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Portfolio optimization models and a tabu search algorithm for the Kuwait Stock Exchange

Abstract

Two mathematical models are developed to study and compare results of two cases of the portfolio selection problem in the Kuwait Stock Exchange (KSE) as an emerging market. The mathematical models are attempting to balance the trade off between risk and return by maximizing the expected return while maintaining the risk measures to certain limits. Model A measures the expected return by the moving average forecasting technique, while Model B measures the expected return by the random walk forecasting technique. Both models, which turned to be non-linear, are solved using a tailored tabu search heuristic algorithm to provide efficient solutions with reasonable amount of computational times. After testing the models using real data from KSE, the results indicated that Model A is able to beat the market in a significant manner.

Keywords: portfolio selection, optimization, mathematical modeling, tabu search.
JEL Classification: G1, C0, C6.

Introduction

Achieving maximum returns with minimum risk is the ultimate goal of investors worldwide. The relationship between return and risk is obvious in stock markets, where most stocks granting high returns are very risky. Therefore, investors sometimes search out portfolios that balance the trade off between risk and return. Markowitz's seminal work on portfolio selection in the 1950s inspired researchers to study the effectiveness of asset portfolio optimization. Special attention was given to stock markets. Theoretically speaking, when the stock market is reasonably efficient, there is little room for ordinary investors to make excess returns as information of any kind, public or private, is of little use in beating the market. However, when the market is inefficient, it is logical to assume the possibility of beating the market through manipulation of public information. The Kuwait Stock Exchange (KSE) is an emerging market that has been found by many studies to have weak efficiency (see for example Al-Loughani, 1995), 2000a and 2000b; Al-Loughani and Chapell, 2000; and Al-Loughani and Moosa, 1999). One study of particular interest for this paper is the work of Al-Loughani, Al-Deehani and Al-Saad (2004) which focuses on portfolio selection. They tested the validity of the *Dow-10* investment portfolio selection strategy in the KSE. The results of their work revealed that the risk-adjusted returns of the *Dow-10* portfolio were much higher than the returns of the market portfolio. By suggesting a simple portfolio selection method that can be used to make returns higher than the market, they actually provided an additional implicit proof of KSE weak efficiency.

In this paper authors intend to contribute to the insufficient literature on portfolio optimization topic in emerging markets. Due to the reasons of accessibility and handiness KSE was selected. Following recent research awareness of new optimization techniques, the tabu search as an approach for investment portfolio selection in the KSE was introduced.

In the remainder of the paper the relevant literature is discussed followed by a description of the distinguishing properties of the KSE. Next the research methodology and the problem formulation are discussed, i.e. how to deal with its main factors followed by the description of the mathematical model. To provide a better understanding of the methodology, a section describing the tabu search heuristic algorithm as an optimization method was intentionally included. In the section that follows the data and the results of the model estimations are discussed. The final section deals with concluding remarks and recommendations.

1. Literature review

Considering the theoretical arguments on the relationship between market efficiency and the possibility to make excess returns, in order to achieve maximum returns researchers have been striving to develop mathematical optimization models to explore such possibilities.

Markowitz (1952) was one of the first who formulate the portfolio selection problem. The introduced model was to minimize the risk, represented by the covariance, subject to a certain bound of expected return (see also Markowitz, 1959). Konno and Yamazaki (1991) were the first to enhance Markwitz work proposing a linear version of the objective function using the simplex methodology. Based on the notion of skewness and semi-variance, both Markowitz et al. (1993) and Konno and Suzuki

(1995) introduced more complex functions to capture the effect of risk. In the subject of tail risk measures, Rockafellar and Uryasev (2000) and Rockafellar and Uryasev (2002) showed new methodologies of using both Value-at Risk (VaR) and Conditional Value-at-Risk (CVaR), which are appropriate to be applied when the portfolios return is not following the normal pattern. Work by Mansini and Speranza (1999) is a good source for reviewing earlier portfolio optimization models. Their paper included some heuristic algorithms. They introduced methods to find a solution close to optimal (heuristic solution) with a reasonable amount of computational time. Similar approach was employed by Aldaihani and Al-Deehani (2004) to solve portfolio selection problems in the emerging market of Kuwait Stock Exchange. Interestingly, this is the only research attempt to model portfolio selection in this market.

Deviating from traditional methods, a practical use of local search techniques for portfolio selection was introduced by Rolland (1997) and then by Chang et al. (2000). In an attempt to improve the work of Rolland and Chang et al., Schaerf (2002) explored the use of several local search techniques. Considering the problem of selecting a portfolio of assets that provides the investor with a suitable balance of expected return and risk, he concluded that tabu search works much better than Hill Climbing and Simulated Annealing techniques.

Other optimization techniques for portfolio selection were analyzed and found to be valid. For example, Alexakis et al. (2007) introduced a dynamic approach to evaluate portfolio performance under risk conditions and applied simulation to recommend the optimal portfolio composition. Yang (2006) used a Genetic Algorithm (GA) approach to improve portfolio efficiency. Lui (2007) explored portfolio selection in a stochastic environment combined bond and stock components.

In emerging markets where weak efficiency is evident little work has been done to explore portfolio selection. Therefore, for reasons mentioned earlier the papers is focused on the KSE. Al-Loughani and Moosa (2000) tested the efficiency of the KSE using a moving average rule. They studied the market for the time periods of 1986-1990 and 1992-1997, and found some evidence demonstrating that the KSE was inefficient. Al-Loughani (1995) studied the application of the Random Walk rule in thinly traded stock markets. Specifically he studied the KSE and showed its inefficiency when sophisticated tests are used. The most part of other researches conducted on the KSE is merely statistical analysis.

Although it has been proved that the KSE market does not enjoy strong efficiency, in the literature there is little work in introducing applications of optimization models for the KSE to explore possibilities of making excess returns. The only known study is the one conducted by Aldaihani and AL-Deehani (2004). Employing an integer programming mathematical modeling, they provided the first evidence of valid optimized portfolio that outran the market while maintaining a level of risk equal to or lower than that of the market. Departing from traditional optimization modeling, we follow in this paper, a new strand of research focusing on local search techniques to model portfolio selection in the KSE.

Before discussing the methodology and analysis, it would be interesting to introduce the properties of the selected stock market in the next section.

2. Kuwait Stock Exchange properties

Although informal trading of stocks in Kuwait had started in 1952, organized and controlled trading did not begin until 1983. Compared to all Arab stock markets, the KSE has the highest turnover ratio. It is ranked second in terms of value traded and third in terms of market capitalization. In the year 2002, the KSE's market capitalization was $35.1 billion representing about 45% of all Gulf Cooperation Council (GCC) countries' stock markets and about 17% of all Arab stock markets. The value trade for the same year was $22.1 billion which represents about 40% of GCC countries and about 34% of all Arab stock markets. At the end of 2002, there were 96 listed companies, 10 of which were not Kuwaiti. In 2004 the total number of listed companies has increased to 111. By the end of 2007 the number has grown to 194 companies, 15 of which are non-Kuwaiti (GCC predominantly).

Common stock is the only financial security traded in the KSE. Short selling is not allowed. Although not practiced by the vast majority of traders, organized margin and call option trading are available through only specific providers. Trading is settled through brokers that are prohibited by law from providing any advice.

Ever since the start of its formal operations, the KSE can only be described as instable. This is due to major financial and political factors. These are, the Iraq-Iran War 1980-1988, and the AL-Manakh financial crisis that started at the end of the 1970s. Its consequences still persist. The Gulf War in 1990 added more to the volatility of the market and still persists. And lately, the consequences of the war against Iraq in 2003 that still persist. These conditions along with other socio-economic factors have made the KSE a manipulative market. Compared to the regional GCC markets (except Oman), the KSE seemed the most volatile (Al-Deehani 2004). Therefore, short-term investment and market manipulation appear to be logical investment strategy for the KSE investors. A comprehensive description of the KSE main characteristics can be found in AL-Loughani and Moosa (1999).

3. Problem formulation and methodology

In this research two mathematical models are developed to determine risk and return balanced portfolios in the KSE. The models, which are solved by Tabu Search (TS) heuristic algorithm, identify the portfolio size in terms of number of stocks as well as the selected stocks in the portfolio. The percentage to be invested in each stock is assumed to be distributed equally among the portfolio components. The main contribution of this research is to check whether there is room for optimization in the KSE or not. The ultimate technical goal of the optimization models is to find a portfolio which maximizes the expected return subject to a certain acceptable limit of risk. Unlike previously employed methods, the proposed model takes into account variety of risk measures such as correlation, variation, and number of stocks. The developed optimization model uses only past real data from the KSE, including stock name, sector name, date, and price. The models are tested using these real data by comparing their portfolios performance measures (actual returns) to the market index. The market is beaten if a model generated portfolio provided greater return than the change in the market index for the same period of time.

The following section represents the mathematical description of the problem. Let $i \in N$ represent the stocks in the market and $s \in M$ represent the sectors in the market. For each stock "i" in sector "s", there is a computed standard deviation $\sigma_{i,s}$ measured according to historical data, and an expected return $r_{i,s}$ determined by a forecasting method. The standard deviation of a stock is measured using the previous eight periods (quarters) and the expected return is computed using forecasting rules. Model A applies the Moving Average (MA) rule using two periods, while Model B employs the Random Walk (RW) rule using eight periods. For each pair of stocks in the market "i" in sector "s" and "j" in sector "g", there is a correlation $\rho^{s,g}{}_{i,j}$ that describes the relationship between the two stocks, which might be in the same sector (s = g) or in different ones (s ≠ g). The decision variable in this problem is $x_{i,s}$ which equals one if the stock "i" in sector "s" is selected in the portfolio and equals zero otherwise. On the other hand, there are certain risk limits included in the models. UB_{sd} represents an identified upper bound for the stock standard deviation to decide whether the stock is eligible to enter the portfolio or not. UB_{ms} denotes the average standard deviation of all the stocks in the market in the previous period (to be used as an upper bound for the portfolio average standard deviation for the current period). UB_{ρ} represents the upper bound for the portfolio correlation. The objective function of the problem is to select a subset "n" stocks from the total N stocks in the market that maximizes the expected return while at the same time satisfies all the risk constraints which are represented by variation, correlation, and number of stocks.

3.1. Risk's constraints. Since the proposed mathematical model is used to balance the trade off between average expected return of selected stocks in a portfolio and the risk associated with selecting these stocks, it is important here to describe the constraints that hold the risk limits.

3.1.1. Correlation. In some instances, the objective of studying the joint behavior of two stocks is not to use one stock to predict the other, but to check whether they are related. Naturally speaking, stock A and stock B would have a positive relationship, if large A's are paired with large B's and small A's are paired with small B's. Similarly, if large A's are paired with small B's and small A's are paired with large B's, then a negative relationship between the stock is implied. This is actually studied by computing the correlation between the stocks that are selected in the portfolio. It is required to limit the selected stocks to predetermined bounds. This definitely helps in avoiding a sudden collapse of the portfolio.

3.1.2. Variation. One way for evaluating the investment risk in a stock is to check its variability. There is no doubt that for two stocks with similar expected returns, it is more safe to invest in the one that has less variation. This is the main reason for restricting the average standard deviation of the portfolio to values less than or equal to the average standard deviation of the market in the previous period. Furthermore, an additional constraint is set to limit the standard deviation of each selected stock in the portfolio.

3.1.3. Portfolio size. It is crucial to identify the required number of stocks that the portfolio contains. The "Dow ten" provides a pattern for this constraint. The number of stocks to be selected in the portfolio is bounded by setting the range from 5 to 15 stocks (+/- 50% of 10 stocks). The model is also capable of bounding the number of stocks in each sector separately.

Before closing this subsection, an important remark must be made concerning the other common risk measures, such as the tail risk measures, that might be used in the portfolio optimization. It is statistically recognized that when the distribution of returns is assumed to follow a normal pattern, the probability that returns will move between the mean and three standard deviations, either positive or negative, is 99.97%. However, the concept of tail risk suggests that the distribution is not normal, but skewed, and has fatter tails. The fatter tails increase the probability that an investment will move beyond three standard deviations. Instead of standard deviation, many other tail risk measures might be applied in our models according to the nature of the stock market in which the portfolio mathematical models are used. Among the most important ones are the Value at Risk (VaR) measure and its updated version – the Conditional Value at Risk (CVaR) measure. CVaR might be incorporated in our model if the objective is to evaluate the portfolios' risk in a very conservative way. Technically speaking, CVaR focuses on the less profitable outcomes by controlling a parameter value of q. For high values of q it ignores the most profitable but unlikely possibilities, for small values of q it focuses on the worst losses. On the other hand, unlike the discounted maximum loss even for lower values of q expected shortfall does not consider only the single most catastrophic outcome. A value of q often used in practice is 5%. Excellent theoretical and practical coverage of both VaR and CVaR concepts is done by Rockafellar and Uryasev (2000) and Rockafellar and Uryasev (2002).

3.2. Mathematical model. Prior to a detailed presentation of the mathematical formulation, we summarize the notations that are used in the model:

i – index of stocks;

s – index of sectors;

N – set of all stocks in KSE;

M – set of sectors in KSE;

M_s – set of stocks in sector "s";

$x_{i,s}$ – decision variable: binary variable which equals 1 if the stock "i" in sector "s" is selected in the portfolio, and equals 0 otherwise;

$r_{i,s}$ – expected return of stock "i" in sector "s" determined by historical data (past 2 periods for the MA technique "Model A" and 8 periods for the RW technique "Model B");

$\rho^{s,g}_{i,j}$ – correlation between the stocks "i" in sector "s" and stock "j" in sector "g";

$\sigma_{i,s}$ – standard deviation of stock "i" in sector "s";

UB_ρ – upper bound for the portfolio correlation;

UB_{ms} – upper bound for the portfolio average standard deviation (market average standard deviation in the previous period);

UB_{sd} – upper bound for the standard deviation of each stock separately;

UB_T & LB_T – maximum and minimum number of stocks in the portfolio;

UB_S & LB_S – maximum and minimum number of stocks in sector "s".

The mathematical formulation is presented below:

$$max \quad z = \frac{\sum_{s=1}^{M} \sum_{i=1}^{Ms} x_{i,s} r_{i,s}}{\sum_{s=1}^{M} \sum_{i=1}^{Ms} x_{i,s}} \tag{1}$$

subject to

$$\sum_{s=1}^{M} \sum_{g \geq s}^{M} \sum_{i=1}^{Ms} \sum_{j \rangle i}^{Mg} x_{i,s} x_{j,g} \rho^{s,g}_{i,j} \leq UB_\rho \sum_{s=1}^{M} \sum_{i=1}^{Ms} x_{i,s}, \tag{2}$$

$$\sum_{s}^{M} \sum_{i}^{Ms} x_{i,s} \sigma_{i,s} \leq UB_{ms} \sum_{s}^{M} \sum_{i}^{Ms} x_{i,s}, \tag{3}$$

$$x_{i,s} \sigma_{i,s} \leq UB_{sd} \qquad \forall i \in N, s \in M, \tag{4}$$

$$LB_T \leq \sum_{s}^{M} \sum_{i}^{Ms} x_{i,s} \leq UB_T, \tag{5}$$

$$LB_s \leq \sum_{i}^{Ms} x_{i,s} \leq UB_s \qquad \forall s \in M, \tag{6}$$

$$x_{i,s} = \{0,1\} \qquad \forall i \in N, s \in M. \tag{7}$$

Equation (1) represents the objective function of the mathematical model to maximize the average expected return. This is achieved by dividing the total portfolio expected return by the number of selected stocks. Constraint (2) guarantees that the average correlation of the selected stocks cannot exceed an upper bound. Constraint (3) assures that the average standard deviation of the selected stocks in the portfolio may not exceed the average standard deviation of the market in the previous period. Constraint (4) identifies a standard deviation limit for each stock separately. Constraint (5) limits the portfolio size in terms of the number of the selected stocks to be between lower and upper bounds. Constraint (6) provides a capability to force the portfolio to include some stocks, between lower and upper bounds, from any sector in the market. Constraint (7) describes the decision variables of the model as binary variables.

Two mathematical models are generated by the above formulation according to the way the expected return is determined. Model A measures the expected return by the moving average forecasting technique, while Model B measures the expected return by the random walk forecasting technique. Both models, which turned to be non-linear, are solved using a tailored tabu search heuristic algorithm to provide efficient solutions with reasonable amount of computational times.

3.3. Tabu search algorithm. As indicated in the flow chart in Figure 1, the tabu search algorithm in the initial iteration (t = 0) gets a feasible portfolio "$P_n(0)$" consisting of "n" stocks, using a quick enumeration. At this stage, the objective function is not given a priority, since the goal is to look for any feasible solution that satisfies all the constraints described in the mathematical model. The feasible solution is then improved by an efficient and smart searching technique utilizing the tabu memory, which is created to avoid getting trapped by any cycles and local minimums. The concept of neighbor solutions (portfolios) plays an important role in TS and it is crucial here to define it. Two portfolios are neighbors to each other when they both contain the same number of stocks (n) and both have (n-1) stocks in common. For example, *P1={a, b, c, d, e}* and *P2={a, h, c, d, e}* are neighbors to each other since they both contain 5 stocks and have 4 stocks in common.

In each iteration, the TS algorithm evaluates all the feasible non-tabu neighbor portfolios to the current one. The best neighbor portfolio is selected for the next iteration regardless whether it is better than the current approximate optimal or not. On the other hand, the approximate optimal is updated if a better portfolio is found in each iteration. To avoid cycling in one region, the tabu memory keeps a record for the latest moves so that they will not be visited again for an identified number of iterations. To evaluate different portfolio sizes, the procedures might be repeated for each size ($LB_T \leq n \leq UB_T$) and ultimately the best portfolio (in terms of size and objective function) is selected.

In addition to the problem size factor in terms of *N* and ($LB_T \leq n \leq UB_T$), the experimental computational time of TS also depends on the identified maximum number of iterations "t_{max}" and the tabu memory maximum iterations "$tabu_{max}$". The experimental tests in this research is done using 1.5 GHz machine with 512 MB of RAM. In all the experimental runs, the PC time did not exceed 5 minutes when we set t_{max} = 100 and $tabu_{max}$ = 10.

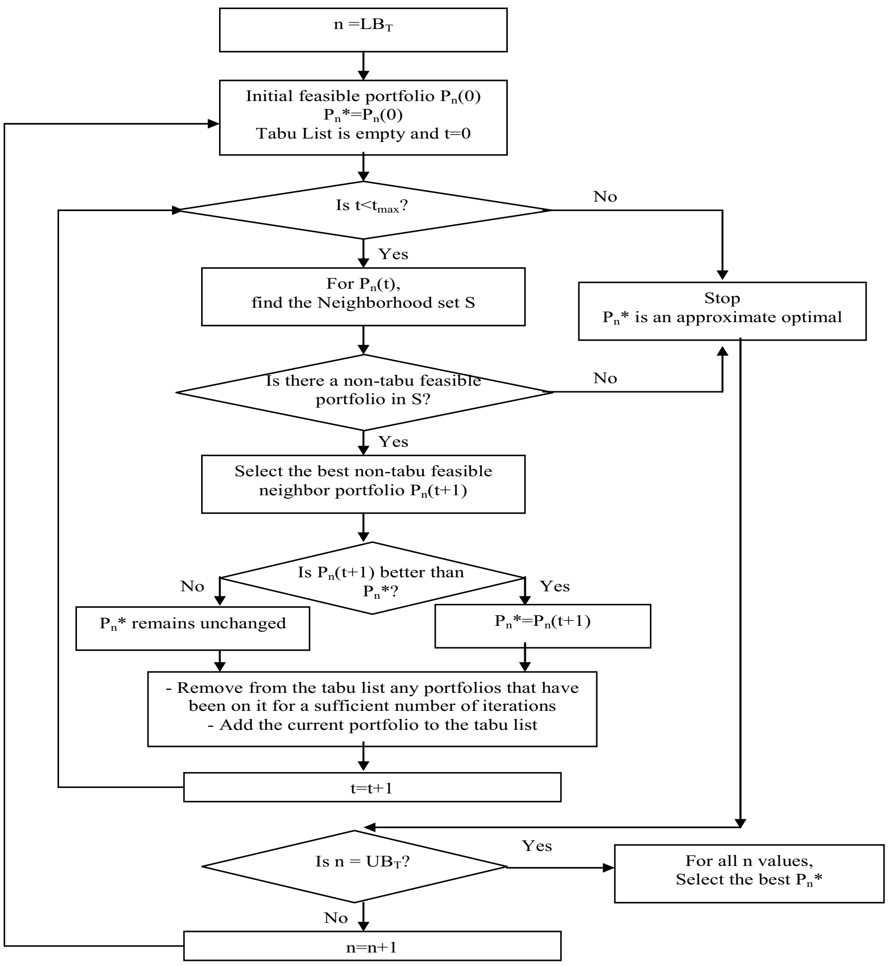

Fig. 1. Tabu search flow chart for the portfolio optimization

4. Experimental tests

Data were collected from the KSE for five historical years. The data included date, stock name, sector name, stock price, and market index. The KSE consists of a number of companies that are categorized, according to their business, under 8 main sectors. The names of the main sectors and the number of companies in each one are shown in Table 1.

Table 1. Sectors and number of companies in the KSE

Sector	Number of companies
$UB_{\square}$	8
Investment	14
Insurance	4
Estate	8
Industry	14
Services	11
Food	4

Table 2. Constraints parameters considered in the models

Parameter	Value
$UB_{\square}$	0.5
UB_{ms}	Market average in the previous period
UB_{sd}	0.3
LB_T & UB_T	5 and 15
LB_S & UB_S	0 and max number of stocks in s

The numbers below vary over time, due to either new companies entering or out of business companies leaving the stock market. The considered companies in this research are the ones that are sufficiently represented, data-wise. The models are tested by applying quarterly basis and annual basis strategies. In each one, the portfolios generated by the models are compared to the market return in the same time interval. The constraints parameters used in the model are shown in Table 2.

4.1. Quarterly basis strategy. The optimization models are tested using real data from the KSE for the time period from year 1994 until year 2001. The models use historical data consisting of 8 periods (quarters) for the eligible companies in the current market (companies with sufficient historical data) in order to estimate the model's parameters such as correlation, standard deviation and expected return. The portfolio is generated on a quarterly basis and compared to the market index. Table 3 shows the standard deviation and actual return for the market, Model A and Model B respectively. As shown in the table, the periods are categorized into over-market periods and under-market periods, according to their performance comparing to the market. The over-market periods are the ones when the generated portfolio provides better return than the market while the under-market periods are the ones when the market provides better return than the selected portfolio. Model A generated 5 over-market periods which are periods 1, 2, 3, 4 and 5, while Model B generated 4 over-market periods which are periods 2, 3, 4 and 6. In addition to the number of times that the market is beaten, it is also important to consider the percentage value over or below the market illustrated by Figure 2. On the other hand, the models provide low risk with respect to the average standard deviation. This does not come as a surprise since there is a constraint in the model, restricting the generated average standard deviation to be less than or equal to the market average standard deviation in the previous period. Another important remark from Figure 2 is that the models are beating the market while at the same time they are following its growing trend.

Table 3. Quarterly basis strategy performance measures

Investment Period	Market		Model A		Model B	
	Standard deviation	Return	Standard deviation	Actual return	Standard deviation	Actual return
Q1: 3/00-6/00	14.2%	2.76%	13.6%	13.60%	10.9%	-1.00%
Q2: 6/00-9/00	15.5%	1.98%	14.17%	2.00%	11.0%	3.00%
Q3: 9/00-12/00	14.3%	-6.65%	15.4%	-4.50%	10.9%	1.00%
Q4: 12/00-3/01	13.9%	7.86%	14.26%	17.30%	9.9%	10.80%
Q5: 3/01-6/01	13.9%	15.89%	13.4%	35.80%	9.9%	14.80%
Q6: 6/01-9/01	16.0%	-5.04%	13.4%	-10.40%	12.3%	-4.00%
Q7:9/01-12/01	16.1%	6.81%	15.7%	3.60%	11.5%	3.60%

4.2. Annual basis strategy. The performance of the models presented in the previous strategy can be improved significantly when the annual basis strategy is used. In the annual basis strategy, the models generate and accumulate four quarterly basis portfolios in the year and the comparison is done annually between the market and the two models. Note that the portfolios are generated in a way similar to that of the quarterly basis strategy. In other words, the money invested at date 3/00 cannot be retrieved until 3/01 even though there are four portfolios that are generated in between. For example, if \$100 is invested in 3/00, it becomes \$113.6 in 06/00, \$115.9 in 09/00, \$110.7 in 12/00, and \$129.8 in 03/01 which is the end of one year. Hence the return of Model A is 29.8% from 3/00 to

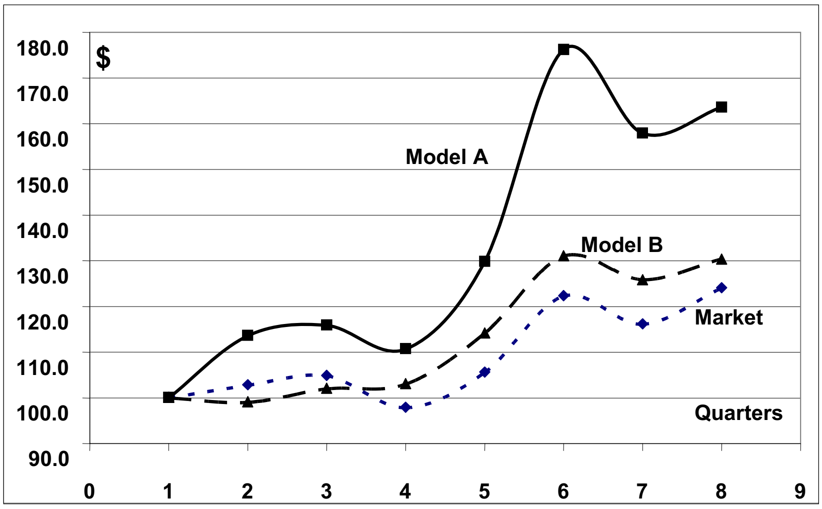

Fig. 2. Accumulating portfolios

3/01 as compared to the market compounded return during the same period of time, which is 5.5%. Table 4 and Figure 3 present a comparison between the two models and the market for 4 investment periods. It is particularly important to note here that the models always provide a portfolio with low risk compared to the market with respect to the average standard deviation since there is a constraint in the model formulated specifically for this purpose. Furthermore, note that the model has other constraints for the purpose of limiting the risk of the generated portfolio with regard to the correlation and number of selected stocks.

Table 4. Annual basis strategy performance measures

Investment period	Market return	Model A return	Model B return
3/00-3/01	5.5%	29.8%	14.1%
6/00-6/01	19.0%	55.2%	32.3%
9/00-9/01	10.8%	36.3%	23.3%
12/00-12/01	26.8%	47.9%	26.5%

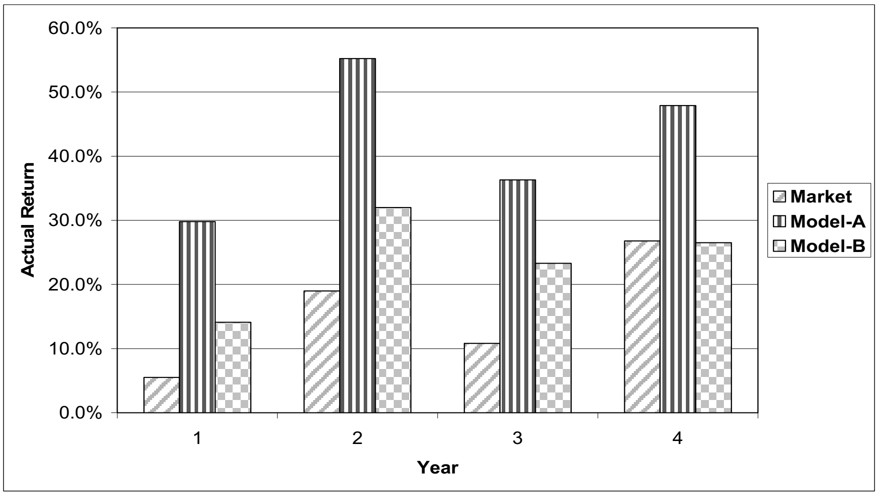

Fig. 3. Annual strategy performance measures

Table 5 shows the selected stocks in each of the seven generated portfolios (Model A). For each quarter, the information includes the sector from which the company is selected, name of the chosen company and expected return of the company during the quarter, using the moving average forecasting technique, and standard deviation during the previous 8 quarters, and actual return of the company for the same quarter. For simpler comparison, we provide, at the bottom of each section of the table, an average of each statistical measures. Moreover, the average correlation of the portfolio is shown in the table.

Table 5. The selected portfolios generated by Model A

Period 1 (03/2000-06/2000) Corr = 0.10	Sector	Company	Expected Return	Standard deviation	Actual return
	Banks	Khalej	0.7%	6.6%	8.9%
	Investment	Sahel	-0.5%	5.8%	5.1%

Table 5 (cont.). The selected portfolios generated by Model A

Period	Sector	Company	Expected Return	Standard deviation	Actual return
Period 1 (03/2000-06/2000) Corr = 0.10	Investment	Al-Awsat	13.7%	21.1%	51.9%
	Industry	Portland	8.0%	15.1%	-8.3%
	Industry	Khrasana	14.4%	20.4%	10.7%
	Average		7.3%	13.8%	13.7%

Period	Sector	Company	Expected return	Standard deviation	Actual return
Period 2 (06/2000-09/2000) Corr = 0.43	Banks	Watani	3.3%	8.5%	17.2%
	Banks	Khalej	3.2%	7.2%	8.1%
	Investment	Sahel	2.0%	6.3%	-3.8%
	Industry	Waraqeia	28.7%	22.2%	-3.0%
	Industry	Khrasana	22.3%	26.5%	-8.4%
	Average		11.9%	14.1%	2.0%

Period	Sector	Company	Expected return	Standard deviation	Actual return
Period 3 (09/2000-12/2000) Corr = 0.47	Banks	Awsat	12.0%	9.7%	-6.1%
	Investment	Sahel	0.6%	6.0%	2.0%
	Industry	Caibellat	-4.8%	10.2%	1.2%
	Services	Petroleia	43.7%	25.5%	-12.0%
	Services	Arabi	34.2%	25.6%	-7.4%
	Average		17.1%	15.4%	-4.5%

Period	Sector	Company	Expected return	Standard deviation	Actual return
Period 4 (12/2000-03/2001) Corr = 0.49	Banks	Watane	7.9%	9.8%	21.6%
	Banks	Khalej	6.8%	7.6%	12.2%
	Banks	Tejari	8.8%	13.4%	26.9%
	Investment	Tashelat	13.6%	11.4%	25.9%
	Industry	Bahreia	-1.5%	29.2%	0.0%
	Average		7.1%	14.3%	17.3%

Period	Sector	Company	Expected return	Standard deviation	Actual return
Period 5 (03/2001-06/2001) Corr = 0.45	Investment	Tashelat	17.6%	11.4%	-8.2%
	Insurance	Ahleia	6.2%	12.8%	-5.3%
	Industry	Khrasana	16.9%	18.6%	11.8%
	Industry	Motaheda	14.5%	19.0%	170.0%
	Food	Agtheia	15.0%	5.2%	10.8%
	Average		14.0%	13.4%	35.8%

Period	Sector	Company	Expected return	Standard deviation	Actual return
Period 6 (06/2001-09/2001) Corr = 0.47	Banks	Awsat	29.2%	16.4%	-16.9%
	Industry	Caibellat	5.7%	8.8%	2.1%
	Industry	Khrasana	22.1%	18.8%	-21.1%
	Food	Mwashe	18.9%	14.6%	-25.0%
	Food	Agtheia	16.5%	8.3%	8.7%
	Average		18.5%	13.4%	-10.4%

Table 5 (cont.). The selected portfolios generated by Model A

	Sector	Company	Expected return	Standard deviation	Actual return
Period 7 (09/2001-12/2001) Corr = 0.30	Bank	Khalej	8.1%	6.5%	1.6%
	Insurance	Ahleia	90.0%	11.5%	-3.9%
	Estate	Motaheda	25.8%	18.7%	10.7%
	Services	Mkhazen	23.5%	18.0%	-5.6%
	Food	Asmak	21.4%	23.9%	15.1%
	Average		33.8%	15.7%	3.6%

Conclusion and future work

A tailored tabu search heuristic algorithm is introduced in this paper to solve two mathematical models for balancing the trade off between the risk and return involved in the portfolio optimization problem in emerging stock markets. It has been shown, as a major contribution of this paper, that a mathematical model can identify a stock portfolio that is able to outperform the KSE market index in terms of risk and return. Another concluding remark of this research is that although Model A in the quarterly basis strategy provided an optimized portfolio that did not outperform the market index in 2 out of 7 quarters, the annual basis strategy outperformed it for all four tested years. The introduced Model A has significantly outperformed the market in all of the tested four years when the annually basis strategy and moving average rule are used. Also, it is noticed that the market is beaten by the models while still maintaining the balance between the risk and return. Consequently, this research suggests that there is room for implementing optimization techniques in the KSE and supports the work done by Al-Loughani (2000) and Al-Loughani et al. (2004), which provide evidence of the weak efficiency of the KSE.

In light of the data used, and before generalizing on the results of this paper, further research on portfolio selection in emerging markets is encouraged to include a larger data sample, different constraints and different markets. Moreover, it would be motivating to witness the models performance after incorporating tail risk measures such as VaR and CVaR.

As a final point to conclude with, the main implication of this research for practitioners is the possibility of using this model to select a portfolio that can produce higher returns without increasing risk.

References

1. Achaerf, A. (2002). Local Search Techniques for Constrained Portfolio Selection Problems, *Computational Economics*, 20, 3, 177-190.
2. Aldaihani, M. and Al-Deehani T. (2004). Modeling and Analysis for Portfolio Optimization in an Emerging Market: The Case of Kuwait, *WSEAS Transactions on Systems*, 7, 3, 2523-2530.
3. Alexakis Christos, Balios Dimitris, and Stavraki Sophia (2007). A Dynamic Approach for the Evaluation of Portfolio Performance Under Risk Conditions, *Investment Management and Financial Innovations*, 4, 4, 16-24.
4. Al-Loughani N.E and Moosa I.A. (1999). Testing the Efficiency of an Emerging Stock Market Using Trading Rules: The Case of Kuwait, *Journal of Gulf and Arabian Peninsula Studies*, 95, 219-237.
5. Al-Loughani, N.E. (1995). Random Walk in Thinly Traded Stock Markets: The Case of Kuwait, *Arab Journal of Administrative Sciences*, 3, 198-209.
6. Al-Loughani, N., Al-Deehani, T. and Al-Saad K. (2004). Stock Dividend Yield and Investment Rates of Return in Kuwait Stock Exchange, *Journal of King Saud University (Administrative Sciences)*, 17, 67-92.
7. Al-Loughani, N.E. (2000a). Recent Trends and Market Inefficiency in the Kuwait Stock Exchange: Evidence from the post-liberation Era. In: Arab Stock Markets: Recent Trends and Performance, Dahel Riad (Ed.), The Arab Planning Institute, Kuwait, The American University in Cairo Press, 2000a, 25-27.
8. Al-Loughani, N.E. (2000b). The Analysis of Causal Relationship between Stock Prices and Trading Volume in the Kuwaiti Stock Market, *Journal of Economic and Administrative Sciences*, 15, 217-237.
9. Al-Loughani, N.E. and Chappell, D. (2001). Modeling the day-of-the-week effect in the Kuwait Stock Exchange: A nonlinear GARCH representation, *Applied Financial Economics*, 11, 4, 353-360.
10. Bulter, K.C. and Malaika, S. J. (1992). Efficiency and Inefficiency in Thinly Traded Stock Markets: Kuwait and Saudi Arabia, Journal of Banking and Finance, 16, 197-210.
11. Konno, H. and Suzuki, K.-I. (1995). A mean-variance-skewness portfolio optimization model, *Journal of the Operations Research Society of Japan*, 38, 2, 173-187.
12. Konno, H. and Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to Tokio stock market, *Management Science*, 37, 5, 519-531.
13. Liu, J.(2007). Portfolio Selection in Stochastic Environment, The *Review of Financial Studies*, 20,1,1-39.

14. Mansini R. and Speranza (1999). Heuristic Algorithms for the Portfolio Selection Problem with Minimum Transaction Lots, *European Journal of Operations Research*, 114, 219-233.
15. Markowitz H. (1952). Portfolio Selection, *Journal of Finance*, 7, 77-91.
16. Markowitz H. (1959). Portfolio Selection: Efficient Diversification of Investments, John Wiley, New York.
17. Markowitz, H. Todd, P., Xu and Yamane, Y. (1993), Computation of mean-semivariance efficient sets by the critical line algorithm, *Annals of Operations Research*, 45, 307-317.
18. Rockafellar, R.J., Uryasev, S.P. (2000). Optimization of Conditional Value-at-Risk, *Journal of Risk*, 2, 21-42.
19. Rockafellar, R.T., Uryasev, S.P. (2002). Conditional Value-at-Risk for General Loss Distribution, *Journal of Banking and Finance*, 26,1443-1471.
20. Rolland, E. (1997). A tabu search method for constrained real number search: Application to portfolio selection. Technical Report, Department of Accounting & Management Information Systems, Ohio State University, Columbus.
21. Yang, X. (2006). Improving Portfolio Efficiency, A Genetic Algorithm Approach, *Computational Economic*, 28, 1-14.