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Identifying time variability in stock and interest rate dependence

Abstract

The correlation between stock markets and interest rates has been discussed in numerous studies in the past, with differing results in strength and direction of the relationship. This paper uses models of the multivariate GARCH type which allow for time-variability and regime changes in correlation. All estimated models allowing for time-varying correlation complement each other in identifying time-varying patterns found in the (co)-movement between the variables. Furthermore, the authors provide evidence for both large changes in correlation, as well as for the existence of regimes between which correlation may move. The result of a dominant time factor indicates a transition in market structures over time which is in line with observations in the markets and which may be seen as the explanation of previously differing results.

Keywords: multivariate GARCH, smooth transition, conditional correlation, regime changes.
JEL Classification: C32, C58.

Introduction

Correlations among the major asset classes take a central role in both theoretical and empirical research, as understanding, estimating and interpreting (co)-movements is crucial for market participants, institutions and policy makers. Considerable effort was spent on explanations and models, and work on identifying the behavior and determinants of correlations is still ongoing: Both the very nature of the financial markets that appear to be increasingly dynamic and the effects of time call for renewed and appropriate dealing with the topic. As not only history is prolonged with every additional trading day but the apparent structures are shifting with more or less strong effects, new insights and interpretations become possible and necessary.

While the possible changes in structures and market behavior may already be a challenge on their own, the expected interaction of asset classes itself is far from clear-cut: Considering the different asset classes, there exist many possible channels through which (co)-movements may be affected, and influences may have both time-varying magnitudes and directions. For the case of interest rates and stocks, negative correlation expectations had to be at least relaxed in recent years. A quote found in Li (2003) delivers a strong yet interesting notion on the previously stated relation between stocks and interest rates, and ultimately bond markets: "In the first version of The Intelligent Investor, published in the 1950s, the author, then investment guru Benjamin Graham, claims that the correlation between stock and bond returns is negative. His argument provides the basis for the asset allocation advice of 50-50 split in stocks and bonds. However, in the second version of this book published in the 1970s, the correlation structure has changed and the argument is dropped. Today, one can randomly search the term "stock and bond correlation" on the Internet, and easily find sharply contradictory opinions among market participants. When it comes to story-telling, one man's story is just as good as others. Most of these opinions are based on causal observations and lack the support of concrete evidence".

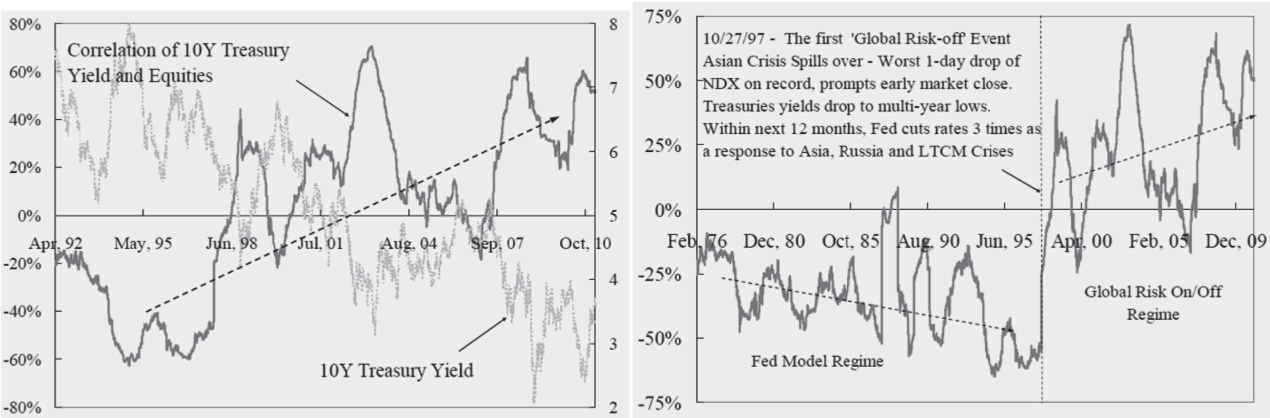
However, there is evidence, even concrete evidence. The problem nevertheless remains: Evidence was provided in either way in the past, and depending on the time span. With Shiller and Beltratti (1992) and Campbell and Ammer (1993) reporting positive correlation between long-term bond returns and stock returns[1], the correlation is constant due to the construction of the analyses. It has been shown in the past that the correlation however, appears to be different between time periods, an observation found by Gulko (2002), Ilmanen (2003), Connolly, Stivers and Sun (2005), Andersson, Krylova and Vahamaa (2008), Aslanidis and Christiansen (2010) and Schopen and Missong (2012) among others. While the studies differ among each other, the most obvious pattern that can be seen is a change from negative to positive correlation between stock returns and bond yields. Accordingly, studies focussing on stock return interaction with bond returns or interest rates mainly show changes from negative to positive correlation over time.

These findings have been discussed in practice as well: The 2011 J.P. Morgan research paper "Rise of Cross-Asset Correlations" by Kolanovic discusses an observable pattern of rising correlations among asset classes. Figure 1 shows the bond-stock correlation graphs of the study. Kolanovic (2011) identifies the abandonment of the so-called "Fed Model"[2] in favor

[1] Studies in the past differ as well regarding the correlation sign as some authors analyze stocks and bond yields, while others use stocks and bond prices.

[2] The name derives from the inaugural mentioning in a Federal Reserve Policy Report of July 1997, although many sources cite Yardeni (1997) and Yardeni (1999), as the reference was first made therein. See Lander, Orphanides, and Douvogiannis (1997a) and Lander, Orphanides, and Douvogiannis (1997b) for the Fed report and a related publication.

of a “risk-on/off” strategy as driving factor of the shift in correlation from negative to positive. The reasoning is that nowadays investors allocating away from risky assets move into riskless assets in the so-called flight to quality, thereby driving bond prices up and yields lower. Furthermore, monetary policy with increasing rates during heated economic phases and low rates to accommodate growth and avoid recession in down-cycles further fuels the positive correlation according to the JP Morgan study. In contrast, the Fed Model would have provided for an explanation of negative correlation: Interest rate yields are expected to be close to equity earning yields, inducing an inverse relationship between stock prices/returns and treasury yields through investors’ comparison of expected gains.



Asset Class	Correlation Between	1990-1995*	Past 5Y	Change
Equity	DM Country Indices	31%	47%	**17%**
Equity	EM Country Indices	23%	45%	**23%**
Equity	DM and EM Indices	38%	74%	**36%**
Equity	Economic Sectors	57%	69%	**12%**
Equity	Individual Stocks	25%	41%	**16%**
Credit	High Yield and Equities	46%	64%	**19%**
Credit	High Yield and VIX	37%	60%	**24%**
Foreign Exch.	DM Currencies and Equities	-1%	28%	**29%**
Foreign Exch.	EM Currencies and Equities	6%	42%	**36%**
Interest Rates	10Y Rate and Equities	-38%	29%	**67%**
Commodity	All Commodities	5%	25%	**21%**
Commodity	Commodities and Equities	-5%	12%	**17%**
Average		19%	45%	**26%**

* For Credit 2002-2005. All Currencies vs. USD

Notes: The plot shows the 10 Year Treasury Yield, the S&P 500 and the CBOE Volatility Index over the entire sample period between January 1990 and February 2012. Data is weekly, differences were taken for the interest rate series, log differences for the stock market and volatility index series. Source: JP Morgan via Chicago Board Options Exchange (http://www.cboe.com/Institutional/JPMCrossAssetCorrelations.pdf).

Fig. 1. Plot of the correlation between Treasury Yields and stocks from Kolanovic (2011)

While the Fed Model became prominent among financial market practitioners, Salomons (2006) is providing evidence against the model at least in the short-term domain of asset allocation. The arguments against the model on both empirical and theoretical grounds were previously raised as well in Asness (2003) and Estrada (2006) and Estrada (2009) among others, with Thomas and Zhang (2007) and Thomas and Zhang (2009) being proponents of the model. Abstaining from a discussion of the model and the critique, we can derive one common and important fact from studies in favor or in dismissal of the model: The observation of a negative correlation between interest rates and stocks and a change into positive dependency before the end of the 20th century.

From the perspective of financial stability, changes in the correlation between the returns of relatively safe assets like US government bonds and of risky financial assets such as stocks are of special interest due to the implications for risk management of financial market participants. Another important question is how changes in correlation depend on risk appetite, which is proxied for by implied volatility of stock returns in this paper. Moreover, changes in correlation might be an indicator of herding of financial market participants particularly in crisis times.

As our focus is to identify if there is time-dependency and how it is structured, it is crucial to know whether this is due to underlying driving factors, or whether structural changes or a shift in structures are the cause. Moreover, we focus on whether the time-variability is identified through regimes between which there may be transitioning or switching. Regarding previous work, the existence of studies with regime-dependent modeling of the correlation between interest rates and stock markets with smooth transition methods and observable transition parameters is limited to the study of Aslanidis and Christiansen (2010). However, their approach contains an estimation of the correlation in the first step to model it with a smooth transition regression model in the following, rather than having the correlation as an integral part of a multivariate estimation. Accordingly, we add to the literature by providing an analysis that allows the correlation to vary over time and to be regime-dependent, while the correlation itself is controlled by observable transition variables.

The structure of the study is as follows. We discuss the methodology in section 1, and present the empirical results in section 2. Implications derived from the results are discussed in section 3. The final section concludes the paper.

1. Correlation estimation in multivariate GARCH models

We employ the class of multivariate models with generalized autoregressive conditional hetero-scedasticity (GARCH). As has become common when analyzing financial data exhibiting time-varying variance and clustering of periods of large movements, GARCH models are capable to account for these effects and depending on the type of specification, allow for several modifications. Regarding multivariate GARCH (MGARCH) models, the fact that the estimation of such processes is exorbitantly demanding, and in many cases simply impossible, has led to different models with restrictions or step-wise procedures emerged in order to make the MGARCH models usable. We briefly review the ones that are used in this study, especially focusing on how the interaction between the variables is modeled.

Bollerslev (1990) decomposes the variance-covariance matrix to separate the conditional correlations from the conditional variances, leading to a parameterization of the conditional covariance and proportionality to the conditional standard deviation. While the decomposition is favorable with respect to dimensionality and estimation, it is a model of constant conditional correlation, and lacks the possibility to model spillover effects and correlations may not change during the course of time. Engle (2002) on the other hand extends the CCC model to allow for time varying correlation, thereby decomposing the GARCH modeling from the correlation specification. Therefore, one obtains a model where univariate GARCH models are linked through a dynamic conditional correlation part.

While the models discussed above have become pretty much standard approaches, the class of GARCH models allowing for smooth transition in the variance or correlation have extended the groups of available multivariate volatility models. With respect to the model selection, we are in line with Aslanidis, Osborn and Sensier (2009) who favor the smooth transition volatility models over Markov-type approaches as used for example in Ang and Bekaert (2002) and Pelletier (2006), as the smooth transition property allows the process to be continuously modeled and observed.

In general the smooth transition volatility models take the smooth transition (auto) regression (ST(A)R) models as defined by Teräsvirta (1994) and Teräsvirta (2004) to the volatility domain. Silvennoinen and Teräsvirta (2005) introduce the smooth transition in the correlation by defining the conditional correlation matrix to be a result of the transition function and two extreme states for the correlation matrices R_1 and R_2. The transition function G is defined as a logistic transition function and the general MGARCH specification as in the previously discussed conditional correlation models, leading to the smooth transition conditional correlation GARCH (STCC-GARCH) model:

$$H_t = D_t^{\frac{1}{2}} \times R_t \times D_t^{\frac{1}{2}},$$

$$R_t = (1 - G_t) \times R_1 + G_t \times R_2.$$

One prominent feature apart from the continuously modeled transition between correlation states is the selection possibility for the transition variable. While Berben and Jansen (2005) in their independently introduced approach have a time transition, one may select transition variables according to aspects of the respective study's aim[1].

Silvennoinen and Teräsvirta (2009) extend the STCC-GARCH model to allow for two transition variables. Accordingly, the specification below leads to the double smooth transition conditional correlation GARCH (DSTCC-GARCH) model with four correlation matrices and two transition functions and transition variables:

[1] Any (D)STCC model that includes a time transition therefore may be seen as a type of time-varying STCC (TV-STCC) model. We stick to the naming as (D)STCC with time transition for the sake of brevity however.

$$R_t = (1 - G_{1t}) \times R_{1t} + G_{1t} \times R_{2t},$$

$$R_{it} = (1 - G_{2t}) \times R_{i1} + G_{1t} \times R_{i2} \text{ with } i = 1, 2,$$

$$G_{it} = (1 + e^{-\gamma_i (s_{it} - c_i)})^{-1}, \gamma_i > 0, i = 1, 2.$$

The DSTCC-GARCH model enables to combine effects of two variables for the conditional correlation, and Silvennoinen and Teräsvirta (2009) note the possibility of using both a variable influence and a time transition, what makes the models highly suitable for our study.

2. Empirical results for identification of correlation changes

2.1. Data and setup. We use data that became standard in the area of analyzing interactions between interest rates and stock markets. Interest rates are measured by the 10 Year Treasury Yield obtained from the U.S. Department of the Treasury, and the US stock market is best represented by the Standard and Poor's 500 Composite Index (S&P 500).

As the STCC and DSTCC models are estimated using transition variables, we need to specify which variables should be used as expected driving factors regarding the correlation. Besides the time as transition variable we include the stock market volatility, measured by the Chicago Board Options Exchange Market Volatility Index (VIX) as second transition variable. By doing so, we are able to see whether there is indeed a risk-on/off structure that shows up in the correlation between interest rates and stock markets. Furthermore, the DSTCC study of stock market correlations by Aslanidis, Osborn and Sensier (2009) finds correlation dependence on the VIX, and Silvennoinen and Teräsvirta (2009) in their bond-stock example identify the VIX as most significant in a test of constant correlation versus alternatives including the STCC and DSTCC model.

As the VIX measures the implied volatility of S&P 500 index options, it serves as a natural measure of volatility that is observable and prevailing at the market. One favorable feature is that the VIX by construction can be interpreted as a forward-looking measure. This is especially suitable when aiming at the identification of regimes that are expected to be driven by a risk-on/off structure.

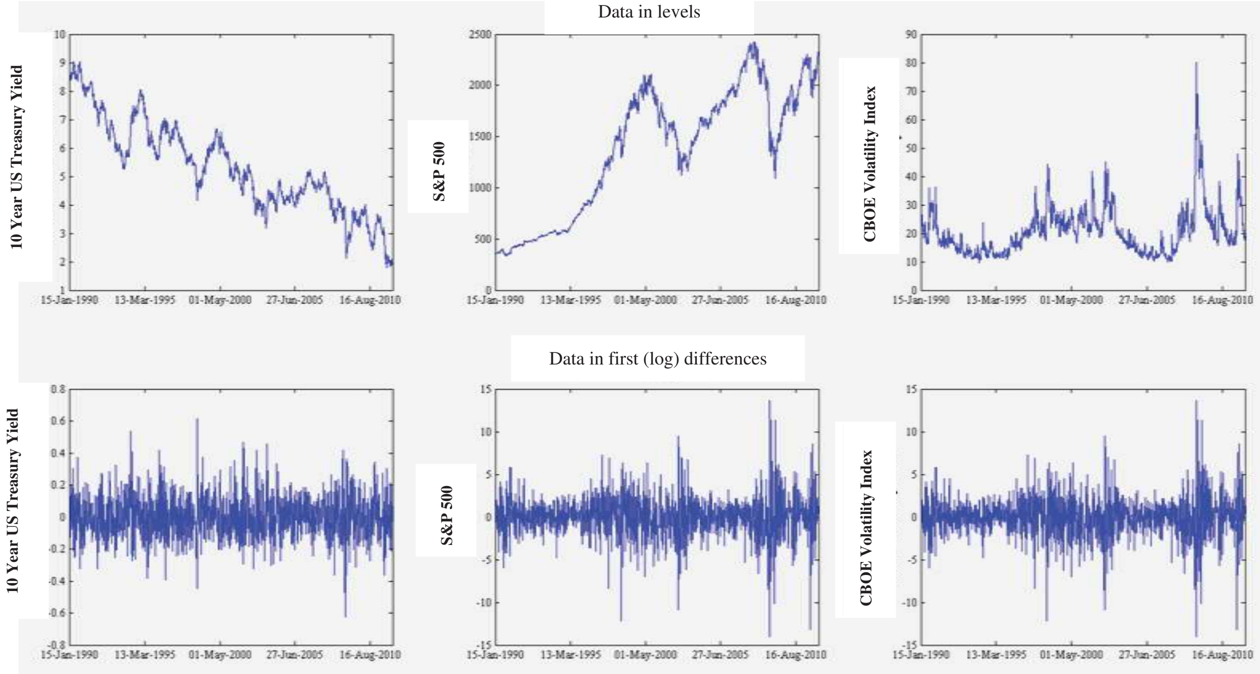
Although data is available on daily frequency, we use weekly data for the sake of estimation. While many GARCH applications have been done on daily data, we are in line with studies employing the smooth transition method to data on lower frequency when long time horizons are considered and where data is heterogeneous. This stems from the fact that albeit the computational burden could be tractable with respect to dimensionality, the large differences in the possible parameter estimates over time hamper algorithm convergence. Fortunately, a switch to weekly frequency was enough, thereby preserving more data information as compared to monthly frequency.

The time-period from the first week of 1990 (the date when the VIX was introduced) until the last week of February 2012 is covered by the sample, resulting in 1156 observations of level data and 1155 observations of return data. Descriptive statistics of the series are presented in Table 1 and all series in levels and returns are depicted in Figure 2. For the analysis we use differenced data, i.e. with Treasury yield changes and stock market returns.

Table 1. Descriptive statistics

	VIX	S&P 500	10-year Treasury Bill
Mean	0.69987	0.19030	-0.00528
Median	0.29922	0.26277	-0.01000
Maximum	102.87405	13.86592	0.61000
Minimum	-33.60417	-13.82788	-0.63000
Std. dev.	12.36719	2.53664	0.13900
Skewness	1.38783	-0.13857	0.24598
Kurtosis	9.83882	6.94352	3.82335
Jarque-Bera	2621.54702	752.10413	44.27143
Probability	0	0	0
Sum	808.35512	219.79893	-6.10000
Sum sq. dev.	176501.30754	7425.46810	22.29758
Observations	1155	1155	1155

Notes: The sample starts in the second week of January 1990 and ends in the last week of February 2012. Descriptive statistics are for the changes of the 10 Year US Treasury Yield, and the log differences of the S&P 500 and the CBOE Volatility Index. All values reported in percent.



Notes: The plot shows the 10 Year Treasury Yield, the S&P 500 and the CBOE Volatility Index over the entire sample period between January 1990 and February 2012. Data is weekly, differences are taken for the interest rate series, log differences for the stock market and volatility index series.

Fig. 2. The data from 1990 to 2012

2.2. CCC-GARCH model and rolling correlations. As a first step in the analysis of interaction between the series, we estimate the CCC-GARCH model on the change in the Treasury rate and the log-returns of the stock market data. Estimated parameters of the CCC-GARCH and test statistics enter Panels 1 and 2 of Table 2. The GARCH estimation for the two series indicates that both the ARCH and GARCH parameters are highly significant and within the necessary restrictions that ensure a non-explosive process. The sum of the GARCH coefficients for the S&P 500 is around 0.98 what displays the common result of considerably high persistence of volatility shocks, albeit the values do not give rise to concerns regarding the model stability. This holds true for all following estimations of MGARCH models so the univariate GARCH model parts need not be discussed in details.

Table 2. Estimates of CCC-, DCC-, STCC-, and DSTCC-GARCH models

Model	CCC	DCC	STCC$_t$	STCC$_v$	DSTCC$_{v,t}$
Constant rate series	0.0014845 (0.00081)	0.00152 (0.00001)	0.00205 (0.00097)	0.00148 (0.00081)	0.00206 (0.00092)
Constant stock series	0.16078 (0.05323)	0.15268 (0.00417)	0.16956 (0.05540)	0.16046 (0.05329)	0.17753 (0.05705)
ARCH (1) rate series	0.06289 (0.01878)	0.06298 (0.00057)	0.06056 (0.01851)	0.06284 (0.01877)	0.06352 (0.01880)
ARCH (1) stock series	0.14982 (0.02529)	0.15029 (0.00133)	0.13671 (0.02358)	0.14972 (0.02532)	0.13963 (0.02412)
GARCH(1) rate series	0.86000 (0.05369)	0.85818 (0.00435)	0.83369 (0.06123)	0.86016 (0.05382)	0.83280 (0.05808)
GARCH(1) stock series	0.83076 (0.02672)	0.83238 (0.00127)	0.83903 (0.02701)	0.83091 (0.02677)	0.83504 (0.02763)
DCC*p*		0.07351 (0.00024)			
DCC*q*		0.91443 (0.00037)			
Correlation *R*	0.03612 (0.02968)	Average: 0.0124			
Correlation *R*11			-0.50859 (0.04082)	0.03317 (0.04900)	-0.57320 (0.04202)
Correlation *R*12			0.39857 (0.03646)	0.03781 (0.03712)	-0.37566 (0.07086)
Correlation *R*21					0.42484 (0.05061)

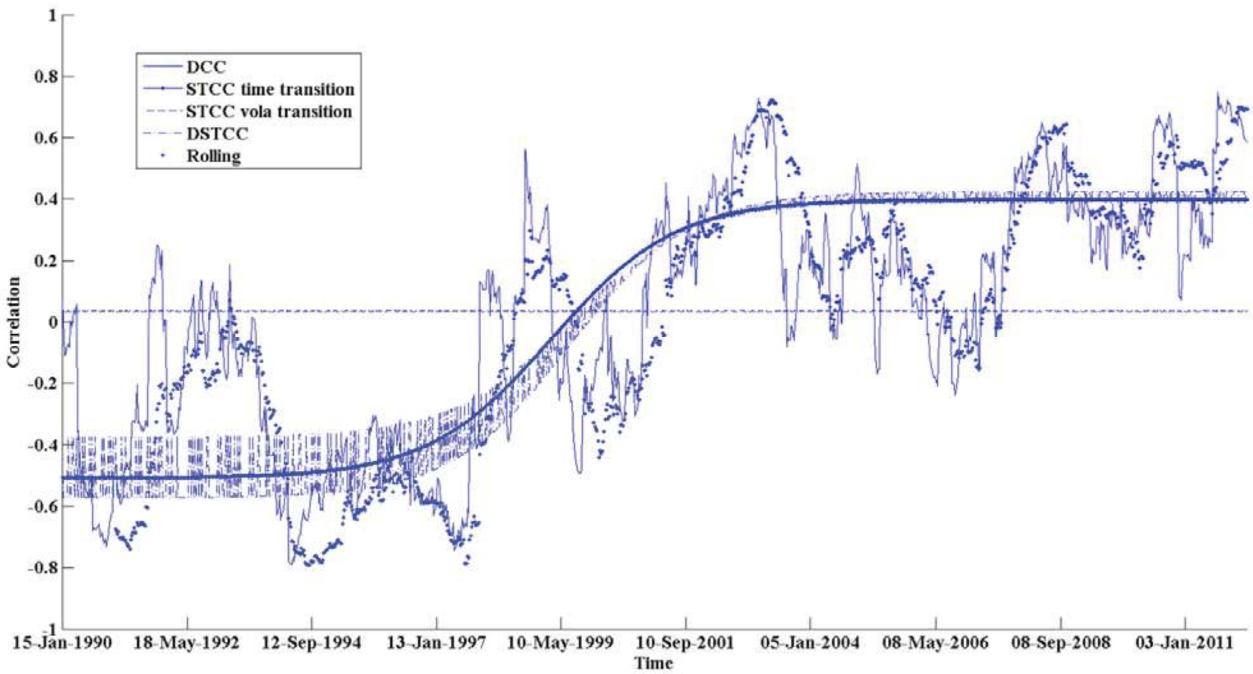
Table 2 (cont.). Estimates of CCC-, DCC-, STCC-, and DSTCC-GARCH models

Model	CCC	DCC	$STCC_t$	$STCC_v$	$DSTCC_{v,t}$
Correlation *R*22					0.38887 (0.04788)
Location transition variable time			0.41358 (0.01778) [March 99]		0.42972 (0.02012) [July 99]
Speed of transition 1			19.218		19.955
Location transition variable volatility				-0.00724 (0.00001)	0.04074 (0.00001)
Speed of transition 2				345.50	231.31

Notes: This table reports the coefficients and test statistics of the different MGARCH models; standard errors for the coefficients are in parentheses if not indicated otherwise. Indexes *t* and *v* represent the models with time-transition or volatility-transition while *v*; *t* indicates the model with time- and volatility-transition.

By construction, the correlation for the CCC-GARCH model remains the same over the whole estimation period. The estimated value of 0.04 corresponds to a conditional correlation of Treasury yield changes and stock market returns that is near zero. While the CCC-GARCH estimation mainly serves as an entry point to the analysis and to compare the models, we check whether the constant correlation of the CCC model holds against alternatives. As discussed above, we use the alternatives of STCC-GARCH with time and with volatility respectively, and DSTCC-GARCH with both variables as transition variables. The test statistics for all three alternatives are included in Panel 2 of Table 2 and indicate a rejection of the null of constant correlation when testing conditioned on the presence of the transition variables time, volatility or both.

As an intermediate step before estimating the various MGARCH models that allow for time varying and/or regime-dependent conditional correlation, we contrast the CCC estimation with rolling correlations. Using a window length of 52 to obtain estimates of the correlation on an annual horizon, we obtain the pattern over time as plotted in Figure 3, with correlations ranging from -0.79 to 0.72. Notably, the average of the rolling correlations with about 0.005 is very near to zero and to the CCC estimate, so the large differences in the correlation appear to be averaged out in the estimation process of the CCC model.



Notes: The plot shows the various estimated conditional and rolling correlations from the time varying MGARCH models. Correlations from the STCC-GARCH with time transition move from -0.509 to 0.399, for the STCC-GARCH with volatility change transition the "extreme" regime correlations are about zero with 0.033 and 0.037. DSTCC-GARCH correlations begin with the range of -0.573 and -0.376 and end with the range of 0.425 and 0.389. DCC-GARCH correlation dynamically moves over time with minimum -0.793 and maximum 0.784, mean about zero with 0.012. For the rolling correlation estimates for preceding 52 weeks, minimum is -0.792 and maximum is 0.722, mean about zero with 0.037. Constant conditional correlation estimate from CCC-GARCH is about zero with 0.036, as can be seen from the straight line.

Fig. 3. Comparison of time-varying correlations

The result of large differences in correlations over time is a well-documented fact for the last 30 years and is found in many studies and with differing approaches and aims, as found by Gulko (2002), Ilmanen (2003), Jones and Wilson (2004), Connolly, Stivers, and Sun (2005) and Connolly, Stivers, and Sun (2007), Cappiello, Engle, and Sheppard (2006), Engle and Colacito (2006), Christiansen and Ranaldo (2007), Andersson, Krylova, and Vahamaa (2008), Aslanidis and Christiansen (2010) and Schopen and Missong (2012). The increasing correlations over time as seen in the rolling window analysis may stem from either a trend in the correlation or from one or more structural breaks/ regime shifts in the interaction between the two series. Furthermore, this finding of rising correlations is in line with the Kolanovic (2011) study.

2.3. Comparison of time-varying conditional correlation GARCH models. Results of the (D)STCC approaches are discussed and compared to the DCC model estimations, the latter being estimated to see how the correlation changes over time after first checks by rolling correlations. All results are presented in Panels 1 and 2 of Table 2.

Table 3. Test statistics: estimated models vs. alternatives

Model	CCC	DCC	$STCC_t$	$STCC_v$	$DSTCC_{v,t}$
CCC vs. $STCC_t$	148.74 (0.00000)				
CCC vs. $STCC_v$	12.532 (0.00040)				
CCC vs. $DSTCC_{v,t}$	236.56 (0.00000)				
$STCC_t$ vs. $DSTCC_{v,t}$			9.0949 (0.0025)		
$STCC_v$ vs. $DSTCC_{v,t}$				148.84 (0.00000)	

Notes: This table reports the coefficients and test statistics of the different MGARCH models; standard errors for the coefficients are in parentheses if not indicated otherwise. Indexes *t* and *v* represent the models with time-transition or volatility-transition while *v*; *t* indicates the model with time- and volatility-transition.

All conditional correlation parameters of the DCC model are highly significant and suffice the parameter restrictions and the dynamic conditional correlation over time is similar to the rolling correlation and to patterns reported in most other recent studies[1]: An increase in the correlation between interest rates and the stock market, with sustained positive correlation following earlier periods of negative interaction. Ranging from -0.793 to 0.748, dynamic conditional correlation is moving within a large span as seen in the rolling correlations as well. In addition, the average of the DCC correlations with a value of 0.012 is approximately zero, as is the CCC estimate and the average of the rolling correlations.

The results from the DCC-GARCH strengthen the notion that estimation of correlations should be done within a time-varying framework, allowing for a correlation that is not fixed to be constant over time. In addition, the results of negative correlations in the beginning and positive correlation in the second half of the estimation period lead to the question about whether there is a trend or break in the correlation, and whether it is possible to detect this with regime-dependent analyses.

Estimating the separate STCC-GARCH models with time and volatility change, we obtain different results regarding the significance of the location parameters and the estimated correlations. While the conditional correlation estimate for both regimes and the location parameter is highly significant for the STCC model with time as transition variable, conditional correlation in the STCC model with volatility change as transition variable is insignificant. The location parameter however, is significant in the STCC model with volatility as well.

In the model with time as transition parameter, conditional correlation is negative with a value of -0.509 at the beginning of the sample period and is turning positive during the estimation sample, with a value of 0.399. This increase from clearly negative to clearly positive is in line with previous findings both in our analysis as well as in related studies. The fact that both correlation values − which can be seen as extreme regimes of correlation between which the process moves smoothly − are highly significant indicates that there is indeed regime-dependence. The transition location is 0.414 (or 41.4% of the sample size), which corresponds to the beginning of March 1999. At this point, the estimated conditional corre-

[1] Naturally, studies analyzing bond returns rather than treasury yields show the respective inverse picture of falling correlations over time, as shown in Andersson, Krylova and Vahamaa (2008) for example, where both rolling windows as well as DCC-GARCH estimation was used.

lation is -0.0538, what is fairly zero what has been observed before in the CCC model and in the averages of the CCC and rolling correlation approach.

When discussing the location parameter of the time transition variable in context of a smooth transition model, we need to take into account that the transition of course begins earlier. How early depends on the speed of transition as measured by the transition parameter. Due to the fact that intuition is hard to extract from the estimated value of 19.218, we depict the transition function in Figure 4: About 80% of the transition to the regime with positive correlations is taking place during August 1996 and September 2001, which is only about 23% of the sample size. Moreover, half of the transition (from 25% to 75%) is happening between December 1997 and June 2000, corresponding to about 11.4% of the analyzed time period.

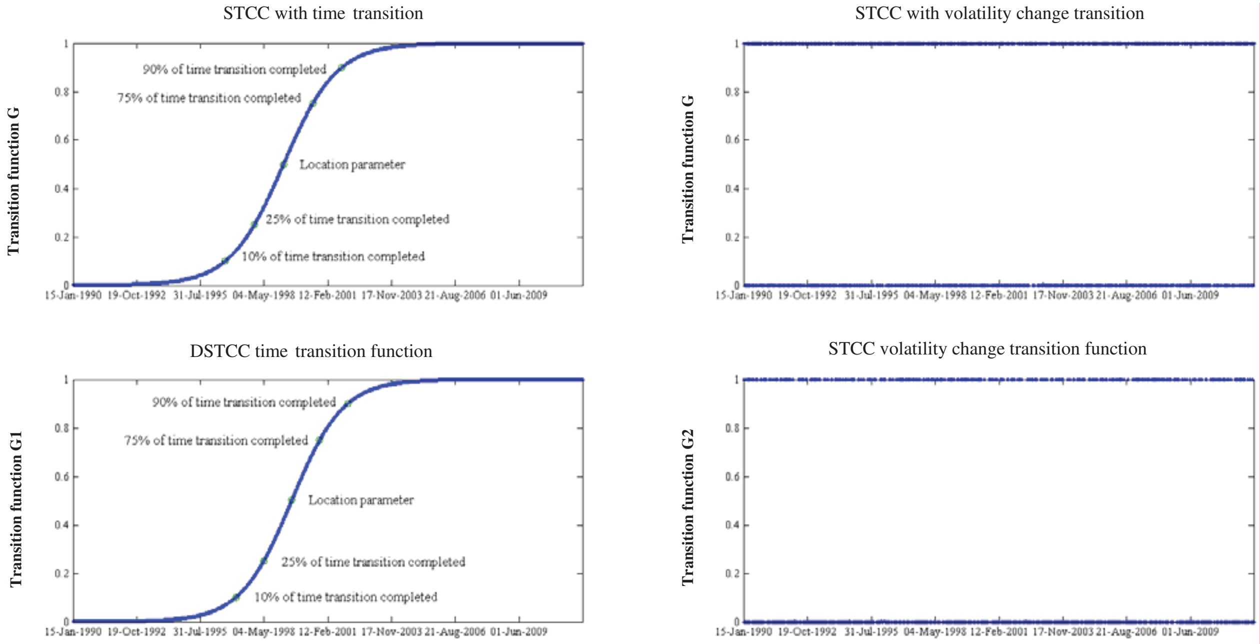
Regarding the correlation changes during the mentioned periods, one obtains -0.417 to 0.308 during the August 1996 and September 2001 span and -0.281 to 0.173 during the December 1997 and June 2000 period. Correlation crossing the line between negative and positive values occurs in June 1999. To compare the results with the DCC-GARCH, we calculate the average of the dynamic conditional correlation for the corresponding time periods. At the point where the transition according to the STCC-GARCH with time is at 10% or -0.417 correlation, the average of the DCC correlation from the sample beginning up to that point in time is -0.347 and at the 25% point the comparison yields -0.281 versus -0.375. Estimated conditional correlations during the phase where we are still in the old regime therefore are in the same area. Averaging the dynamic condition correlation from the 75% point and 90% point forward until the estimation's end, we obtain 0.283 and 0.309 compared to 0.173 and 0.308 in the STCC. We conclude that despite the swings in conditional correlation that are possible in the DCC model, the STCC with only time as transition variable is near the average of those dynamic conditional correlations when the sub-periods are considered. This can be seen from Figure 3 as well, with the DCC correlation roughly swinging around the correlation as estimated using the STCC with time transition.

As for the CCC model, we ran the test whether the conditional correlation should be modeled with an alternative; here we remain with a possible extension to the DSTCC model with both time and volatility change transition. The test statistic as shown in Panel 2 of Table 2 indicates clearly that the specification with only time as transition variable should be forfeited in favor of the richer model with both transition variables.

Before we discuss the DSTCC model however, we need to focus on the STCC with volatility change as transition variable. As mentioned above, the estimation of this model yields a significant location parameter (a change of 0.7% in the VIX) but insignificant correlation estimates of around zero for both regimes. Insignificance of correlation estimates using volatility as transition is less surprising when considering the fact that correlation changes so much over time. Accordingly, correlation estimates driven by a single transition variable may be less robust even if they were significant, leading us to the DSTCC model with results again being reported in Panels 1 and 2 of Table 2.

Now all four correlations are highly significant, as are the location parameters for both transition functions. The location of 0.429 (July 1999) for the time transition function takes on a value that is approximately the same as in the STCC with time transition (0.414 or March 1999). In the transition function dependent on the volatility change, the location parameter is 0.040, meaning that the regime is changing at an increase of about 4% of the implied market volatility. Apparently, the fact that volatility is now significantly changing the correlation and that the correlations estimated are all significant strengthens the notion that in the STCC model with volatility as transition the strong time dependence hampers identification of the respective correlations.

DSTCC correlation over time is depicted in Figure 3 and is similar to the STCC with time transition, only that now the correlation can vary not only over time, but within a range over time – due to the second transition variable. At the beginning of the sample when the time transition has not yet taken effect, correlation varies between -0.573 and -0.376, depending on the value of the change in stock market volatility as the second transition variable. Accordingly, now not only the parameters of the volatility transition function are significant, but the volatility change effect is observable and significant for the correlation values as well. Transitions are considerably fast, as indicated by the speed of transition parameter value of 231.31 and as seen from the graph of the value of the transition function in Figure 4. At the end of the sample period, when time transition is completed and the conditional correlation of this transition function is fully in its new regime, the correlation remains within a tighter range from 0.425 to 0.389.



Notes: The plots show the estimated transition functions over time for the STCC-GARCH with time as transition variable, STCC-GARCH with volatility change as transition variable and the DSTCC-GARCH with both as transition variables. Time transition and volatility transition functions of the DSTCC-GARCH are close to those obtained when estimating the respective STCC-GARCH models. In STCC-GARCH with time, 80% of transition to the new regime between August 1996 and September 2001 and 50% between December 1997 and June 2000. In DTSCC-GARCH, 80% of transition to the new regime between February 1997 and December 2001 and 50% between April 1998 and October 2000.

Fig. 4. Comparison of transition functions

One striking result of the DSTCC estimation is that the time transition part is almost the same as in the STCC with time as single transition variable. This is underpinning the results in terms of robustness. Additionally, with volatility change entering the model significantly and all correlation estimates being significant, we conclude that the variable itself indeed belongs to the estimation – only that without taking the time transition into account its effects are less visible. In addition, the strong time trend may be responsible for the slim band in which the correlation remains in the second half of the sample, with volatility being still significant, but having a smaller impact on the correlation itself.

3. Implications

From the estimations, one can clearly see that the correlation is highly time-varying and regime-dependent. From both the STCC models and the DSTCC model it is evident that there is a strong effect of time, whereas the volatility influence is less clear-cut at first glance. But considering the results from the DSTCC and the apparently strong time effect, these results have a natural interpretation: The insignificance of some parameters and the conditional correlation of about zero in the STCC model with volatility change transition may be due to identification problems caused by the strong shift in dependence over time, and an averaging out of the correlation, as the correlation normally could be clearly negative or positive, *depending on the period of time*. Therefore, without the possibility to move from negative to positive in the course of time, there may be no distinction of the regimes due to volatility change.

Above interpretation is crucial in light of the discussion of a change from a Fed Model structure to as risk-on/off world. The fact that volatility is not the driving factor of a major change is by no means evidence against this possible structural development. It simply states that the effect of stock market volatility had a stronger impact before the transition to a new regime – as indicated by the time transition – took effect, and the possibility that the main effects are captured through the time factor. This implies a risk-on/off behavior that is marked by volatility considerations in earlier periods but becomes a structural factor in later time periods.

Moreover, as the recent years with the unfolding of the sub-prime crisis and what followed were marked by numerous phases of market turmoil, extreme changes and financial market deteriorations, correlation's relation to several otherwise identifiable drivers may have changed or simply been in disproportion and buried in the noise of the markets. Meanwhile, the time transition was in full effect with over 90% of the transition already being completed some years before the stock markets peaked in 2007 – and the estimated correlation sufficiently captures the nowadays positive relation between interest rates and stock markets.

Another more technical consideration is one that is focusing on the sensitivity of correlation estimates. As Füss and Glück (2012) point out, DCC models tend to exhibit highly unstable conditional correlation patterns and erratic behavior. They propose confidence intervals to identify fundamental changes in the conditional correlation process. This can be interpreted as being a technical correspondence to a theoretical notion of a more stable and medium- to long-term consideration of correlation, i.e. an expectation that correlation structures in an economically meaningful way do not change at high frequency – and therefore not due to quick changes in a possible transition variable either.

Apart from the discussion regarding impacts of time and volatility transition, the economic perspective that is related to the differing assumptions of yield comparisons in the Fed Model and the risk-on/off approach is interesting regarding the fact that there might indeed be a change and that the change took place at the end of the 20th century. While several studies identified different factors that may be driving correlation, our study is not providing evidence against those, but is in complement to others, due to the following reasons. Given the assumption that there is interplay between variables that changed over the course of time, the effects of those may be non-identifiable when a strong structural effect emerges from that interplay. Furthermore, the analysis identified a strong change from 1997 to 2001, and while other studies that focus on explaining correlation may find that the correlation is driven by factors that changed during that time, the time effect itself can be seen as the dominant driver that captures the effects in the shift towards a new regime with positive correlations. These implications and the fact that the time transition is far from being linear but steepest around the expected time period at the end of the 20th century, leads us to the conclusion that the (D)STCC model with time transition is correctly identifying a structural shift into a new regime of positive correlation in the estimated time period.

Conclussion

We identify a strong and significant time transition in the correlation between interest rates and the stock market using both STCC models with time transition and DSTCC models where the change in market volatility is added as a second transition variable. The time where the transition occurs is in line with both anecdotal evidence in the markets and previous research. Most crucial in our point of view is the existence of a regime change, indicating that the positive correlation between rates and stocks in recent years indeed is an effect of a changed structure prevalent in financial markets.

Apparently, the time effect is so strong and robust that both in the STCC model with time as single transition variable and accompanied by the volatility change, the transition function is almost the same. While the volatility change is influencing the estimation and all estimated correlation regimes are significant, the role of changing volatility as a transition variable is less strong than that of time regarding the correlation. This may be either due to the fact of the dominating influence of structural change that is identified through the time factor, or may arise from the fact that the volatility itself has been influenced heavily by the forces that drove the structural change at hand – because the time effect already captured much of the effects otherwise associated with volatility.

Regarding further research it will be interesting to identify whether the structural change that apparently occurred may be disentangled using different market factors and how sustainable the new regime is. Furthermore, correlation dependence on other factors may be interesting to study and to compare whether the time trend remains as strong when other factors that possibly drove the structural shift are added to the analysis.

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