"Optimal exercise and profit sharing of joint real investments in energy industry"

AUTHORS	David Müller
ARTICLE INFO	David Müller (2013). Optimal exercise and profit sharing of joint real investments in energy industry. *Problems and Perspectives in Management*, *11*(3)
RELEASED ON	Tuesday, 24 September 2013
JOURNAL	"Problems and Perspectives in Management"
FOUNDER	LLC "Consulting Publishing Company "Business Perspectives"

NUMBER OF REFERENCES

0

NUMBER OF FIGURES

0

NUMBER OF TABLES

0

SECTION 2. Management in firms and organizations

David Müller (Germany)

Optimal exercise and profit sharing of joint real investments in the energy industry

Abstract

Large and risky investments in energy are characterized by three crucial attributes: high level of uncertainty, the opportunity to postpone and the possibility to invest together with other firms, so that a joint investment results. Two questions arise from these attributes: when it is optimal to invest and how to share the profits of the joint investment. To answer the first question, the paper applies real option theory. As not only the future cash flows from the project but also the investment costs are uncertain, such investment opportunities are analogous to exchange options. The paper presents a new, integrated approach for evaluating and defining optimal exercise of such real investment opportunities for the first time. It will be shown that investment options are a special case of exchange options. The second question is answerable by a mechanism which is judged as fair and is therefore accepted by the partners. Cooperative game theory deals with this problem in different ways. The nature and calculation of two well-known mechanisms are discussed, and a newer solution concept is presented in the paper for the first time.

Introduction

The fact that decisions regarding investments and/or divestments in the energy industry can be interpreted and modelled as real options is often emphasized and commonly accepted, so there is a wide range of models and ways of interpreting a real scenario as an option available (Guj, 2011; Chorn & Shokor, 2006; Rammerstorfer & Eisl, 2011; Abadie & Chamorro, 2008; Keppo & Lu, 2003; Benthem, Kramer & Ramer, 2006; Cartea & González-Pedraz, 2012; Lin, Ko & Yeh, 2007; Fan & Zhu, 2010). The scenario which has got the most attention is the question of if and when to invest – the classical investment decision. An investment opportunity is – technically speaking – the opportunity to exchange the investment amount for the cash flows resulting from the project. The investment amount is assumed in the overwhelming amount of available real option models to be certain, but in many real cases this assumption does not hold. Therefore, an irreversible investment under uncertainty has to be interpreted as an exchange of one uncertain asset for another uncertain asset, opening up a very fundamental way of valuing the majority of investment decisions. Beside the question of evaluating such an option, the problem of optimal exercising has to be solved.

In addition, it has to be pointed out that investments in energy business often require large amounts and run over several years. For these reasons it is advantageous to bear such investment jointly. In this case the question arises of how to share the jointly generated benefit of the investment between the

partners. Several concepts of cooperative game theory are intended to solve this problem. The two best-known concepts – Shapley value and nucleolus – have been discussed previously in terms of questions of fair distribution of network costs or profits in the energy business (cf. Stamtsis & Erlich, 2004; Songhuai et al., 2006; Bhakar et al., 2010; Junqueira et al., 2007; Lima, Contreras & Padilha-Feltrin, 2008; Massol & Tchung-Ming, 2010). As these suffer from some limitations, another solution concept is introduced. The paper is organized as follows. Analogies between real investment opportunity and financial options are briefly discussed in the first part of section 1, followed by presentation of a model for evaluating American exchange options in the second part. Moreover, how to find the trigger value for optimal exercise of such options is discussed. The first part of section 2 presents the background of cooperative game theory, creating the basis for the following chapter, which presents the specific solution concepts of Shapley value and nucleolus. Ways to overcome the disadvantages of these concepts are demonstrated in the last part of this section by presenting and discussing the τ-value. An example is presented in section 3. Characteristics and differences between the solution concepts of game theory are demonstrated. Every theory is based on assumptions. The main propositions of the two applied theories are the focus of attention of section 4. The final section concludes and presents the main findings.

1. Investment decisions as real options

1.1. Nature and interpretation of real options. Real options in their actual shape have been dis-

cussed in the economic literature for nearly three decades (Myers, 1977; Emery et al., 1978; Rao & Martin, 1981), grounded in early analysis in the area of environmental conservation (Weisbrod, 1964, p. 472; Krutilla, 1967, p. 780; Arrow & Fisher, 1974, pp. 312-319; Henry, 1974). Among the four basic characterizations of options as a component of total firm value, as specific projects, as choices and as a heuristic for strategic investment (McGrath, Ferrier & Mendelow, 2004, pp. 86-88), only the characterization as a specific investment proposal with option-like properties is of interest for further consideration. Irreversibility of technical investments and decision-maker's flexibility allow interpretation of the investment opportunity as an option. The decision-maker has the opportunity to pay the certain investment cost and to acquire the uncertain cash flows of the project. Deciding when to invest is equivalent to the question of when to exercise this option (McDonald & Siegel, 1986, p. 721). The decision-maker may realize the investment today, later or never, so he/she has a real option at his disposal. If he/she makes the investment expenditure, this option is gone. The decision-maker loses the flexibility to wait for additional information and decide later. Valuing this situation using the option analogy leads to the following main results (Pindyck, 1991, p. 1116; Dixit & Pindyck, 1994, p. 141):

♦ value of the investment opportunity under uncertainty and flexibility is higher than without flexibility and uncertainty; and

♦ full cost of investment consists of the traditional investment cost plus the lost option value.

Of these two statements, the last one is the centrepiece of real option valuation: the critical value or the trigger value for exercising the real option to invest under uncertainty is higher than the trigger value without uncertainty due to the option value which will be lost by exercising the option. This explains deferring investment in times of high uncertainty: the higher the uncertainty, the higher the trigger value for realizing an investment is (for such relation in the case of new technologies cf. Zhu & Fan, 2011; for such a relation of oil price uncertainty and industrial activity cf. Dunne & Mu, 2010; Bredin, Elder & Fountas, 2011).

The structure and usefulness of applying real options for investment decisions in the energy sector has been demonstrated in many cases. The overwhelming majority of these contributions have analyzed the investment option (also described as option to defer) and have modelled the investor's situation as an American call (Szolgayova, Fuss & Obersteiner, 2008; Benthem, Kramer & Ramer, 2006; Lin, Ko & Yeh, 2007; Rammerstorfer & Eisl, 2011; Fuss et al.,

2011; Abadie & Chamorro, 2008; Keppo & Lu, 2003; Fan & Zhu, 2010; Takizawa et al., 2001; Kiriyama & Suzuki, 2004), whereas some papers model European-style options (Cartea & González-Pedraz, 2012; Rodríguez, 2008). Modelling investment decisions as call options is justified for situations in which the investment costs are certain. In the case of large and complex investments which require several years of construction and large financial resources, investment costs are not certain. 'However, the strike price of a proven undeveloped reserve – that is, the expected present value of development costs – is uncertain. Moreover, because the prices of drilling rigs and oilfield services tend to rise as oil and gas prices increase, the strike price of a proven undeveloped reserve is often correlated with the value of the developed project' (McCormack, Sick & Calistrate, 2002, p. 489). Other examples of the existence of real exchange options can be found in the electric power industry. Power companies have the opportunity to produce and sell electricity based on oil, gas or coal. Electricity prices as well as commodities' prices oscillate, so that the companies exchange one uncertain asset for another (Rosenberg et al., 2002, pp. 323-332).

Deepening the option analogy, such an investment opportunity may be interpreted as an exchange opportunity – the decision maker has the opportunity to exchange one asset for another. In the strictest sense, every investment is an exchange of two assets: investment costs are exchanged for cash flows. The opportunity to invest is equivalent to the option to exchange. The decision maker may change investment costs (I), which he/she owns, for the value of the projects' cash flows, which he/she will acquire (V) during a space of time (T). Both assets underlie an uncertain future development, which is incurporated by the volatility of the assets' relative price change σ_I and σ_V. This price paths may be correlated weakly, very strong, inversely or not at all, a fact which is expressed by the correlation $\rho_{I,V}$. While postponing investment, i.e. not exchanging the assets, the decision maker earns from his investment amount a benefit, e.g. risk-free interest rate or another value of benefits that accrue to the owner. In the case of a financial option this is the dividend yield of the stock in place and is, therefore, denoted as δ_I. The owner of this stock renounces the benefits from the project, the cash flows resp. rate of return shortfall δ_V in this time. The counterpart of this 'cost of waiting' is the 'benefit of waiting', which results from the remaining flexibility and additional further information. Hence, analogies with and charac-teristics of real investment opportunities and financial exchange options become apparent (cf. Table 1).

Table 1. Characteristics of financial and real exchange option (own representation)

Financial exchange option (FEO)	Symbol	Real exchange option (REO)
Right to exchange one stock for another		Right to exchange one uncertain asset for another
Current price of the stock to be acquired (asset price)	V	Real asset which can be acquired, e.g. cash flows from a project
Current price of the stock to be delivered (strike price)	I	Real asset which has to be delivered, e.g. necessary investment amount
Risk-free interest rate	r	Risk-free interest rate
Time to expiration	T	Period within which the exchange may be realized
Dividend yield rate of the two stocks	$\delta_I; \delta_V$	Rate of return shortfall for the two assets
Correlation between the two stocks	$\rho_{I;V}$	Correlation between the two assets
Volatility of the stocks relative price change	$\sigma_I; \sigma_V$	Volatility of the assets' relative value change

It is a little bit surprising that interpreting a real investment decision as a real exchange option has received scarce attention in the literature despite this fundamental analogy (cf. for an exception Armada, Kryzanowski & Pereira, 2007). All the more so as there exist very early contributions in this field, which have shown the principal building bricks (McDonald & Siegel, 1986; Carr, 1995).

1.2. Valuation and optimal exercise of exchange options. Valuation of real options is the basis for identifying the optimal investment rule and is based on the models of financial options' valuation. Option valuation is grounded in two main sources: analytical and numerical modelling. The first valuation of an exchange option was an analytical model and was suggested in 1978 (Margrabe, 1978). This model was limited to European exchange options. As the majority of real options are of American style, the following discussion is confined to models for American exchange options. Whereas the numerical valuation of American exchange option with finite lifetime is possible with numerical procedures, analytical methods only approximate the numerical values. From the different approximation approaches (McDonald & Siegel, 1986; Carr, 1995; Armada, Kryzanowski & Pereira, 2007; Andrikopoulos, 2010), the following one created by Bjerksund and Stensland in 1993 is discussed as it provides, beside

the option value, a closed-form approximation of the very important critical trigger value and is characterized by sufficient accuracy (Hoffmann, 2001, p. 14; Broadie & Detemple, 1996, pp. 1221-1232; Andrikopoulos, 2010). Suppose that the asset price of the project cash flows V and the price for realizing the investment I follow a geometric Brownian motion (Abadie & Chamorro, 2008, p. 1857; Carr, 1995, pp. 110-111) of the form:

$$\frac{d\tilde{I}}{I} = \alpha_I dt + \sigma_I dz_I,$$

$$\frac{d\tilde{V}}{V} = \alpha_V dt + \sigma_V dz_V.$$

The drift rate α is the expected rate of return from holding this asset. The total expected return on the asset is given by μ as the expected percentage rate of change of the assets value and by δ as continuous dividend yield on this asset (Pindyck, 1991, p. 1119; Bjerksund & Stensland, 1993, pp. 762-763). dz_V and dz_I are increments of standard Wiener processes and the correlation between these two processes is ρdt. Valuing the option is done by defining the trigger value at which exercise of the option becomes optimal. The trigger value of an American exchange option $V^*_{BJRKSTEN}$ results in (Bjerksund & Stensland, 1993, p. 91):

$$V^*_{BJRKSTEN} = \max\left[I,\left(\frac{\delta_I}{\delta_V}\right)I\right] + \left(\frac{\beta}{\beta-1}I - \max\left[I,\left(\frac{\delta_I}{\delta_V}\right)I\right]\right)\left(1 - e^{-\left((\delta_I-\delta_V)T + 2\sigma\sqrt{T}\right)\left(\frac{V_0}{\frac{\beta}{\beta-1}I - V_0}\right)}\right) \quad (1)$$

with $\beta = \frac{1}{2} - \frac{(\delta_I - \delta_V)}{\sigma^2} + \sqrt{\left(\frac{(\delta_I - \delta_V)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\delta_I}{\sigma^2}}$ and $\sigma = \sqrt{\sigma_V^2 + \sigma_I^2 - 2\rho_{VI}\sigma_V\sigma_I}$.

It is interesting that in the formula, two fundamental fractional values are incorporated. One is the value which would result if the option would be exercised immediately. This value is defined by the maximum of two possible values, is denoted by V_0 and is calculated by the following equation:

$$V_0 = \max\left[I,\left(\frac{\delta_I}{\delta_V}\right)I\right] \quad (2)$$

The other value is the value which would be necessary to exercise a perpetual option. That means this is the trigger value, which is already known (Pindyck, 1991) and is calculated by:

$$V_\infty = \frac{\beta}{\beta - 1} I \ . \tag{3}$$

The value of the real exchange option REO_{approx} follows with these results from Bjerksund & Stensland (1993, pp. 90-92):

$$REO_{approx} = \alpha V^\beta - \alpha \Phi\left(V, T \mid \beta, V^*, V^*\right) +$$

$$+ \Phi\left(V, T \mid 1, V^*, V^*\right) - \Phi\left(V, T \mid 1, I, V^*\right)$$

$$- I\ \Phi\left(V, T \mid 0, V^*, V^*\right) + I\ \Phi\left(V, T \mid 0, I, V^*\right)$$

with $\alpha = \left(V^* - I\right) V^{*(-\beta)}$ \hfill (4)

where Φ is a function which evaluates different parts of the options, according to Bjerksund and Stensland (1993). This approximation has the advantage that the option's value and its optimal exercise is calculated simultaneously. Trigger value indicates whether it is optimal to exercise the option immediately or not. If the amount of the expected cash flows from the project is under this value it is not optimal to exercise the option. With this approximation, the two fundamental questions appearing in a real-option context can be answered: the value of the investment opportunity *and* the optimal exercise value.

In practice, the decision-maker sometimes has the opportunity to choose between a stand-alone investment and sharing such a project with other companies. The question of how to share jointly burdened costs or jointly generated profits will be discussed in the following.

2. Joint investments and the question of profit sharing

2.1. Joint investments as a cooperative game and their characteristics. Investments often tend to require large amounts and/or a broad knowledge basis and tend to have a complex structure. Realizing such investment together with other companies offers the possibility to reach the defined goal in a manner which leads to a better result than the sum of individual gains. This effect is called synergy-effect or synergy-gain and may consist of, for example, lower investment cost, higher cash flows and better solutions. The result – a higher benefit for the partners – has to be allocated in a way that the collaboration represents the best alternative for the companies. Interpretation of joint investments in the energy sector as a cooperative game is not new (Massol & Tchung-Ming, 2010; Stamtsis & Erlich, 2004; Bhakar et al., 2010; Junqueira et al., 2007; Lima, Contreras & Padilha-Feltrin, 2008; Tsukamoto & Iyoda, 1996), so that a short introduction is sufficient for further discussion.

A cooperative game Γ is the pair (N,v), where $N = \{1,2,\ldots,n\}$ denotes the set of players. Not only the amount of all players, N, is important here, but also all subsets of N. Such a subset $S \subseteq N$ is referred to as coalition S, whereas N itself is described as the grand coalition. Each coalition $S \in P(N)$ is marked by a value function $v\ (S)$ (Neumann & Morgenstern, 1947, p. 238). The function v assigns a value to each subset S, which represents the economic performance of this coalition. Each firm $i \in N$ seeks to maximize the benefit which it can obtain from belonging to a coalition. The result of an empty coalition is zero, therefore: $v(\varnothing) = 0$. A characteristic for further consideration is the fact that the benefit generated by a coalition is completely transferable, so the benefit can entirely be divided between the members of the coalition. The aim of joint investment is to generate synergy effects. This is taken into account by the characteristic of superadditivity (Driessen, 1988, p. 11).

Property 1: A characteristic function v with transferable utility is a superadditive function if for all coalitions $R, S \subseteq N$ out of $R \cap S = \varnothing$ follows: $v(R) + v(S) \leq v(R \cup S)$.

This property requires following explanation - the benefit of a joint investment may be twofold: on the one hand, benefit may be generated by increased revenues, consistent with the superadditivity property. On the other, the jointly gained benefit may be caused by reduced costs as a result of shared resources. In this case, the superadditivity property of characteristic revenue function is replaced by the subadditivity property of characteristic cost function (Moulin, 1988, pp. 89-93; Young, 1994, pp. 1197-1198; Fiestras-Janeiro, García-Jurado & Mosquera, 2011, p. 4). Assuming fixed revenues, the subadditivity of costs leads to a superadditivity of profits. Only games with a superadditive charac-teristic profit function – regardless of whether caused by increased revenues or reduced costs – stand at the center of the following discussion.

The aim of the cooperative game theory is the development of a solution mechanism which leads to a fair allocation of the jointly realized outcomes. Within the context of cooperative game theory, a profit allocation is then referred to as fair if each company accepts the allocation and if it is lucrative for the players to cooperate with each other. Within cooperative game theory, several concepts concerning the way that such an allocation may be defined as fair have been established (Curiel, 1997, pp. 2-15) which are not elaborated in detail here. Instead, the following discussion is limited to presenting the principles of the core and derived solution concepts.

The possible solutions of a game are restricted to the set of imputations. An imputation describes an allocation which fulfils two requirements: individual rationality and group rationality.

A payoff vector is individually rational, if only the generated profit will be divided by the payoff vector $x = (x_1, x_2, \ldots, x_n)$, so that for the value of the game, $v(N)$ results in $\sum_{i \in N} x_i \leq v(N)$. Moreover, it has to be assured that there exists no veto right and every player gets at least the profit which he would gain in a stand-alone coalition, leading to $x_i \geq v(\{i\})$, $\forall i \in N$. A payoff vector is group rational and therefore Pareto-optimal if there exists no other distribution in which one player gets a higher share of the joint profit without another player getting less, so that results in: $\sum_{i \in N} x_i \geq v(N)$. The sum of these requirements leads to the imputation of a game (Curiel, 1997, p. 5).

Definition 1: A solution is called an imputation and is thus defined by:

(a) $\sum_{i \in N} x_i = v(N)$ *as well as*

(b) $x_i \geq v(\{i\})$, $\forall i \in N$.

To solve a distribution problem only these imputations are of interest which are not dominated by other imputations, as only the undominated imputations incorporate an incentive for joining the coalition. The set of undominated imputations form the core (Moulin, 1988, pp. 94-95).

Definition 2: The core of a cooperative game C(N,v) is formed by all undominated imputations and therefore results from:

$$C(N,v) = \left\{ x : \sum_{i \in S} x_i \geq v(S) \; \forall S \subseteq N \right\}.$$

The core of a game may be very large or may be empty. The core of a special sub-class of games – convex games – is never empty (Shapley, 1971, p. 24).

Property 2: The game (N,v) is convex, if for all coalitions S and R results in:

$$v(S \cup \{i\}) - v(S) \leq v(R \cup \{i\}) - v(R),$$

$$\forall S \subseteq R \subseteq N \setminus \{i\}, \forall i \in N.$$

Every convex game is superadditive, but not every superadditive game is convex. From the existing concepts (for an overview cf. Fiestras-Janeiro, García-Jurado & Mosquera, 2011), the Shapley value, the nucleolus and the τ-value will be discussed in the following.

2.2. Characterization of two established concepts. The concept of the Shapley value is widely spread and has already been used several times in the energy sector for fair allocation of network costs (Stamtsis & Erlich, 2004; Bhakar et al., 2010; Junqueira et al., 2007; Lima, Contreras & Padilha-Feltrin, 2008; Songhuai et al., 2006; Tsukamoto & Iyoda, 1996; Tan & Lie, 2002; Pierru, 2007; Moghaddam, 2010) or profits (Massol & Tchung-Ming, 2010). In order to determine share of the synergy profit for a company i, the following thought is noted: each player receives a payoff depending on his contributions to the theoretically possible, thus imaginable coalitions. The contribution of the company consists in the increase in value caused by its participation in the coalition. The question that has to be answered is which value the coalition has with company i and which it would have without company i.

Definition 3: The marginal contribution $mc_{i;S}$ of the player i for the coalition $S \subseteq N$ is determined by: $mc_{i;S}(v) = v(S\{i\}) - v(S \setminus \{i\})$.

This marginal contribution $mc_{i;S}(v)$ of a company will show a different value for different coalitions. It is to be pointed out that in the case of a marginal contribution defined in such a way, it is irrelevant whether the coalition S actually contains the company i, since for every coalition the following is valid: $mc_{i;S\{i\}}(v) = mc_{i;S\setminus\{i\}}(v)$.

Definition 4: The weighted marginal contribution of the company $\varphi_i(v)$ results from

$$\varphi_i(v) = \sum_{S \subseteq N \setminus i} \frac{|S|!(n-1-|S|)!}{n!} \big(v(S \cup \{i\}) - v(S) \big).$$

This solution principle is commonly described as Shapley value (Shapley, 1953, p. 311; Bilbao, 2000; p. 170).

Another well-known and important solution concept of cooperative game theory is the nucleolus, which was introduced by Schmeidler (1969). The nucleolus involves searching for fair distribution by minimizing the maximal dissatisfaction of every player. For this, the dissatisfaction of a coalition with a concrete payoff vector is named in this connection as excess (Moulin, 1988, pp. 121-123; Driessen, 1988, pp. 37-38). It has to be calculated, how unhappy a coalition would be with a payoff vector.

Definition 5: The excess (unhappiness) ex (S, x) of a coalition S with a payoff vector x is derived by $ex(S, x) = v(S) - \sum_{i \in S} x_i$.

To derive the nucleolus, the payoff vectors with the highest unhappiness for every player are searched in the next step. To do so these complaint vectors are ordered lexicographically. The solution of the problem is the order, which minimizes the unhappiness.

Definition 6: The lexicographic order of the excess values $\Theta(x) = \left(ex(S_1, x), ..., ex(S_{2^N}, x) \right)$ *is built by*

$$ex(S_j, x) \geq ex(S_{j+1}, x) \forall j = 1, ..., 2^N - 1.$$

To compare two possible payoffs, their lexicographic orders are compared and the payoff which is lexicographically smaller than the other one is chosen.

Definition 7: If two imputations x and y are compared, then x is called to be lexicographically smaller ($\prec_L$) than y if there exists an index m, with which results:

(a) $\Theta_k(x) = \Theta_k(y) \quad \forall 1 \leq k < m$ and

(b) $\Theta_m(x) < \Theta_m(y)$.

With these explanations the nucleolus of a game can be defined as follows.

Definition 8: In a game (N,v) the nucleolus nuc (v) is defined with respect to the number of imputations I (v) by:

$$nuc(v) = \left\{ x \in I(v) \big| \Theta(x) \prec_L \Theta(y) \right\} \quad \forall y \in I(v).$$

The nucleolus is established in the field of game theory and has been used for decision making in the energy sector (Stamtsis & Erlich, 2004; Bhakar et al., 2010; Songhuai et al., 2006; Tsukamoto & Iyoda, 1996; Massol & Tchung-Ming, 2010), but not as often as the Shapley value.

These two solution concepts suffer from the following main limitation: the Shapley value is based on the expected marginal contribution of each player from its merely theoretically realizable participation in the sum of all possible – which means imaginable – coalitions. The nucleolus – by determining the lexicographic order of unhappiness – is referring to the sum of all possible coalitions too. A solution concept which uses another interpretation of the collaborative situation is discussed in the following.

2.3. τ-value as improved solution concept. The concept of the τ-value was developed some time ago (Tijs, 1981; Driessen & Tijs, 1982) and is alternatively described as Tijs value (Bilbao, 2000, p. 6). This concept has been known in the research field of game theory for many years (Driessen, 1983; Tijs & Driessen, 1986; Driessen, 1988) but has only recently

been put to use for solving management issues (Zelewski & Peters, 2010; Jene & Zelewski, 2011).

The τ-value is characterized by the fact that it has been developed on the basis of the logical negotiation situation which is presented to the cooperation companies. In this concept an upper and lower limit are specified in a first step. As the upper limit, the vector of the marginal contributions of the individual companies to the grand coalition is determined. No higher payoff is granted to the company than the value which it contributes by its participation in the grand coalition.

Definition 9: In an n-person game $\Gamma = (N,v)$ the vector b, defined by $b = \begin{pmatrix} b_1 \\ ... \\ b_n \end{pmatrix}$ *and* $b_i = v(N) - v(N\setminus\{i\})$ *, is referred to as the upper vector.*

The ith coordinate b_i of this upper vector is the marginal contribution of the company i with regard to the grand coalition (Tijs, 1981, p. 1; Curiel, 1997, p. 13).

To determine the lower limit, the following consideration is used: in the case that the company i does not participate in the grand coalition, there is the opportunity for i to participate in another coalition or form this one – the so-called outsider coalition. In this constellation, the company i receives not less than that amount with which it is able to threaten credibly by founding at least one outsider coalition. However, in order to attract other companies to the outsider coalition, the company i has to offer each of those members at least the amount they would be able to achieve in the best case within the grand coalition. The amount resulting after these side payments remains for the company i and represents the lower limit – also referred to as threat point or concession limit. The company i would strive for that outsider coalition in which the residual income is maximum.

Definition 10: In an n-person game $\Gamma = (N,v)$ the vector a, which is defined by $a = \begin{pmatrix} a_1 \\ ... \\ a_n \end{pmatrix}$ *and*

$a_i = \max_{S \subseteq N: i \in S} \left\{ v(S) - \sum_{j \in S\setminus\{i\}} b_j \right\}$, *is referred to as the lower vector.*

In an outsider coalition S, which i might enter, the remaining companies could each maximally claim their marginal contributions. In the worst case, the payment $v(S) - \sum_{j \in S\setminus\{i\}} b_j$ would be due to the

company *i*. The company *i* rationally strives for that coalition in which this difference is at the maximum (Tijs, 1981, p. 2; Curiel, 1997, p. 13).

The vectors *a* and *b* do not necessarily represent imputations. If there should be an imputation between these two vectors, the following condition is to be introduced:

Property 3: An n-person game Γ = (N,v) is quasi-balanced if the following condition holds: $\sum_{i \in N} a_i \le v(N) \le \sum_{i \in N} b_i$.

Therefore, the sum of a_i may not exceed the synergy profit while the sum of b_i may not fall below that value. For quasi-balanced games one clearly determined imputation exists, which is between the upper and the lower vector and which is referred to as *τ*-value (Tijs, 1986, p. 4).

Definition 11: The τ-value of a quasi-balanced game (N,v) is defined by $\tau = a + \dfrac{v(N) - \sum_{i \in N} a_i}{\sum_{i \in N} b_i - \sum_{i \in N} a_i}(b - a)$,

and leads to the τ-value of the company i:

$$\tau_i = \max_{S \subseteq N: i \in S}\left\{v(S) - \sum_{j \in S\setminus\{i\}} b_j\right\} + \frac{v(N) - \sum_{i \in N} \max_{S \subseteq N: i \in S}\left\{v(S) - \sum_{j \in S\setminus\{i\}} b_j\right\}}{\sum_{i \in N} v(N) - v(N\setminus\{i\}) - \sum_{i \in N} \max_{S \subseteq N: i \in S}\left\{v(S) - \sum_{j \in S\setminus\{i\}} b_j\right\}}\left[v(N) - v(N\setminus\{i\}) - \left(\max_{S \subseteq N: i \in S}\left\{v(S) - \sum_{j \in S\setminus\{i\}} b_j\right\}\right)\right] \forall i \in N.$$

By fulfilling these requirements, the probability is increased that the participating companies classify the allocation as fair and will accept it. The *τ*-value proves to be superior to the Shapley value insofar as the argumentation on which its determination is based better reflects the real decision situations. This improved reflection affects two issues: an investment coalition cannot be formed between all imaginable companies, but is restricted to specific combinations. Those companies which do not participate in a grand coalition can form one or more outsider coalitions. With the threat of this outsider coalition, a company can increase its profit share in the grand coalition. The coalitions are formed through negotiations.

3. Valuation and interpretation example

3.1. Exchange options as fundamental type of all investment decisions. To demonstrate and discuss the nature of the two different methods of option pricing – numerical vs. analytical model – an example from the energy branch is introduced which is adapted from some early real option cases in this field, which have demonstrated the existence and valuation of real options in this branch in principle (Paddock, Siegel & Smith, 1988; Kemna, 1993; Bjerksund & Ekern, 2001; Fan & Zhu, 2010). As none of these contributions discusses the case of American exchange options, this is done in the following. The decision-maker has the opportunity to

develop and explore a petroleum resource. The present value of the cash flows is expected at *V* = = 1000000, – €, whereas the necessary investment is supposed to be *I* = 900000, – €. Without uncertainty and without postponement opportunity the net present value results with *NPV* = *V* – *I* = 100000, – €. The trigger value for immediately exercising this opportunity is equal to the investment costs and results in *V** = 900000, – €. This indicates an immediate investment as optimal decision.

Cash flows' value is supposed to be uncertain due to oil prices' development in the next step. The degree of uncertainty can be derived from the market prices of different oil products, e.g. unleaded gasoline, heating oil and crude oil respectively (Dunne & Mu, 2010, p. 197). The benefit of owning the oil – also called convenience yield – does not accrue to the decision-maker while postponing the investment. The convenience yield for different commodities may be extracted based on market observations (Hochradl & Rammerstorfer, 2012) and is denoted as δ_V. Options lifetime is restricted to only two years due to technological and judicial restrictions. Investment costs are supposed to be uncertain as well, and they are calculated with a volatility of 10% per year. The opportunity to invest or postpone corresponds to an American exchange option with input parameters from Table 2.

Table 2. Input data for the real exchange option (own representation)

Parameters description	Value	Symbol
Cash flows from the project (real asset, which can be acquired)	1000000, – €	*V*
Annual rate of return shortfall as a percentage of the total value *V*	5%/a	δ_V
Annual volatility of cash flows	20%/a	σ_V
Investment amount	900000, – €	*I*
Annual rate of return shortfall as a percentage of the total value *I*	5%/a	δ_I
Annual volatility of investment expenditure	10%/a	σ_I

Table 2 (cont.). Input data for the real exchange option (own representation)

Parameters description	Value	Symbol
Time to expiration	2a	T
Correlation	0	$\rho_{t,V}$

The fractional values for the *BJERKSUND & STENSLAND* approximation follow from equations (1) to (3):

$$V_{\infty} = 1800000, - \text{€};$$

$$V_{BJRKSTEN}^{*} = 1322000, - \text{€};$$

$V_0 = 900000, - \text{€}; \beta = 2; h(T) = -0.632455; \alpha = 2.41 \times \times 10^{-7}.$

The value of the real exchange option is approximated with equation (4): $REO_{approx} = 164000, - \text{€}$. The trigger value of exercising the real option immediately accounts for $V^{*}_{BJRKSTEN} = 1322000, - \text{€}$. This is a higher value in comparison with the trigger value

without flexibility $V = 900000, - \text{€}$, and indicates that it is not optimal to exercise the option immediately. The value of this real American exchange option is shown as well as the critical value of optimal exercise in Figure 1. It is visible, that at the trigger point $V^{*}_{BJRKSTEN}$ the value of the standard *NPV* equals the value of the American option, which indicates immediate exercise as optimal if – and only if – the value of the cash flows from the project reach this critical value. At the trigger point, not only the absolute values of both functions are matching (value matching condition), but also their slopes are matching (smooth pasting condition; Dixit & Pindyck, 1994, p. 141). The fulfilment of these requirements ensures that one could not do better by exercising the option at another value of cash flows.
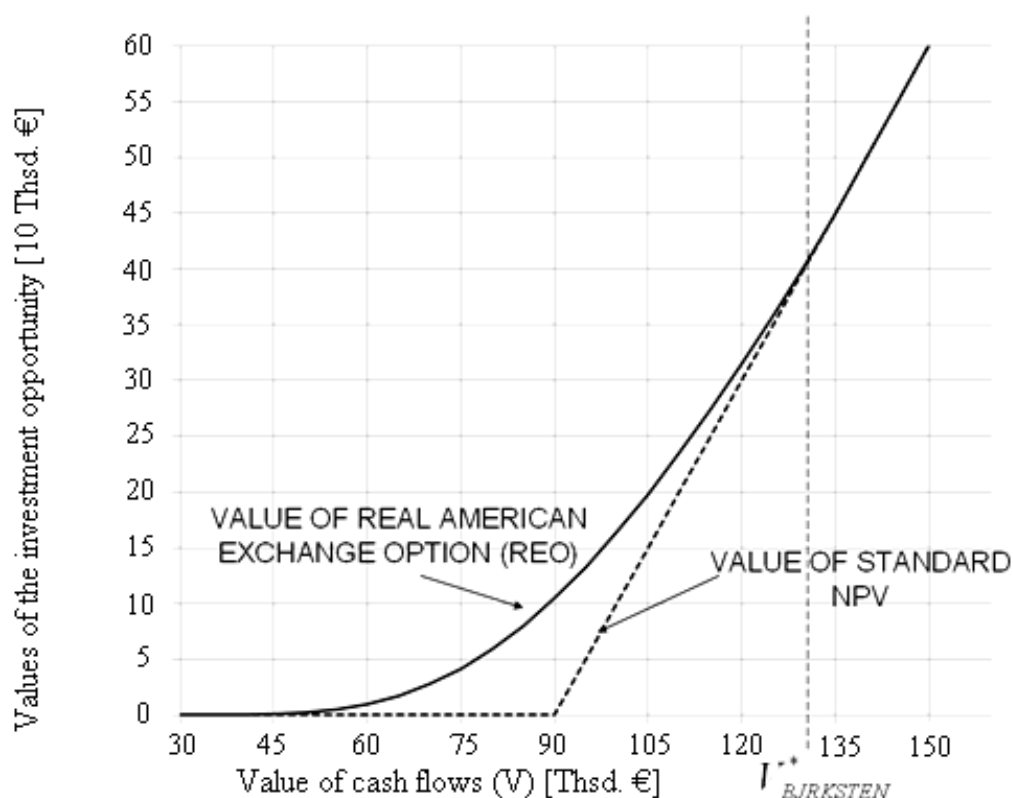

Fig. 1. Value of investment opportunity value matching and smooth pasting (own representation)

The net value of optimally exercising the option immediately results from the difference between the critical trigger value and investment costs with 422000,- € and is referred to as optimal net value.

Stochastic modeling of investment costs leads to the relevance of the following two parameters: volatility of investment costs and the correlation

between uncertain investment costs and uncertain cash flows as stated as to be characteristic for the energy sector by McCormack, Sick & Calistrate (cf. page 4 of this paper). The value of such an option subject to these two parameters is shown in Figure 2. The higher the correlation, the lower the option value.
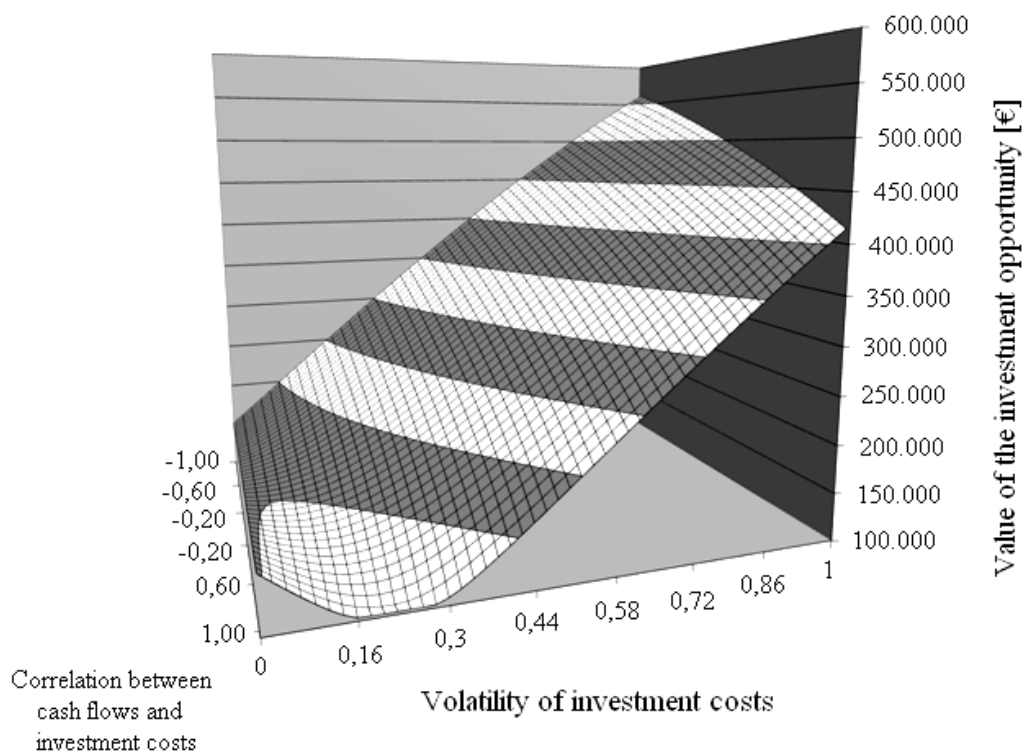
Fig. 2. Value of American exchange option as a function of correlation and volatility of investment costs (own representation based on the BJERKSUND & STENSLAND approximation)

Advanced analysis shows the influence of the volatility of investment costs in Figure 3. If volatility of investment cost is equal to zero – that means if there is no uncertainty concerning the investment costs – the value of the exchange option equals the value of a simple call option (153000, – €).

The often discussed investment option is therefore a *special case* of the exchange option presented here. Every investment oppor-tunity is an exchange option. This exchange option turns into a call option if – and only if – the investment costs are certain.
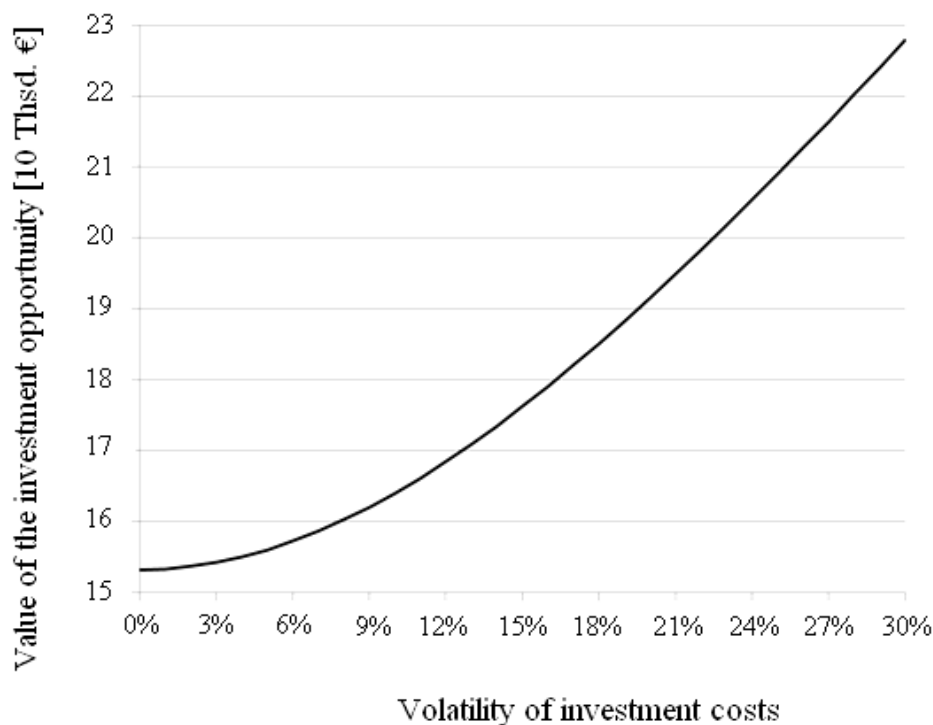
Fig. 3. Value of American exchange option as a function of volatility of investment costs (own representation based on the BJERKSUND/STENSLAND-approximation)

After defining the critical value of cash flows, the open question of how to share this amount between potential partners will be answered in the next chapter.

3.2. Jointly burdened investment and possible profit sharing rules. In the next step it is assumed that the above discussed investment can be realized with four different firms. Investment is only reali-

zable by at least two companies; the large coalition has to gain at least the above calculated optimal net value of 422000,– €. Smaller coalitions result in smaller net values. The bigger the coalition, the greater the resulting net value has to be, which is necessary for optimal exercising of the option. The characteristic function based on different coalitions' optimal net values is shown in Table 3.

Table 3. Characteristic function values based on optimal net values

S	v(S)	S	v(S)	S	v(S)	S	v(S)
Ø	0	{D}	0	{B,C}	100000	{A,B,D}	220000
{A}	0	{A,B}	100000	{B,D}	90000	{A,C,D}	240000
{B}	0	{A,C}	100000	{C,D}	110000	{B,C,D}	280000
{C}	0	{A,D}	100000	{A,B,C}	200000	{A,B,C,D}	422000

This constellation is categorized as a cooperative game, which is convex and superadditive. The question of how to share the net value of optimally exercising the American exchange option of 422000 € is answered by the three different solution concepts. The 422000 € are divided between the four companies according to the Shapley value which results from Definition 4 as follows:

$$\varphi_A = \frac{271500}{3} = 90500 \; € ; \varphi_B = \frac{306500}{3} \approx 102167 \; € ;$$

$$\varphi_C = \frac{336500}{3} \approx 112167 \; € ; \; \varphi_D = \frac{351500}{3} \approx 117166 \; € .$$

The firms' shares on the 422000 € based on the nucleolus result from Definition 8 amount to:

$$nuc_A = 71000 € ; nuc_B = 105500 € ;$$

$$nuc_C = 115500 \; € ; nuc_D = 130000 € .$$

The shares for the four companies according to the τ-value result, according to Definition 11, are as follows:

$$\tau_A = \frac{14981000}{187} \approx 80112 \; € ; \tau_B = \frac{19201000}{187} \approx 102679 \; € ;$$

$$\tau_C = \frac{21311000}{187} \approx 113963 € ; \; \tau_D = \frac{23421000}{187} \approx 125246 € .$$

All of these solutions are in the core of the game. It becomes apparent that with every solution the whole amount of 422000 € is divided fairly, but the shares for the companies differ in a range, depending on the chosen solution concept. All concepts allocate the highest share to company D, followed by company C and then B and A, but with different absolute values.

4. Limitations

There are two main points of criticism: real option thinking and cooperative game theory. Firstly, it has

to be questioned whether the real situation is analogous to a financial option. The main limitations between real and financial options result from not contracting real investment opportunities and therefore not defining the value drivers of real options in a contract. So it is possible that a real option exists, but is not identified by the decision-maker. As it is not possible to exercise an option optimally without identifying it, the first step in real option valuation/exercise consists of appropriate identification. In addition, in a real situation there is no writer of the real option as legal counterpart, who guarantees the fulfilment of the contract. This leads to the fact that important value drivers, e.g. time to expiration and strike price, are not fixed, complicating the option's valuation and exercise. Moreover, real options are not traded on a market, which impedes verification of the option valuation.

If the fundamental question of option analogy can be answered in the affirmative, the question emerges of which valuation model to use. Option pricing models and their assumptions (e.g. the existence of a perfect market, knowledge of future developments, totally rational actors) have been criticized since their development (Figlewski, 1989; Campbell, Lo & MacKinlay, 1997, pp. 391-393). Beside these fundamental critics some specific assumptions have to be discussed. The discussed method of modelling uses a common but special assumption concerning the underlying stochastic process – geometric Brownian motion. It has to be questioned if real assets follow such a path. Another problem is calculating and deriving the volatility as well as the correlation of the underlying assets. Both values are derived from historical data and are prolonged as constant values into the future. This procedure ignores the stochastic nature of both parameters.

Secondly, the other pillar of presented approach – cooperative game theory – is based on assumptions

which also call for criticism. The basis of this theory is the fact that firms are planning and acting together to jointly generate a positive result. It is assumed that the participating companies reach binding agreements which are met in the future. This assumption can be justified by interpreting these agreements as a cooperation agreement, which can indeed be considered as realistic. In addition, the possibility of side payments between the participating companies is of crucial importance, and seems to be less simple in cooperation in real-life circumstances. It has to be ensured that whole synergy profits are revealed, i.e. each coalitionist must know and disclose them. In addition, it is to be guaranteed that this profit is collected and allocated according to the respective solution concept.

The discussion has illustrated the fact that the different solution concepts are based on the postulation of individual and collective rationality. These claims are specified by the different solution concepts in such a way that a fair profit allocation results. By aiming at fulfilling the fairness criterion, the stability of the cooperation is ensured. In addition, each of the presented concepts has to be checked for validity concerning modelling real-life cooperation. Differences between the τ-value, the Shapley value and the nucleolus have been demonstrated.

Summary and conclusion

Decisions concerning real investments in the energy branch are often characterized by the fact that the investment costs and revenues are uncertain – that investment does not have to be realized today and can be postponed. This opportunity incorporates flexibility, which has to be taken into account. Real option theory offers a well-accepted way of valuing such flexibility, so the paper is based on this methodology. Against this background, investment decisions are interpreted as exchange options. It is shown that every investment opportunity is an exchange opportunity. From this it has been concluded that every investment option is a special class of exchange options – an exchange option with a certain exercise price.

Beside pure valuation of such investment opportunity, its optimal exercise is of crucial importance. The paper presents a model for valuing real exchange options and for determining conditions of optimal exercise for the first time. With this model at hand, the trigger price can be calculated, which allows a decision on the optimal exercise and the resulting optimal net value.

In addition, it has to be taken into consideration that large and risky investments can be realized by joint ventures. This may lead to a dilemma, as every company strives for that cooperation which allows the largest possible share of the largest possible net value. Thereby, the partners are in a quandary, as on the one hand the optimal net value is only realizable through cooperation while on the other, firms are interested in increasing their share of this net value, which is only possible at the expense of the other participants. Cooperative game theory allows an allocation of synergy profits in such a way that this allocation is referred to as fair and accepted by the companies. Three of the available solution concepts are discussed and the differences in applying these models are demonstrated in this article.

References

1. Abadie, L.M., Chamorro, J.M. (2008). Valuing flexibility: the case of an integrated gasification combined cycle power plant, *Energy Economics*, Vol. 30 (4), pp. 1850-1881.
2. Andrikopoulos, A. (2010). On the valuation of American exchange options: an analytical approximation, *Applied Economic Letters*, Vol. 17 (14), pp. 1429-1435.
3. Armada, M.R., Kryzanowski, L., Pereira, P.J. (2007). A modified finite-lived American ex-change option methodology applied to real option valuation, *Global Finance Journal*, Vol. 17 (3), pp. 419-438.
4. Arrow, K.J., Fisher, A.C. (1974). Environmental preservation, uncertainty and irreversibility, *Quarterly Journal of Economics*, Vol. 88 (2), pp. 312-319.
5. Bhakar, R. et al. (2010). Probabilistic game approaches for network cost allocation, *IEEE Transaction on Power Systems*, Vol. 25 (1), pp. 51-58.
6. Benthem, van A.A., Kramer, G.J., Ramer, R. (2006). An options approach to investment in a hydrogen infrastructure, *Energy Policy*, Vol. 34 (17), pp. 2949-2963.
7. Bilbao, J.M. (2000). *Cooperative games on combinatorial structures*, Kluwer Acad. Publ./Boston.
8. Bjerksund, P., Stensland, G. (1993). Closed-form approximation of American options, *Scandinavian Journal of Management*, Vol. 9 (Suppl.), pp. 87-99.
9. Bjerksund, P., Stensland, G. (1993). American Exchange Options and a Put-Call Transformation: A Note, *Journal of Business Finance and Accounting*, Vol. 20 (5), pp. 761-764.
10. Bjerksund, P., Ekern, S. (2001). Managing investment opportunities under price uncertainty: from "last chance" to "wait and see" strategies, Trigeorgis, L., Schwartz, E.S. (eds.): *Real options and investment under uncertainty: classical readings and recent contributions*. MIT Press, Cambridge, pp. 687-718.
11. Bredin, D., Elder, J., Fountas, S. (2011). Oil volatility and the option value of waiting: an analysis of the G-7, *The Journal of Future Markets*, Vol. 31 (7), pp. 679-703.

12. Broadie, M., Detemple, J. (1996). American option valuation: new bounds, approximations and a comparison of existing methods, *The Review of Financial Studies*, Vol. 9 (4), pp. 1211-1250.
13. Campbell, J.Y., Lo, A.W., MacKinlay, A.C. (1997). *The Econometrics of Financial Markets*, Princeton: Princeton University Press.
14. Carr, P. (1995). *The valuation of American exchange options with application to real options*, In: Trigeorgis, L. (ed.): *Real options in capital investment: models, strategies, and applications*, London, pp. 109-120.
15. Cartea, Á., González-Pedras, C. (2012). How much should we pay for interconnecting electricity markets? A real options approach, *Energy Economics*, Vol. 34 (1), pp. 14-31.
16. Chorn, L.G., Shokor, S. (2006). Real options for risk management in petroleum development investments, *Energy Economics*, Vol. 28 (4), pp. 489-505.
17. Curiel, I. (1997). Cooperative game theory and applications: cooperative games arising from combinatorial optimization problems, Kluwer Acad. Publ./Boston.
18. Dixit, A.K., Pindyck, R.S. (1994). *Investment under uncertainty*, Princeton.
19. Driessen, T., Tijs, S. (1982). The [tau]-value, the nucleolus and the core for a subclass of games, Nijmegen.
20. Driessen, T. (1983). *Game-theoretic solutions for some economic situations*, Nijmegen.
21. Driessen, T. (1988). *Cooperative games, solutions and applications*, Kluwer, Dordrecht.
22. Dunne, T., Mu, X. (2010). Investment spikes and uncertainty in the petroleum refining industry, *Journal of Industrial Economics*, Vol. 58 (1), pp. 190-213.
23. Emery, D.R. et al. (1978). An investigation of real investment decision making with the options pricing model, *Journal of Business Finance & Accounting*, Vol. 5 (4), pp. 363-369.
24. Fan, Y., Zhu, L. (2010). A real options based model and its application to China's overseas oil investment decisions, *Energy Economics*, Vol. 32 (3), pp. 627-637.
25. Fiestras-Janeiro, M., García-Jurado, I., Mosquera, M. (2011). Cooperative games and cost allocation problems, *TOP*, Vol. 19 (1), pp. 1-22.
26. Figlewski, S. (1989). What does an option pricing model tell us about option prices? *Financial Analyses Journal*, Vol. 45 (5), pp. 12-15.
27. Fuss, S., Szolgayova, J., Golub, A., Obersteiner, M. (2011). Options on low-cost abatement and investment in the energy sector: new perspectives on REDD, *Environment and Development Economics*, Vol. 16 (4), pp. 507-525.
28. Guj, P. (2011). A practical real option methodology for the evaluation of farm-in/out joint venture agreements in mineral exploration, *Resources Policy*, Vol. 36 (1), pp. 80-91.
29. Henry, C. (1974). Option values in the economics of irreplaceable assets, *Review of Economic Studies*, Vol. 41 (5), pp. 89-104.
30. Hochradl, M., Rammerstorfer, M. (2012). The convenience yield implied in European natural gas hub trading, *The Journal of Future Markets*, Vol. 32 (5), pp. 459-480.
31. Hoffmann, C. (2001). *Valuation of American Options*, Oxford.
32. Jene, S., Zelewski, S. (2011). Fair distribution of added values in networks of autonom actors. In: Jodlbauer, H., Olhager, J./Schonberger, R.J. (eds.): *Modelling value*, Shaker, Aachen, pp. 317-337.
33. Junqueira, M. et al. (2007). An Aumann-Shapley approach to allocate transmission service cost among network users in electricity markets, *IEEE Transactions on Power Systems*, Vol. 22 (4), pp. 1532-1546.
34. Kemna, A.G.Z. (1993). Case studies on real options, *Financial Management*, Vol. 22 (3), pp. 259-270.
35. Keppo, J., Lu, H. (2003). Real options and large producer: the case of electricity markets, *Energy Economics*, Vol. 25 (5), pp. 459-472.
36. Kiriyama, E., Suzuki, A. (2004). Use of real options in nuclear power plant valuation in the presence of uncertainty with CO_2 emission credit, *Journal of Nuclear Science and Technology*, Vol. 41 (7), pp. 756-764.
37. Krutilla, J.V. (1967). Conservations reconsidered, *American Economic Review*, Vol. 57 (4), pp. 777-786.
38. Lima, D.A., Contreras, J., Padilha-Feltrin, A. (2008). A cooperative game theory analysis for transmission loss allocation, *Electric Power Systems Research*, Vol. 78 (2), pp. 264-275.
39. Lin, T.T., Ko, C.-C., Yeh, H.-N. (2007). Applying real options in investment decisions relating to environmental pollution, *Energy Policy*, Vol. 35 (4), pp. 2426-2432.
40. Margrabe, W. (1978). The Value of an Option to Exchange one Asset for Another, *The Journal of Finance*, Vol. 33 (1), pp. 177-186
41. Massol, O., Tchung-Ming, S. (2010). Cooperation among liquefied natural gas suppliers: is rationalization the sole objective? *Energy Economics*, Vol. 32 (4), pp. 933-948.
42. McCormack, J., Sick, G., Calistrate, D. (2002). Valuing proven undeveloped petroleum reserves using real options. In: Ronn, E. (ed.): *Real Options and Energy Management*, London, pp. 485-503.
43. McDonald, R.L., Siegel, D.R. (1986). The value of waiting to invest, *The Quarterly Journal of Economics*, Vol. 101 (4), pp. 707-727.
44. McGrath, R., Ferrier, W., Mendelow, A. (2004). Real options as engines of choice and heterogeneity, *Academy of Management Review*, Vol. 29 (1), pp. 86-101.
45. Moghaddam, A.T.N. (2010). Allocating the CO_2 emissions of an oil refinery with Aumann-Shapley prices: Comment, *Energy Economics*, Vol. 32 (1), pp. 243-255.
46. Moulin, H. (1988). *Axioms of cooperative decision making*, Cambridge University Press, Cambridge.
47. Myers, S.C. (1977). Determinants of corporate borrowing, *Journal of Financial Economics*, Vol. 5 (2), pp. 147-175.

48. Neumann, V.J.M., Morgenstern, O. (1947). *Theory of games and economic behaviour*, Princeton University Press, Princeton.
49. Paddock, J.L., Siegel, D.R., Smith, J.L. (1988). Option valuation of claims on real assets: the case of offshore petroleum leases, *Quarterly Journal of Economics*, Vol. 103 (3), pp. 479-508.
50. Pierru, A. (2007). Allocating the CO_2 emissions of an oil refinery with Aumann-Shapley prices, *Energy Economics*, Vol. 29 (3), pp. 563-577.
51. Pindyck, R.S. (1991). Irreversibility, uncertainty, and investment, *Journal of Economic Literature*, Vol. 29 (3), pp. 1110-1148.
52. Rammerstorfer, M., Eisl, R. (2011). Carbon capture and storage – investment strategies for the future, *Energy Policy*, Vol. 39 (11), pp. 7103-7112.
53. Rao, R.K.S., Martin, J.D. (1981). Another look at the use of options pricing theory to evaluate real asset investment opportunities, *Journal of Business Finance & Accounting*, Vol. 8 (3), pp. 421-429.
54. Rodríguez, R.Y. (2008). Real option valuation of free destination in long-term liquefied natural gas supplies, *Energy Economics*, Vol. 30 (4), pp. 1909-1933.
55. Rosenberg, M. et al. (2002). Price Spikes and Real Options: Transmission Valuation. In: Ronn, E. (ed.): *Real Options and Energy Management*, London, pp. 323-369.
56. Schmeidler, D. (1969). The nucleolus of a characteristic function game, *Journal of Applied Mathematics*, Vol. 17 (6), pp. 1163-1170.
57. Shapley, L.S. (1953). A value for n-person games. In: Kuhn, H.W., Tucker, A.W. (eds.): *Contributions to the theory of games*, Volume II, Princeton Univ. Press, Princeton, pp. 307-317.
58. Shapley, L.S. (1971). *Cores of convex games*, Santa Monica, Rand corp.
59. Songhuai, D., Xinghua, Z., Lu, M., Hui, X. (2006). A novel nucleolus-based loss allocation method in bilateral electricity markets, *IEEE Transactions on Power Systems*, Vol. 21 (1), pp. 28-33.
60. Stamtsis, G.C. and Erlich, I. (2004). Use of cooperative game theory in power system fixed-cost allocation, *Institution of Electrical Engineers*, Vol. 151 (3), pp. 401-406.
61. Szolgayova, J., Fuss, S., Obersteiner, M. (2008). Assessing the effects of CO_2 price caps on electricity investments – a real options analysis, *Energy Policy*, Vol. 36 (10), pp. 3974-3981.
62. Takizawa, S., Omori, R., Suzuki, A., Ono, K. (2001). Analysis of critical electricity price for the investment for constructing a nuclear power plant using real options approach, *Journal of Nuclear Science and Technology*, Vol. 38 (10), pp. 907-909.
63. Tan, X., Lie, T.T. (2002). Application of the Shapley Value on transmission cost allocation in the competitive power market environment, *IEE Proceedings*, Vol. 149 (1), pp. 15-20.
64. Tijs, S.H. (1981). Bounds for the core of a game and the [tau]-value for quasi-balanced games, Nijmegen.
65. Tijs, S.H. (1986). An axiomatization of the [tau]-value, Nijmegen.
66. Tijs, S.H., Driessen, T.S.H. (1986). Game theory and cost allocation problems, *Management Science*, Vol. 32 (8), pp. 1015-1028.
67. Tsukamoto, Y., Iyoda, I. (1996). Allocation of fixed transmission cost to wheeling transactions by cooperative game theory, *IEEE Transactions on Power Systems*, Vol. 11 (2), pp. 620-629.
68. Weisbrod, B.A. (1964). Collective-consumption services of individual-consumption goods, *Quarterly Journal of Economics*, Vol. 78 (3), pp. 471-477.
69. Young, H.P. (1994). Cost allocation. In: Aumann, R.J./Hart, S. (eds.), *Handbook of game theory with economic application*, Vol. II, Elsevier, Amsterdam, pp. 1193-1235.
70. Zelewski, S., Peters, M. (2010). Fair distribution of efficiency gains in supply networks from a cooperative game theory point of view, *International Journal of Information Systems and Supply Chain Management*, Vol. 3 (2), pp. 1-24.
71. Zhu, L., Fan, Y. (2011). A real options-based CCS investment evaluation model: Case study of China's power generation sector, *Applied Energy*, Vol. 88 (12), pp. 4320-4334.