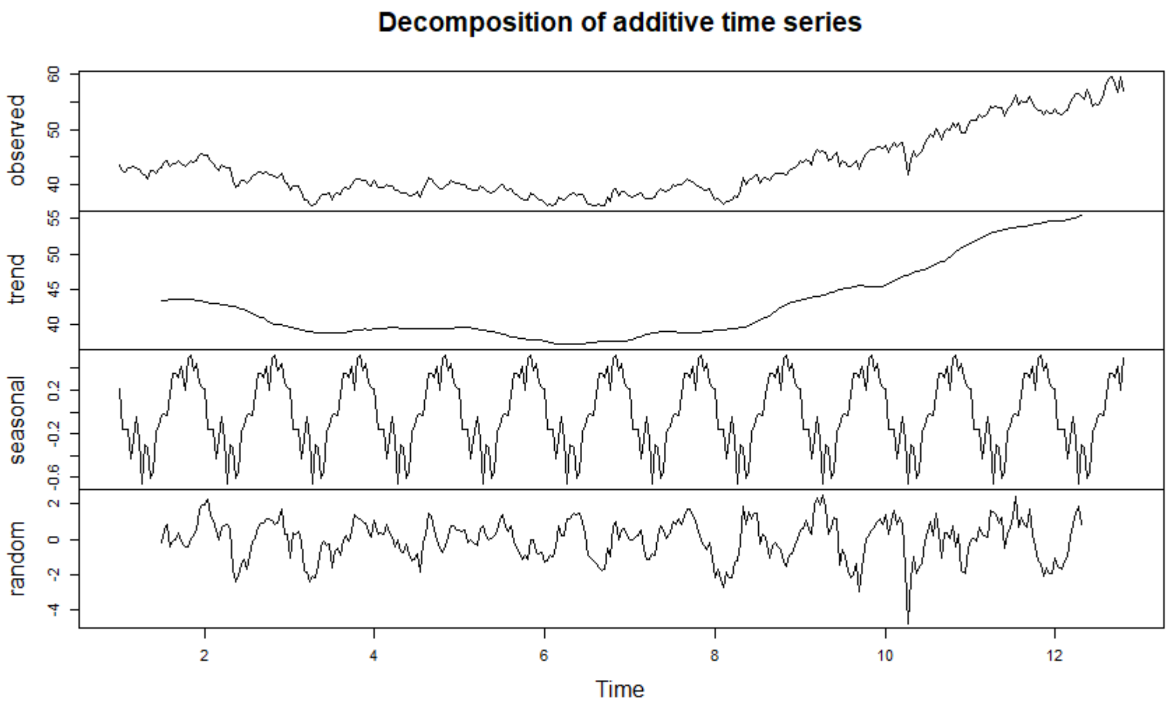

Figure 1. Decomposition of Aurubis stock price

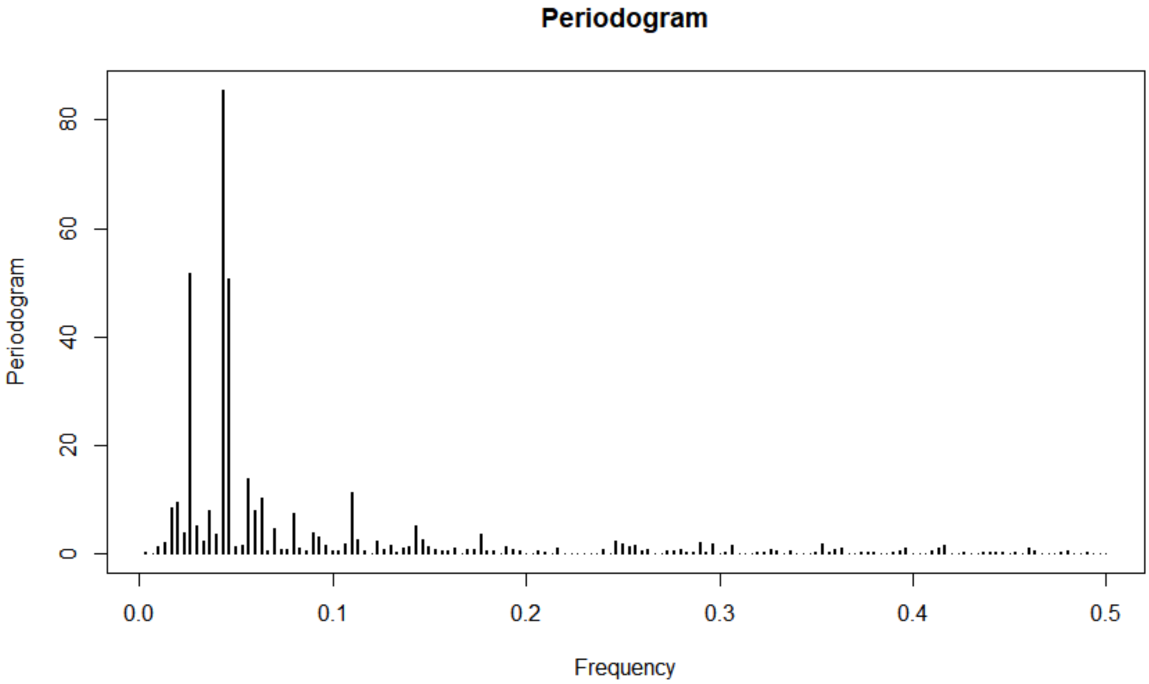

Figure 2. Periodogram

proposed by Montgomery et al. (2015) and transforming the following would be achieved for the first three Fourier components (see Table 1).

Table 1. Fourier components

Component	Amplitude	Frequency	Phase
f_0	0.63816	–	–
f_2	4.28328	0.00170	–2.44645
f_5	1.7028	0.00086	1.01149

According to Fumi et al. (2013), the forecast can be obtained using the formula

$$y(t) = amplitude_0 + amplitude_1 \times$$
$$\times \cos\left(2\pi \cdot freguency \cdot t - phase\right).$$

Applied on the Aurubis stock data set, this would mean the following:

$$y(t) = 0.63816 + 4.28328 \times$$
$$\times \cos\left(2\pi \cdot 0.0017 \cdot t + 2.44645\right) +$$
$$+1.7028 \cdot \cos\left(2\pi \cdot 0.00086 \cdot t - 1.01149\right).$$

Since the forecast was only performed on the stationary residuals, the re-application of trend and seasonal component is required. Using 20 harmonics, the forecast would draw the following picture for the first 5 forecasts (starting from 11 September 2018).

Summing up the results, the mean absolute percentage error looking at a time window of 137 trading days (from 11 September 2018 to 28 March 2019) would be roughly 5.0129% when using the spectral time series analysis. This result was achieved using 20 harmonics. When simulating the stock price returns under the Wiener process,

Table 2. Aurubis stock forecasts

Date	Residual forecast	Trend component	Seasonal component	Sum
11 September 2018	–0.36165	59.72654	1.41122	60.77612
12 September 2018	–0.49306	59,63038	2.07489	61.21221
13 September 2018	–0.61332	59,53126	2.25107	61.16900
14 September 2018	–0.72159	59,43728	2.83664	61.55232
17 September 2018	–0.81746	59,33934	2.96759	61.48946

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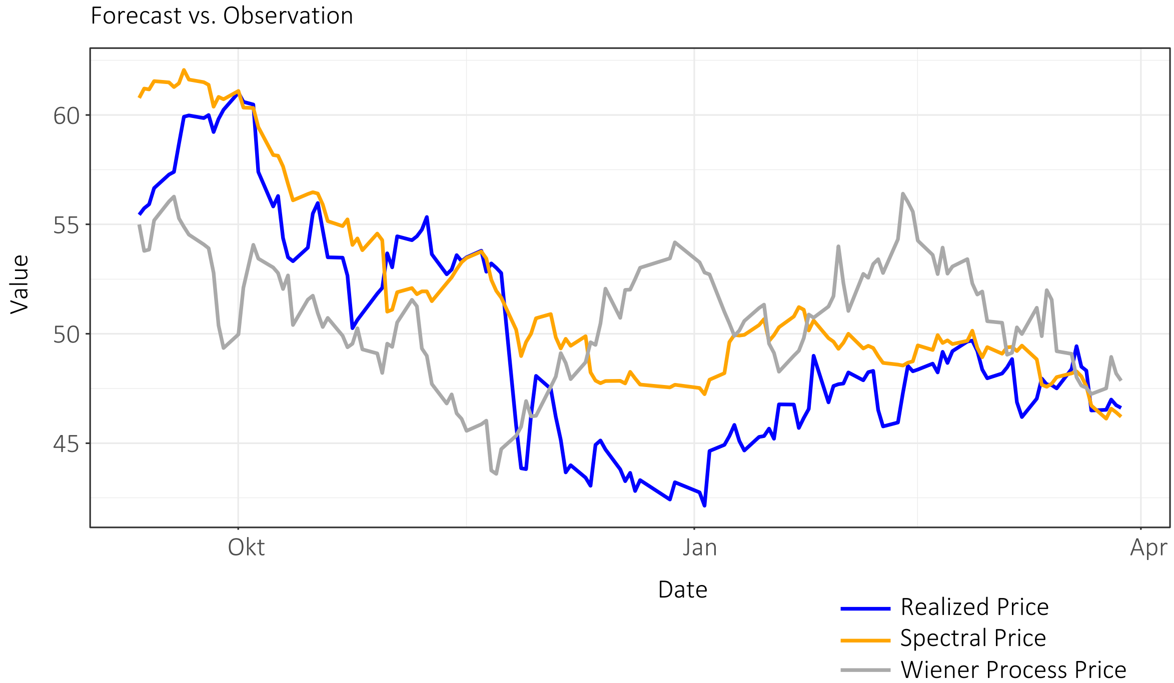

Figure 3. Comparison of simulation results vs the realized stock price

a mean absolute percentage error of about 6.2103% was achieved.

When plotting the obtained forecast results against the observed variables, the forecast would be pictured, which is presented in Figure 3.

5. DISCUSSION

Fumi et al. (2013) showed that Fourier transform leads to an improvement of around 30% regarding forecasting quality compared to exponential smoothing and moving average. Nonetheless, the MAPE remains at around 74% and 43% for both forecasted and tested categories. The study concluded that spectral analysis using the Fourier transform provides advantages over traditional ways of forecasting stock prices. These advantages include its little implementation effort and the entirely non-parametric approach. Further, the achieved results were considered satisfactory. Grzesica and Wiecek (2016) indicated that spectral analysis provides better estimates than the Brown model and the classical ARMA model. Further, they proved that using spectral time series analysis to analyze a time series in the frequency domain leads to the most accurate results based on the average forecast error results. The MAPE using spectral analysis was around 0.39%. Using the singular spectrum analysis, Hassani (2007) predicted the number of accidental deaths in the USA. His work showed that a mean average percentage error of around 2% could be achieved using spectral analysis. Spectral analysis outperformed the other forecasting techniques, including the Holt-Winter algorithm and the SARIMA approach introduced by Box and Jenkins in 1970. Marques et al. (2006) also forecasted a hydrological univariate time series using spectral analysis. The SSA was applied to find and extract the trend and oscillatory component from the series to produce high-quality predictions. The paper discovered and supported as well that spectral analysis could forecast the extracted components accurately. All the studies mentioned above considered their achievements reasonable, since spectral time series performed better than other forecasting techniques. In contrast, the 0.39% MAPE of Grzesica and Wiecek (2016) and the 2% MAPE attained by Hassani (2007) can be considered satisfactory regarding stock price forecasting, the 74% and 43% MAPE achieved by Fumi et al. (2013) are dissatisfactory.

Summing up the results, within this study, a MAPE of about 5% was achieved when considering a trading window of 137 days under spectral time series analysis. When employing a Wiener process under the same conditions, i.e., for the same calibration and validation dataset, a MAPE of about 6% is achieved. Hence, the spectral time series analysis delivered better results regarding the forecast of the Aurubis stock price.

The results support both the findings of Hassani (2007) and Grzesica and Wiecek (2016) that spectral time series analysis poses a suitable tool for the analysis of financial data. While the Wiener process has received great attention regarding forecasting stock price returns, spectral time series analysis was not employed to predict stock price movements. The low MAPE indicates that spectral time series analysis poses an adequate means for creating stock price return forecasts while delivering superior results when comparing against the widely used Wiener process.

Forecasting stock prices can be considered an important task in financial investing since it helps investors make superior investment decisions. In this regard, Aurubis has delivered strong financial ratios for 2018 achieving ROA of 5.91%, ROE of 10.63%, and EPS of 4.6. Additionally, the company delivered a strong EBITDA margin of 4.41% and a dividend yield of 3.32% (Bloomberg, 2018), driven by superior management decisions. In this regard, Kim et al. (2016) have shown that overconfidence of the top management is negatively correlated with the stock price performance. This is because overconfident management overestimates future cash flows of investments. Ultimately, the overconfidence poses a bigger threat for stock price crashes (Kim et al., 2016). The study of Fujianti (2018) has shown that top managers' age is also influencing the company's performance tremendously. Older managers seem to lead to a better stock price performance due to enhanced experience. Additionally, Fujianti (2018) found that neither the educational background nor gender influences the company value significantly. Cooper et al. (2016) found that excess compensation on the management level negatively influences its stock price performance.

CONCLUSION

This paper's main objective was to evaluate whether spectral time series analysis poses an adequate tool for creating stock price forecasts. This can be considered indispensable for an investor's risk management as well-grounded investment performance forecasts lead to superior risk management and realistic anticipation of the expected return. To judge its ability, the obtained results were compared against the results obtained under a Wiener process.

The paper used the Aurubis stock price as a proxy for evaluating spectral analysis as a forecasting model. Aurubis is the largest copper recycler in the EU, employing 50,000 people to attain economic importance. Additionally, its steady dividend policy and economic and financial soundness make it an attractive investment for investors who seek to increase their portfolio performance while diversifying their exposure among different industries.

The deviation achieved within this study when comparing realized returns against observed returns was roughly 5.0129% under the usage of spectral time series analysis and 6.2103% when using a Wiener process. Achieving a low MAPE of around 5.0129% when comparing the forecasted Aurubis stock price against the observed stock price and the Wiener process, the results can be considered adequate. From this, it follows that spectral time series analysis poses an adequate means to produce stock price forecasts.

Based on the results, which reveal superiority of the spectral time series analysis over the Wiener process, it is concluded that spectral analysis using the Fourier transform provides advantages over traditional ways of forecasting stock prices. These advantages include its little implementation effort and the entirely non-parametric approach. Lastly, the low MAPE also supports the usage of spectral time series analysis as a forecasting tool.

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