“Modelling dependence between risk profiles through the Farlie-Gumbel-Morgenstern family in the compound Poisson-Lindley risk model”

Maria Martel (Spain), Agustín Hernández (Spain), Francisco José Vázquez Polo (Spain)

Modeling dependence between risk profiles through the Farlie-Gumbel-Morgenstern family in the compound Poisson-Lindley risk model

Abstract

This paper considers the case of dependence between risk profiles in compound collective risk model, where the dependence is modeled by making the prior densities of the model parameters, which belong to a Farlie-Gumbel-Morgenstern family, and a particular way to measure departures from independence is used. The authors analyze the consequences of the dependence, using the Bayes premium. The paper concludes that even for a coefficient with a constant linear correlation the consequences of the dependence on the Bayes premium may vary considerably, in magnitudes greatly superior to those found with respect to the linear correlation and the divergence.

Keywords: Bayesian analysis, Bayes premium, Farlie-Gumbel-Morgenstern distributions, risk profile dependence.

Introduction

In actuarial risk theory, the collective risk model is described by a frequency distribution for the number of claims N and a sequence of independent and identically distributed random variables representing the size of the single claims X_i. Frequency N and severity X_i are assumed to be independent, conditional on distribution parameters.

One of the most usual model types consists in considering that the distribution of the variable N (primary distribution), follows a Poisson law (Goovaerts and Kass, 1991) and that the secondary one, i.e., the severity of the claims, follows an exponential distribution (Panjer and Willmot, 1981; Sarabia and Guillén, 2008). Our interest is then focused in $S = X_1 + \ldots + X_N$ which denotes the aggregate losses or the total cost over a period.

Although the above modelization is often utilized, some aspects deserve special mention. Actuarial data often present positively skewed and overdispersion. Consequently, alternative distributions to Poisson are sometimes required. Certainly, due to the popularity of the Poisson distribution (with or without extra zeroes) some other sampling distributions have been somewhat neglected in the literature. In this paper, we propose a simple and applicable alternative distribution to model variable N, the Poisson-Lindely distribution (Sankaran, 1970). This distribution provides a statistical model that is more flexible for fitting data and which empirically fits many kinds of loss and/or actuarial data with a strong asymmetry presence (Ghitany et al., 2008) and where some other properties as overdispersion and zero-inflated are usually present in sample observations. Recently, Hernández-Bastida et al. (2011) derived Bayesian premium under the collective risk model using Poisson-Lindley and exponential distributions. Ghitany and Al-Mutairi (2009) provided a comprehensive treatment of statistical behavior of the Poisson-Lindley distribution and its parameter estimation.

If a random variable N follows a discrete Poisson-Lindley distribution with parameter λ, its probability density function is given by:

$$\Pr\left(N = k \mid \lambda\right) = \frac{\lambda^2(\lambda+2+k)}{\left(\lambda+1\right)^{k+3}}, \tag{1}$$

$k = 0,1,2,\ldots \quad \lambda > 0.$

It follows immediately from equation (1) that the following properties hold:

1. The moment generating function is:

$$M_1(t;\lambda) = \frac{\lambda^2\left(\exp(t)-\lambda-2\right)}{(\lambda+1)^2(\exp(t)-\lambda-1)}.$$

2. The mean and the variance are given by:

$$\mathrm{E}\left[N\right] = \frac{\lambda+2}{\lambda(\lambda+1)}, \text{ and}$$

$$\mathrm{Var}\left(\mathrm{N}\right) = \frac{\lambda^3+4\lambda^2+6\lambda+2}{\lambda^2(\lambda+1)^2}.$$

3. The coefficient of variation is:

$$\frac{1}{\lambda+2}\sqrt{\lambda^3+4\lambda^2+6\lambda+2},$$

and the Poisson-Lindley distribution presents overdispersion.

The following example shows that the Poisson-Lindley model is an alternative model to fit insurance data.

Example 1. The data for this example are taken from Ghitany and Al-Muatairi (2009) corresponding to the distribution of accidents to 647 women working on high explosive shells in five weeks, data that were previously used in Sankaran (1970). The sample dispersion index (1.485) is slightly bigger than 1, and sample tail index is negative (-0.33), indicating that there is overdispersion relative to Poisson but short tail relative to NB. Table 1 shows fits from the Poisson, Negative Binomial and Poisson-Lindley models to data set. Observe that all models are uniparametric except the Negative Binomial, which is biparametric. For comparative and illustrative purposes, all the usual measures, such as *p*-values, log likelihood, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are used to compare the estimated models. As is well known, a model with a minimum BIC value is to be preferred.

Table 1. Estimated parameters, *p*-value, log likelihood, x^2-statistic, AIC and BIC for Example 1

		Fitting distribution (expected frquency)		
# of accidents	Observed frequency	Poisson	Neg. Bin.	Poisson-Lindley
0	447	406.31	445.89	439.45
1	132	189.03	134.90	142.76
2	42	43.97	44.00	44.96
3	21	6.82[a]	14.70	13.85
4	3	079[a]	4.96[a]	4.20[a]
5+	2	0.07[a]	1.69[a]	1.25[a]
Parameters		$\lambda = 0.4652$	$\rho = 0.6503$	$\lambda = 2.729$
χ^2			$r = 0.8651$	
(d.f.)		65.01 (2)	3.27 (2)	4.86 (3)
p-values		<1%	0.1945	0.182
Log likelihood		617.184	592.267	592.67
AIC		1236.37	1188.53	1187.42
BIC		1240.84	1197.48	1191.89

Notes: [a]Expected frequencies have been combined for the calculation of χ^2.

The estimation of the parameters λ's, p's and/or *r*'s by the maximum likelihood method is presented in Table 1, where numerical routines are used to solve the non-linear systems presented in the normal equations. The Mathematica® software package was used to code these computations. The maximum likelihood estimator ($\hat{\lambda}$) for the Poisson-Lindley model was obtained following the method described by Ghitany and Al-Mutairi (2009) and tested simultaneously by the EM algorithm from Karlis (2005). Sankaran's moment estimate (i.e., $\tilde{\lambda} = \dfrac{-(\bar{x}-1)+\sqrt{(\bar{x}-1)^2+8\bar{x}}}{2\bar{x}}$) was also obtained and its value is $\tilde{\lambda} = 2.726$. Observe that the maximum likelihood estimator (MLE) and that obtained from the method of moments (MOM) are very close. In fact, the observed distribution provided by the Poisson-Lindley model under the MOM estimator is similar to that from MLE, thus verifying Sankaran's conjecture that the moment estimate is close to the MLE (Karlis, 2005). Table 1 shows that the Poisson-Lindley model performs very well in fitting the distribution, compared to other uniparametric models, and provides a fit as good as that of the biparametric Negative Binomial model. Based on the AIC and BIC, the PL distribution fits the data better than NB, and NB distribution is better than Poisson. Furthermore, the Poisson-Lindley model presented is somewhat simpler than the NB and therefore it might appear to be preferable as a less complex model, taking into account the Ockham's razor principle.

On the other hand, the computations required to obtain *S* under the different models above cited are difficult to perform without the independence hypothesis. In the absence of this independence hypothesis, it may be necessary to elicit the bidimensional prior distribution without any subjective meaning. Peters et al. (2008) proposed that this kind of independence assumption in operational risk models should be investigated further.

Using the Poisson-Lindley distribution as a primary distribution, the main objectives of this article are: (1) to carry out an easy implementable statistical procedure to investigate the importance of the independence assumption; and (2) to produce an application that involves computational aspects and a simulated data analysis based on this procedure.

Certainly, in actuarial practice, the final purpose of the analysis is to provide a good estimate of the premium to be charged. In our opinion, this paper could be considered as a previous step in this final objective. We try to answer to the general questions: How realistic the model's hypothesis in practice is? Is the choice of independence hypothesis motivated by mathematical tractability rather than by theoretical justification? This paper presents a particular use of the Farlie-Gumbel-Morgenstern (FGM) family classes of priors which can be used to determine whether there are large deviations in final insurance decisions when the assumption of prior independence does not hold.

With respect to previous studies, we address the problem of independence from a different standpoints. First, we focus on the hypothesis of the independence of risk profiles as an indirect way of analyzing the independence between claim frequency and claim severity. Subsequently, we propose a model of (weak) dependence between the prior densities of these risk profiles, including the case of independence as a particular case using the FGM family of distributions (Morgenstern, 1956). By means of these tools, it is a straightforward matter to study how the independence hypothesis affects actuarial decisions. By setting a measure of comparison (for example, the Bayes premium), it suffices to compare this measure over the entire class under consideration with the one that would be obtained under independence.

The article is organized as follows. Section 1 provides details of the statistical model proposed containing the likelihood derived from the choice of a Poisson-Lindley count distribution and exponential severities, and the class of priors considered to develop a Bayesian analysis. Section 1 also presents the derivation of the (prior) joint moments of parameters, including the covariance and correlation coefficients. In Section 2 we describe how the models react to variations in the independence of the risk profile priors with respect to the Bayes premium, and how the results obtained can be used in practice. Some conclusions are drawn and final comments made in the last section.

1. The statistical model

1.1. The likelihood. As commented in section 1, we consider that N follows a Poisson-Lindley distribution with parameter λ and in order to complete our model we need a distributional assumption on severities. We suppose that random variables X_i, $i = 0,1,\ldots$ follow an Exponential distribution of parameter $\theta \geq 0$,

$$f_2(x_i|\theta) = \theta e^{-\theta x_i},\ x_i > 0. \tag{2}$$

Its moment generating function is given by $M_2(t;\theta) = \dfrac{\theta}{\theta - t}$. The mean and variance, respectively, for each i are then:

$$\mathrm{E}[X_i] = \frac{1}{\theta},\ \text{and}\ \mathrm{Var}(X_i) = \frac{1}{\theta^2}.$$

We assume conditional independence between claim amounts and claim numbers. Then, in the compound collective model our interest is focused on the random variable "total cost or aggregate loss", S, where its probability density function is defined by:

$$f(s\mid\lambda,\theta) = \sum_{n=0}^{\infty}\Pr(N = n\mid\lambda)\cdot f_2^{ne}(x\mid\theta),$$

where $\Pr(N=n|\lambda)$ denotes the probability that n claims have occurred and f_2^{ne} is the n-th convolution of the $f_2(x\mid\theta)$ function in equation (2).

Proposition 1. The probability density function (pdf) and moment generating function, respectively, of the random variable aggregate losses S are given by respectively:

$$f(s\mid\lambda,\theta) = \begin{cases} \lambda^2(\lambda+1)^{-5}\cdot\theta\cdot\left(\theta s + (\lambda+1)(\lambda+3)\right)\exp\left(-\dfrac{\lambda\theta}{\lambda+1}s\right), & \text{if } s > 0; \\ \lambda^2(\lambda+1)^{-3}\cdot(\lambda+2), & s = 0. \end{cases} \tag{3}$$

$$M(t;\lambda,\theta) = \frac{\lambda^2\left[\theta^2(\lambda+1) - \theta\lambda(2\lambda+3) + t^2(\lambda+2)\right]}{(\lambda+1)(\theta\lambda - t(\lambda+1))^2},\quad t > 0. \tag{4}$$

Proof. Using equations (1) and (2) the corresponding aggregated loss distribution (the likelihood function when variable S is observed) density function is obtained after some straightforward calculations. Furthermore, it is obvious that for the case $s = 0$,

$$f(0\mid\lambda,\theta) = \Pr(N = 0\mid\lambda) = \frac{\lambda^2(\lambda+2)}{(\lambda+1)^3}.$$

On the other hand, $M(t;\lambda,\theta)$ is derived as in Klugman et al. (2004):

$$M(t;\lambda,\theta) = M_1(\log M_2(t;\theta);\lambda).$$

As a consequence of the above result, the first moment (i.e., the mean) and variance of variable S are given as:

$$\mathrm{E}(S) = \frac{\lambda+2}{\theta\lambda(\lambda+1)},$$

$$\mathrm{E}(S^2) = \frac{2(\lambda^2+3\lambda+3)}{\theta^2\lambda^2(1+\lambda)},$$

and $\mathrm{Var}(S) = \dfrac{2\lambda^3+7\lambda^2+8\lambda+2}{\theta^2\lambda^2(\lambda+1)^2}$.

1.2. The priors. Under a Bayesian viewpoint, the parameters of interest of the problem can be estimated by using our state of knowledge about them. A natural conjugate prior for a parameter θ under Poisson or exponential sampling is the Gamma $G(\alpha,\beta)$ density:

$$\pi(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\cdot\theta^{\alpha-1}\cdot\exp(-\beta\theta), \tag{5}$$

$\theta > 0,\quad c,d > 0.$

We consider values of α such that $\alpha > 1$. The prior mean and variance for θ are given by $\mathrm{E}(\theta) = \dfrac{\alpha}{\beta}$ and $\mathrm{Var}(\theta) = \dfrac{\alpha}{\beta^2}$. The corresponding prior mode for the parameter θ is $\mathrm{Mo}(\theta) = \dfrac{\alpha-1}{\beta}$. If we have very little prior information concerning θ, then the selection $(\alpha = 2, \beta = \frac{1}{4})$ is usually satisfactory (Scollnik, 1995).

In actuarial literature it is normally assumed that the parameters λ and θ are independent. Although there has been significant theoretical development of the sensitivity procedures in Bayesian statistics for prior independence (Lavine et al., 1991; Wasserman et al., 1993; Berger and Moreno, 1994) relevant applications have been less forthcoming. In this paper we propose to introduce some dependence between the risk profiles θ and λ through the Farlie-Gumbel-Morgenstern (FGM) system. The FGM family has been used recently in Cossette et al. (2008) as a tool to introduce dependence between claim amounts and the interclaim time, but in the context of copula theory.

The bivariate FGM system of distributions (originally introduced by Morgenstern, 1956) has a joint cumulative distribution function of the form

$$H(x,y) = F(x)G(y)\left[1+\omega\left(1-F(x)\right)\left(1-G(y)\right)\right], \qquad |\omega| \le 1, \tag{6}$$

where F and G are the marginal cumulatives. The joint density corresponding to (6) is:

$$f(x,y) = f(x)g(y)[1+\omega(1-2F(x))(1-2G(y))] = f(x)g(y) + \omega[f(x)(2F(x)-1)][g(y)(2G(y)-1)]. \tag{7}$$

Equation (7) is an easy and attractive method for constructing the joint distribution with specified marginals F and G (or equivalently, f and g). Lai (1978) showed that the parameter ω is directly proportional to the correlation coefficient. The usefulness of the system, however, is marred by the fact that it is restricted to describing relatively weak dependence. The product moment correlation coefficient for all the FGM distributions with continuous marginals can never exceed $\frac{1}{3}$.

Assuming that the two prior distributions are $\lambda : G(\alpha_\lambda, \beta_\lambda)$, and $\theta : G(\alpha_\theta, \beta_\theta)$, we proceed now to obtain the correlation coefficient over the FGM family of priors in order to obtain a measure of dependence between risk profile.

Observe that:

$$\mathrm{E}\left(\lambda\cdot\theta\right) = \frac{1}{\beta_\lambda \beta_\theta}\mathrm{E}\left(\xi_1\xi_2\right), \tag{8}$$

where $\xi_1 = \beta_\lambda \lambda : G(\alpha_\lambda, 1)$ and

$\xi_2 = \beta_\theta \theta : G(\alpha_\theta, 1)$.

Then, similarly to D'Este (1981) we obtain:

$$\mathrm{E}\left(\lambda\cdot\theta\right) = \beta_\lambda \cdot \beta_\theta \cdot \mathrm{E}\left(\xi_1\right)\cdot \mathrm{E}\left(\xi_2\right)\cdot\left(1+\omega\left\{2\frac{I(\alpha_\lambda)}{B(\alpha_\lambda)}-1\right\}\left\{2\frac{I(\alpha_\theta)}{B(\alpha_\theta)}-1\right\}\right), \tag{9}$$

where $I(\upsilon) = \int_0^1 \frac{z^{\upsilon-1}}{(1+z)^{2\upsilon+1}}dz$ and $B(\upsilon) = \frac{\Gamma(\upsilon)\Gamma(\upsilon+1)}{\Gamma(2\upsilon+1)}$ and function $I(\cdot)$ satisfy the relation

$$2\frac{I(\upsilon)}{B(\upsilon)} - 1 = \frac{1}{\upsilon}\cdot 2^{-2\upsilon}\cdot\frac{\Gamma(2\upsilon+1)}{\Gamma(\upsilon)\Gamma(\upsilon+1)}.$$

Therefore, from (9) we have the following:

$$\mathrm{E}\left(\lambda\cdot\theta\right) = \frac{\alpha_\lambda\alpha_\theta}{\beta_\lambda\beta_\theta}\left(1+\frac{\omega 2^{-2(\alpha_\lambda+\alpha_\theta)}}{\alpha_\lambda\alpha_\theta}\cdot\frac{\Gamma(2\alpha_\lambda+1)\Gamma(2\alpha_\theta+1)}{\Gamma(\alpha_\lambda)\Gamma(\alpha_\lambda+1)\Gamma(\alpha_\theta)\Gamma(\alpha_\theta+1)}\right), \tag{10}$$

and the (a priori) structure of dependence between the risk profiles λ and θ measured by the correlation coefficient is given by

$$\mathrm{Corr}(\lambda,\theta) = \frac{\mathrm{Cov}(\lambda,\theta)}{\sqrt{\mathrm{Var}(\lambda)\mathrm{Var}(\theta)}} = \frac{\omega}{\sqrt{\alpha_\lambda\alpha_\theta}}\cdot\frac{1}{2^{2(\alpha_\lambda+\alpha_\theta)}\cdot B(\alpha_\lambda,\alpha_\lambda+1)\cdot B(\alpha_\theta,\alpha_\theta+1)}. \tag{11}$$

Thus, since $|\omega| \le 1$, we obtain the bound:

$$|\mathrm{Corr}(\lambda,\theta)| \le \frac{1}{\sqrt{\alpha_\lambda\alpha_\theta}}\cdot\frac{1}{2^{2(\alpha_\lambda+\alpha_\theta)}\cdot B(\alpha_\lambda,\alpha_\lambda+1)\cdot B(\alpha_\theta,\alpha_\theta+1)}. \tag{12}$$

For the special case, $\alpha_\lambda = \alpha_\theta = 1$ it follows that

$$|\mathrm{Corr}(\lambda,\theta)| = \frac{|\omega|}{4} \le \frac{1}{4}. \qquad (13)$$

2. Analyzing the robustness of the independence hypothesis between risk profile parameters

Let $\pi_\omega(\lambda,\theta) = \pi_1(\lambda)\cdot\pi_2(\theta) +$
$+\,\omega[\pi_1(\lambda)(1-2F_1(\lambda))\cdot\pi_2(\theta)\cdot(1-2F_2(\theta))]$

be a prior density in the FGM family with $\pi_1(\lambda)$: $G(a,b)$ and $\pi_2(\theta)$: $G(c,d)$ fixed marginals, where $\omega = 1(-1)$ represents the maximum (minimum) degree of positive (negative) dependence allowed in the model.

Following De la Horra and Fernández (1995), there exists a simple way to test the assumption of prior independence between the λ and θ risk profiles. Consider the class of priors

$\Pi = \Pi_{(\omega>0)} \cup \Pi_{(\omega<0)}$, where

$\pi_I(\lambda,\theta) = \pi_1(\lambda)\cdot\pi_2(\theta)$ is the prior density obtained under independenceand $\pi_{(\omega=1,-1)}(\lambda,\theta)$ in the FGM family with $\omega = 1\,(-1)$ are fixed densities with marginals $\pi_1(\lambda)$ and $\pi_2(\theta)$, representing the larger of positive (negative) dependence.

$$\Pi_{(\omega>0)} = \left\{\pi(\lambda,\theta) = (1-\omega)\pi_I(\lambda,\theta) + \omega\pi_{(\omega=1)}(\lambda,\theta), \varepsilon\in[0,1]\right\},$$
$$\Pi_{(\omega<0)} = \left\{\pi(\lambda,\theta) = (1-\omega)\pi_I(\lambda,\theta) + \omega\pi_{(\omega=-1)}(\lambda,\theta), \varepsilon\in[0,1]\right\},$$

In order to test the influence of the independence hypothesis on posterior decisions, we focus on the problem in the following way. The Bayesian premium (i.e., the posterior mean of the true individual premium) plays an important role in ratemaking. As we know,

$$\mathrm{E}(S) = \frac{\lambda+2}{\lambda(\lambda+1)\theta}.$$

Then, the Bayes premium is obtained as the (posterior) expected value,

$$\mathrm{E}_{\pi_\omega(\cdot|s)}\left(\frac{\lambda+2}{\lambda(\lambda+1)}\frac{1}{\theta}\right) = \int_0^\infty\int_0^\infty \frac{\lambda+2}{\lambda(\lambda+1)}\frac{1}{\theta}\pi_\omega(\lambda,\theta\,|\,s)\,d\lambda\,d\theta. \qquad (14)$$

We present a particular way of determining whether there are large departures from premium measures when the assumption of prior independence is relaxed, and we find a method to account for such consequences in several common situations. That is, once the data are observed, we are interested in upper and lower bounds of these posterior quantities in (14) over class Π.

As in De la Horra and Fernández (1995), differentiating with respect to ω, the above bounds are calculated comparing only the three following quantities:

$$\frac{\iint h(\lambda,\theta)\cdot f(s|\lambda,\theta)\cdot\pi_{(\omega=i)}(\lambda,\theta)d\lambda d\theta}{\iint f(s|\lambda,\theta)\cdot\pi_{(\omega=i)}(\lambda,\theta)d\lambda d\theta},\ i=-1,1, \qquad (15)$$

and $$\frac{\iint h(\lambda,\theta)\cdot f(s|\lambda,\theta)\cdot\pi_I(\lambda,\theta)d\lambda d\theta}{\iint f(s|\lambda,\theta)\cdot\pi_I(\lambda,\theta)d\lambda d\theta}, \qquad (16)$$

where $h(\lambda,\theta) = \frac{\lambda+2}{\lambda(\lambda+1)}\frac{1}{\theta}$ and $f(s\,|\,\lambda,\theta)$ is the likelihood function given in equation (3).

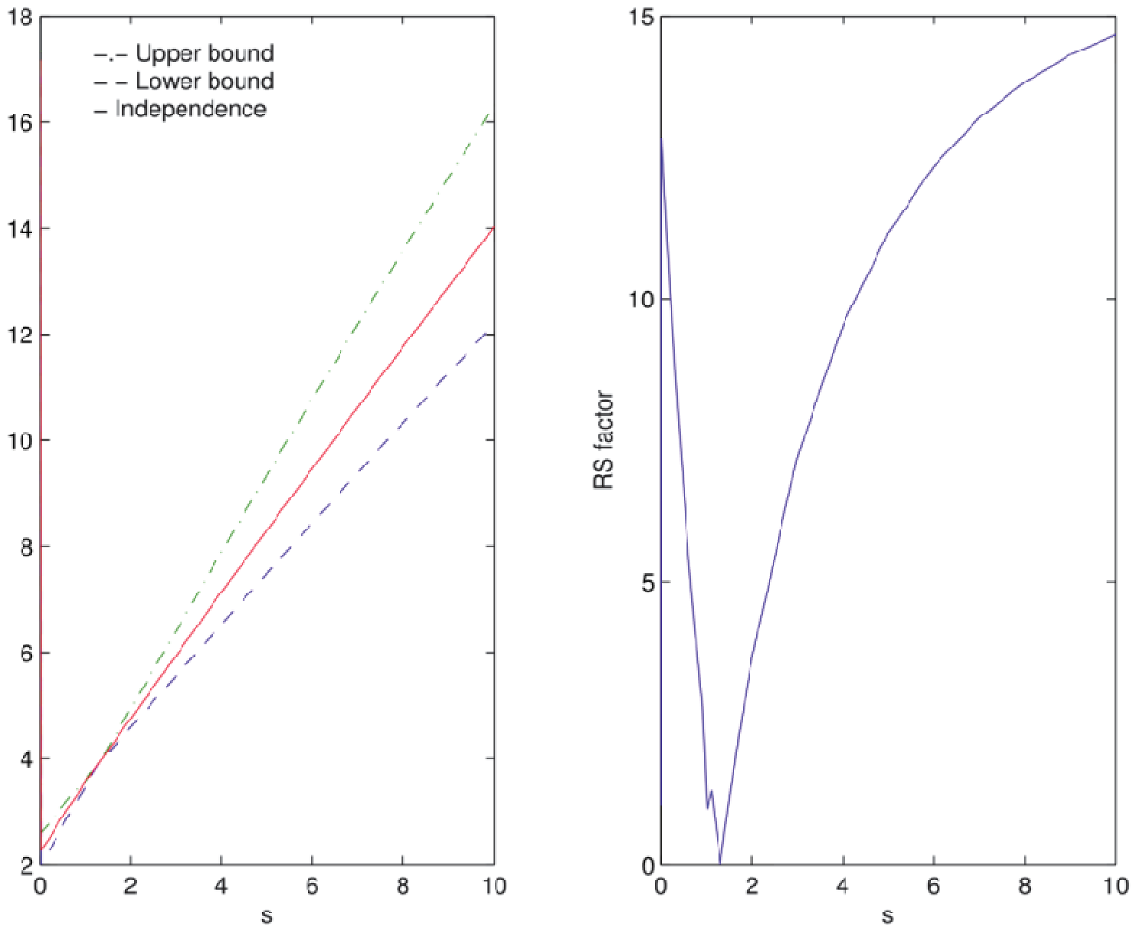
The difference between the upper and lower bound obtained from equations (15) and (16), denoted by $U - L$, is a measure of the robustness (or its absence, i.e., sensitivity) of the prior independence, for different values of s. In order to standardize this measure, we use a slight modification of the RS factor (Sivaganesan, 1991) defined by:

$$RS = 100\frac{U-L}{2\mathrm{E}_{\pi_I}(h(\lambda,\theta))}. \qquad (17)$$

RS is a standardized factor which can be thought of as the percentage variation in the Bayes premium as π_α varies over Π on either side of $\mathrm{E}_{\pi_I}(h(\lambda,\theta)\,|\,s)$, which is used as a pattern (independence scene), like the center of the variation interval (L,U).

Example 2. Consider an insurance business where the number of claims *N* has a Poisson-Lindley distribution with the parameter λ. Suppose also that each single claim size distribution is exponential with parameter θ. As commented in previous sections, one of the most useful compound collective risk models consists in assuming a Gamma prior distribution over λ (and θ). This is reasonable, since the shape of the Gamma density is very flexible (Miller and Hickman, 1974; Scollnik, 1995; among others). Let us now consider a numerical illustration. Supposing the actuary assumes the expected frequency to be E (λ) = 1 with "no claim" as the most frequent event. Hence, with these two items of partial prior information, it is reasonable to assume that the base prior $\pi_1(\lambda)$ is G (1,1) (with this elicitation the actuary knows that the mode is around 0). Using similar reasoning, suppose that the prior density for θ is also G (1,1) (i.e., the expert expects a claim size of 1 monetary unit). Due to the exponential behavior of the above priors, we can consider this elicited prior within a weak prior information context.

The ranges of the Bayes premium and the RS factor, for various values of s (from 0 to 10 by steps of 0.01) are shown in Figure 1. Similar conclusions may be obtained with greater values of s. The computations involved in equations (14)-(16) were carried out using Mathematica software. Several minutes of CPU time were needed to complete the calculations. The sensitivity of the answer to independence departures was measured by considering the RS factor in (17).

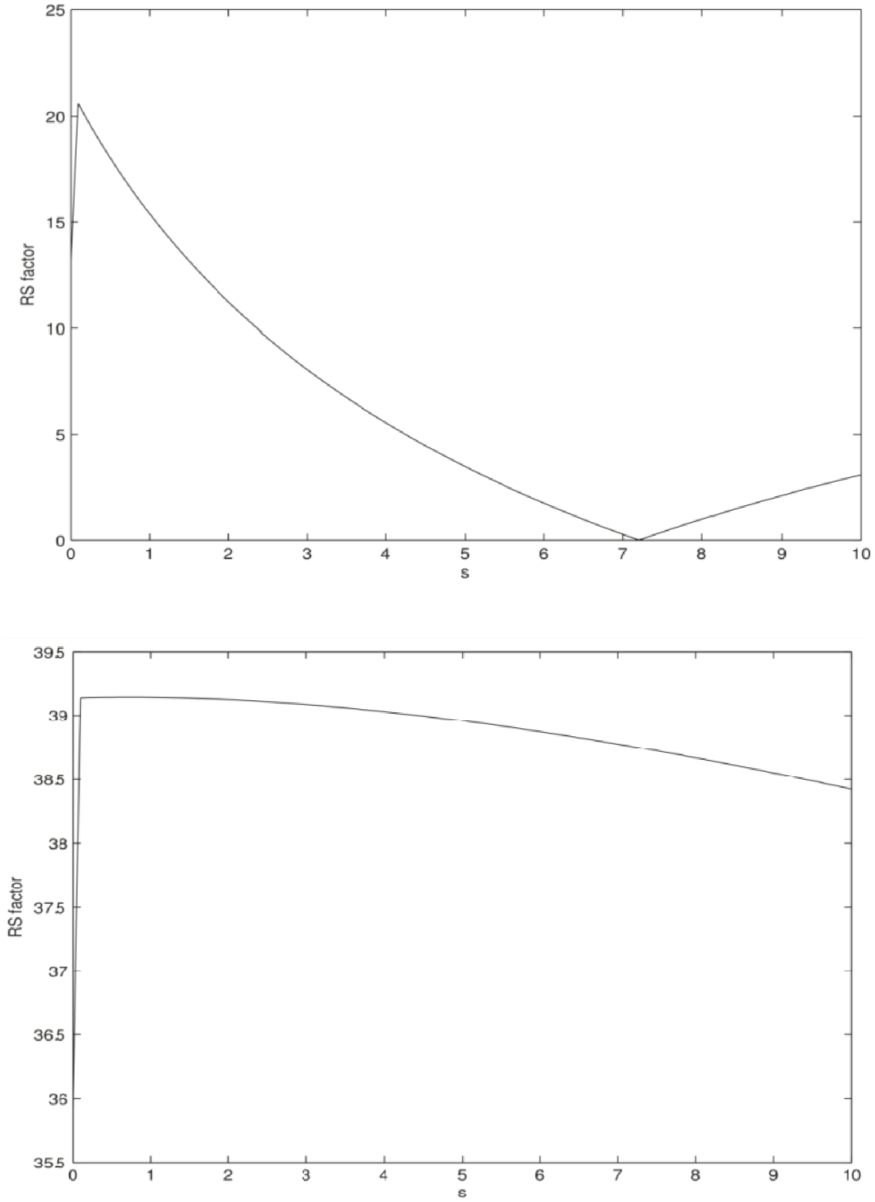


Notes: Left: Upper (dash-point line) and Lower (dashed line) bounds for the Bayes premium in the FGM family with both fixed marginals G (1,1). Continuous line represents the Bayes premium under independence hypothesis. Right: RS factor.

Fig. 1. Bounds for the Bayes premium in the FGM family

Some interesting general points emerge from Figure 1. First, observe that for the weak prior information ($\alpha_\lambda = \beta_\lambda = \alpha_\theta = \beta_\theta = 1$) scene considered, the correlation between the risk profiles in equation (13) is bounded by $\frac{1}{4}$. Then, we would expect a similar behavior of the RS -factor over the class. If ω is used to control the confidence level of practitioners concerning the independence assumption, similar behavior would be expected for the RS factor, for example, but this does not hold.

Also for illustrative purposes, Figure 2 shows two additional simulated situations ($\alpha_\lambda = \alpha_\theta = 1$ and $\beta_\lambda = \beta_\theta = 2$ and 20, respectively).



Notes: Upper: *RS* factor for the case $\alpha_\lambda = \alpha_\theta = 1$ and $\beta_\lambda = \beta_\theta = 2$. Lower: *RS* factor for the case $\alpha_\lambda = \alpha_\theta = 1$ and $\beta_\lambda = \beta_\theta = 20$.

Fig. 2. Sensitivity of the Bayes premium in the FGM family

Figure 2 shows the notable differences between the maximum ranges of variation of the *RS* factor, almost 20% in the first case, and 39% in the second, and also between the profiles, where a value of 20% is recorded in the first case, for a small range of values of *s*, while in the second case, 39% is recorded very frequently. In summary, the example examined presents a lack of robustness with respect to the hypothesis of independence between the two parameters.

Conclusions

In this paper we have examined the hypothesis of independence between the risk profiles (the parameters of the problem). To do this, firstly we propose an alternative close-fitting collective risk model where the primary and secondary distributions are Poisson-Lindley and exponential, respectively. Secondly, dependence was modeled using the Farlie-Gumbel-Morgenstern family, and the coefficient of linear correlation was determined with respect to the prior bidimensional distribution, in the case of independence, which is also known as the index of mutual dependence. Subsequently, we set out to analyze the robustness of a posterior magnitude of interest, the Bayes premium, with respect to variations from independence. An analytic path was developed, and various specific contexts examined. The numerical conclusions obtained reveal considerable, and very marked, differences between the values of the Bayes premiums within contexts of equal linear correlation.

Acknowledgments

Research partially funded by grant ECO2009-14152 (MICINN, Spain).

References

1. Berger, J.O. and Moreno, E. (1994). Bayesian robustness in bidimensional models: prior independence (with discussion), *Journal of Statistical Planning and Inference*, 40, pp. 161-176.
2. Cossette, H., Marceau, E. and Marri, F. (2008). On the compound Poisson risk model with dependence based on a generalized Farlie-Gumbel-Morgenstern copula, *Insurance: Mathematics and Economics*, 43 (3), pp. 444-455.
3. D'Este G.M. (1981). A Morgenstern-type bivariate gamma distribution, *Biometrika*, 68, pp. 339-340.
4. De la Horra, J. and Fernández, C. (1995). Sensitivity to prior independence via Farlie-Gumbel-Morgenstern model, *Communications in Statistics: Theory and Methods*, 24 (4), pp. 987-996.
5. Ghitany, M.E., Al-Mutairi, D.K. and Nadarajah, S. (2008). Zero-truncated Poisson-Lindley distribution and its applications, *Mathematics and Computers in Simulation*, 79, pp. 279-287.
6. Ghitany, M.E. and Al-Mutairi, D.K. (2009). Estimation methods for the discrete Poisson-Lindley distribution, *Journal of Statistical Computation and Simulation*, 79, pp. 1-9.
7. Goovaerts, M.J. and Kaas, R. (1991). Evaluating compound generalized Poisson distributions recursively, *ASTIN Bulletin*, 21, pp. 193-197.
8. Hernández-Bastida, A., Fernández-Sánchez, M.P. and Gómez-Déniz, E. (2011). Collective risk model: Poisson-Lindley and exponential distributions for Bayes premium and operational risk, *Journal of Statistical Computation and Simulation*, 81 (6), pp. 759-778.
9. Karlis, D. (2005). EM algorithm for mixed Poisson and other discrete distributions, *ASTIN Bulletin*, 35 (1), pp. 3-24.
10. Klugman, S., Panjer, H. and Willmot, G. (2004). Loss Models: From Data to Decisions (2nd edition), John Wiley and Sons.
11. Lai, C.D. (1978). Morgenstern's Bivariate Distribution and its Applications to Point Processes, *Journal of Mathematical Analysis and Applications*, 65, pp. 247-256.
12. Lavine, M., Wasserman, L. and Wolpert, R. (1991). Bayesian inference with specified prior marginals, *Journal of the American Statistical Association*, 86, pp. 964-971.
13. Miller, R. and Hickman, J. (1974). Insurance credibility theory and Bayesian estimation, In Kahn, P.M. (ed.) Credibility theory and applications, New York: Academic Press, pp. 249-270.
14. Morgenstern, D. (1956). Einfache beispiele zweidimensionaler verteilungen, *Mitt. Math. Statistik*, 8, pp. 234-235.
15. Panjer, H.H. and Willmot, G.E. (1981). Finite sum evaluation of the negative binomial-exponential model, *ASTIN Bulletin*, 12, pp. 133-137.
16. Peters, G.W., Shevchenko, P.V. and Wüthrich, M.V. (2008). Dynamic operational risk: modelling dependence and combining different sources of information, Preprint, CSIRO, call number CMIS2741.
17. Sankaran, M. (1970). The discrete Poisson-Lindley distribution, *Biometrics*, 26, pp. 145-149.
18. Sarabia, J.M. and Guillén, M. (2008). Joint modelling of the total amount and the number of claims by conditionals, *Insurance: Mathematics and Economics*, 43 (3), pp. 466-473.
19. Scollnik, D.P.M. (1995). Bayesian analysis of two overdispersed Poisson regression models, *Communications in Statistics: Theory and Methods*, 24 (11), pp. 2901-2918.
20. Sivaganesan, S. (1991). Sensitivity of some posterior summaries when the prior is unimodal with specified quantiles, *Canadian Journal of Statistics*, 19, pp. 57-65.
21. Wasserman, L., Lavine, M. and Wolpert, R. (1993). Linearization of Bayesian robustness problems, *Journal of Statistical Planning and Inference*, 37, pp. 307-316.