“Broad market risk for sector fund of funds: a copula-based dependence approach”

Michael Stein (Germany), Svetlozar T. Rachev (Germany), Stoyan V. Stoyanov (USA)

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Abstract

A crucial problem for institutional money managers that are focussed on one sector or sub-sector of financial markets is to know to what degree they depend on the broad markets they aim at diversifying away from. This is a special problem for fund of fund (FoF) managers because with an increasing number of target funds, the marginal contribution from diversification decreases and active bets of target funds may be cancelled out. Furthermore, when appropriate tools to hedge or reduce risks are unavailable for the respective sectors, investments in derivatives on a more general universe or index may become necessary. Both problems make an appropriate method for estimating sector FoF risk exposure to the general markets necessary. We provide a solution for sector portfolios that is especially comforting when being applied to small datasets. Our parsimonious approach of using only short time spans for estimating broad market dependence of the sector portfolio is particularly interesting for practical applications, as it is in line with requirements in the industry where very recent and frequently updated risk measures are used and demanded for by regulators.

Keywords: copula, asymmetric t copula, stable distributions, risk, funds of funds, tail events, tail dependence, sector, hedging.
JEL Classification: G11.

Introduction

If there were still doubts concerning the dependence of sectors in broad market downturns, the recent crisis following the sub-prime meltdown and the so-called credit crunch have erased those in an impressive manner. While sectors or industries may be largely affected by the fundamentals and structures in their very own part of the global economy or sub-sectors of markets, disruptions and downturns in the general financial markets affect them, too. For this reason, it is crucial for managers of sector funds or sector fund of funds (FoF) to take into account the dependence structure of their underlying industry portfolio on broad market movements. The impact of economic and political changes that affect all markets and sub-sectors impose a certain minimum of similarity in the behavior of stock markets in different aggregation levels (say from the very specialized sub-part of an industry up to the MSCI World). These minimum similarities are pronounced when financial market effects lead to broad market movements that show up in all industries and sectors, for example through flow-effects, market sentiment, de-leveraging and flights to substitute asset classes. While these effects are not new in nature, appropriate approaches to deal with them are still scarce in nature, and often include strong assumptions or non-flexible concepts.

As the degree to which a sector portfolio is affected by market movements is a problem of measuring the interdependence between financial variables, it is a part of research that has undergone tremendous developments in recent decades, from correlation or covariance-based methods to the use of more sophisticated multivariate distribution functions and copulas. We combine an asymmetric *t* copula and stable marginals to measure the dependence of a sector FoF on the broad stock market, thereby modelling the univariate randomness of the variables adequately as well. As information on investment or market risks must be updated in high frequencies and on a regular basis, we show how the modelling of the sector exposure to broad market risk can be done with a very parsimonious approach that reduces the dimensionality of the problem at hand, thereby using all relevant information available. A slim approach that is applicable even in the presence of few data is of special interest nowadays with the industry being highly dynamic and financial assets being generated very quickly.

The estimation procedure has one crucial benefit in practical applications, as it may be used on both sides of a FoF, meaning that FoF managers may use the approach to model their own broad market dependence structure on the one hand, and investors in a specific sector FoF may use the approach to model their investment risks with respect to the index which they are willing to diversify away from.

Employing a copula approach with an asymmetric *t* copula as chosen form for the dependence modelling and stable distributions for the marginal distributions of the variables respectively, we generate simulations for the market index as well as for the synthetic FoFs of the sector under consideration. Both the dependence structure and the univariate randomness appear to be modelled very well with our approach, showing the need to apply the right sophisticated concepts for modelling financial assets prone to tail events, and even more important, tail dependence. From the time-varying, rolling window estimations we can see that increases in broad market tail risk lead to increases in sector portfolio tail risk, but not vice versa, indicating a good and unbiased representation of the dependence structure as well as the simulation of the realizations for each period under consideration.

The fact that simulations are generated using the combination of methods at hand is especially comfortable when it comes to the calculation of measures that demand a lot of observations and do not possess closed-form solutions. In addition, the fact that the asymmetric approach allows for differing tail-dependencies on the up-side and the down-side suits the analysis for a FoF very well, as the dependence may be skewed due to industry-specific characteristics as well as by fund characteristics. Furthermore, changes in those characteristics are well tracked by the approach because estimations are done using very recent data and therefore short memory.

Knowing the broad market exposure is especially important for managers or shareholders of sector FoFs in industries for which derivatives are either not available or scarce, as in these cases it is especially difficult to reduce risk and market exposures. Unfortunately, for some industries, hedging considerations therefore simply fail due to the lack of hedging products. Employing an approach to measure the joint risks with the general stock market for which myriads of derivatives are available may enable sector-exposed portfolios to be isolated from the broad market movements or at least dampen the effects of extreme events.

Our parsimonious approach for measuring (inter)dependence between financial markets and assets where the data input must be very up to date or where only a short history of data is available is not limited to FoFs of course. However, we consider it especially appealing for the FoF class for the following reasons. While many funds are allowed to invest in derivatives to hedge their risks, they often abstain from doing so. Reasons for doing so include the lack of adequate tools (if the fund is sector focused for example, as discussed above), the costs of hedging may be too high or the use of derivatives is regarded as being too exotic a tool in classical asset management. However, if the risks are not hedged on the fund level, but merely dampened by holding cash positions during downturns (thereby forfeiting partial exposures that would be beneficial and incurring a considerable inertia into the fund), the FoFs may fail to get the benefit of diversification and risk reduction by spreading their allocation over the target funds. This is a special problem for FoFs, because with an increasing number of target funds, the marginal contribution from diversification is decreasing and characteristics may cancel each other out. With reliable measurement of the risks and exposures of the FoF and the market, this problem of practical portfolio management may be easily overcome and therefore the approach presented in this paper should be used in practical applications not only for risk measurement but for risk management and hedging on the FoF level as well.

The organization of this paper is as follows. In the next section we review the methods used, namely the skewed *t* copula, stable distributions, and risk measures. In Section 2, we discuss the approach of the study and the data. The empirical results are presented in Section 3, showing the application of our framework to synthetic technology sector FoFs, and their dependence on the broad market represented by the S&P 500. The last section concludes the paper.

1. Skewed *t* copulas and stable Paretian distributions

In this section, we explain the method that we propose to model sector FoF dependence on broad market movements, as well as the type of distribution that we employ to model the univariate randomness of the single variables.

To model the dependence structure between the FoF and the index, we use a copula function. Copulas have found increasing attention first in academic research on financial markets and have made their way to Wall Street and many other parts of finance in the following. While the use of copulas brings a substantial improvement to the toolboxes that are available for financial and economic research, the methods have been discussed in heated debates in the financial industry as well[1]. We take the view that it is merely the application of the right concept for a problem at hand and the difficulty of choosing the right form of the copula that is decisive on the way a copula model suits the needs of the researcher or practitioner (see Rachev et al., 2009). Thus, the use of copulas is advantageous to all currently existing methods for measuring dependence if the right concept is applied.

Generally, the concept of copulas enables one to separate the univariate randomness of any variable from the multivariate dependencies by means of factorization. A copula represents the true interdependence structure between variables while the marginal distribution is informative on the univariate randomness of these variables. Therefore, a standardized measure of the purely joint features of a multivariate distribution is generated by using copulas. We briefly discuss the copula definitions

[1] See Whitehouse (2005) and Salmon (2009) for example.

below[1]. The cumulative distribution function of a one-dimensional random variable is called the *grade of a random variable* (uniformly distributed between 0 and 1), and the copula is the distribution of these grades, such that an n-Copula $C:[0,1]^n \rightarrow [0,1]$ is an n dimensional distribution function with univariate marginal distributions $U(0,1)$.

$$C_p(u_1,...,u_n) = P(U_1 \leq u_1,...,U_n \leq u_n) \quad (1)$$

$$H(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n)), \quad (2)$$

where H is an n-dimensional distribution function with marginal distributions F_i.

We will focus on the most common and influential types of copulas and will compare them with each other in Section 3 where we present our empirical results. Archimedean (for example Clayton, Frank or Gumbel) copulas are calculated over a closed-form solution (being very hard to derive for multivariate applications beyond two dimensions however) and do not need to be represented by an application of well-known families of multivariate distributions using the theorem of Sklar (1959 and 1973). In contrast, elliptical (for example, Gaussian or Student *t*) copulas can be derived via simulating these multivariate distributions taking advantage of their simple stochastic representations. In the recent past, the focus in both academia and practice turned to the elliptical class of copula forms. However, a caveat of general elliptical copulas is that the upper and lower tail dependence, being informative on joint extreme realizations, is identical, due to the radial symmetric shape of the elliptical copulas. In addition, a Gaussian copula has no tail dependence at all (see Bradley and Taqqu, 2003), and this is the main argument against its use in financial market applications from our point of view.

That the Gaussian copula is inappropriate for most financial applications due to the aforementioned inability of measuring tail dependence is especially interesting in light of the ongoing debate surrounding copula functions in financial markets and especially during the current credit crisis (see Rachev et al., 2009). The fact that the Gaussian copula has no tail dependence at all is stemming from the fact that a multivariate Gaussian distribution is the n-dimensional version of a Gaussian distribution, which assigns too low probabilities to extreme outcomes. While the use of Gaussian distributions in financial market applications is widely accepted as being flawed due to the fact that this distribution type attributes too low probabilities to extreme observations, the multivariate version still is frequently used in copula applications.

The *t* copula, or Student copula, does not share the shortcoming of the normal copula concerning the tail dependence and enables the modelling of joint extreme market outcomes. However, the radial symmetric shape of the t copula still leaves a concern regarding the use for financial market data, as the upper and lower tail dependence is identical. Thus, the probabilities of joint tail events on the downside are equally distributed as the ones on the upside. In reality, this may pose problems when modelling markets or assets for which this assumption may not hold.

Improving the features of copula models is the use of asymmetric *t* copulas, which in contrast to the general elliptical copula forms discussed above allow for differing tail dependencies as well. Especially in our application of a sector FoF and the broad market, this feature is highly desirable as the dependence of the FoF may be different when considering upside and downside events. The multivariate t distribution that is used takes the following form:

$$X := \mu + \gamma W + Z\sqrt{W} \quad (3)$$

with $W \in IG\left(\frac{\nu}{2},\frac{\nu}{2}\right)$ and $Z \in N(0,\Sigma)$, the latter being independent of the former. The parameter vector $\gamma = (\gamma_1,.....,\gamma_n)$ defines the skewness of each variable n, while $\mu = (\mu_1,.....,\mu_n)$ is the vector of location parameters in the same dimension. Denoting the distribution as $X \in t_n(\nu,\mu,\Sigma,\gamma)$, ν is the degrees of freedom that define the inverse gamma distribution sub-part with $\frac{\nu}{2}$ and Σ is the covariance matrix of the zero-mean normal distributed sub-part. Using Sklar's theorem, the skewed Student's t copula is defined as the copula of the multivariate distribution of X. Therefore, the copula function is obtained as:

$$C(u_1,....,u_n) = F_X\left(F_1^{-1}(u_1),....,F_n^{-1}(u_n)\right) \quad (4)$$

with F_X being the multivariate distribution function of X and $F_k^{-1}(u_k)$ being the inverse of the cumulative density function of all marginals k (for k ranging 1 to n). Therefore, the above notation is defining the copula for all dimensions using the multivariate skewed t distribution. As this approach is fully general to the type of marginals

[1] See Embrechts et al. (2003), Cherubini et al. (2004), Meucci (2006) and Nelsen (2006) for thorough discussions of copulas and their applications in finance.

used, the randomness of each univariate entry to the multivariate distribution can be modelled by the choice of distribution type.

Using the asymmetric *t* copula, we generate a large number of copula scenarios, thereby taking into account the dependence between the assets. These copula scenarios are then used to generate univariate scenarios for each variable, thereby making use of the inverse of the cumulative distribution function of the marginal distribution used for the univariate modelling.

The marginal distribution for the univariate randomness of each asset is modelled using the stable Paretian distribution type, in the following simply called stable distribution. Basically, the stable distributions generalize the normal distribution. While the normal or Gaussian distribution is determined by the two parameters, location and dispersion, i.e mean and standard deviation, the stable distributions are defined through four parameters.

First, the characteristic exponent ($0<\alpha<2$), called the index of stability or stable index, determines the weight of the distribution's tails. For lower values of α, the shape of the distribution is more peaked at the location parameter and exhibits fatter tails, parameter value 2 corresponds to the normal distribution. Second, the parameter β, which is bounded between -1 (skewed to the left) and +1 (skewed to the right) determines the distribution's skewness and is informative on whether the occurrence of returns is more probable for negative or positive realizations. Third, the parameter σ is scaling the distribution. Fourth, as for any other type of commonly used distributions, the location parameter is responsible for the shift of the distribution's peak to the left ($\mu<0$) or to the right ($\mu>0$).

The fact that stable distributions are described by four parameters and may take a large variety of shapes is an advantage over other distribution types, with the fact that asymmetric probability distributions and heavy tails are featured being very favorable. Especially when being compared to the normal distribution function, the stable models show up as being more in line with real market observations, as the probabilities of occurrence of extreme observations far away from the mean of a variable are heavily underestimated by the normal distribution.

More detailed discussions and overviews on the properties and applications of stable distributions in finance are provided in Mittnik and Rachev (1993), Samorodnitsky and Taqqu (1994), Rachev and Han (2000), Rachev and Mittnik (2000) and in Ortobelli et al. (2002 and 2003), while the stable property's importance for financial data has been initially discussed by Mandelbrot (1963).

2. Asymmetric copula, heavy-tailed marginals and tail risk valuation; example with FoF's data

As the properties of both the interdependence and the univariate randomness are changing over time and therefore should be estimated on a regular basis, we use a short time span for the estimations in this study. Thus, the data set is chosen to reflect the very recent realizations of the variables under consideration, mirroring the need of up to date estimations that are crucial in financial market applications for which often only a limited data span is available. Using a window of 100 trading days that is rolled through the whole data sample is beneficial, on the one hand, as the estimations are always very focused on recent realizations but is resulting in a small sample for each estimation, on the other hand.

This classical trade off is losing its severity in our approach, as we reduce the dimension of the problem to a bivariate one. We use all available funds at each time point to build a synthetic equal weighted FoF. If a fund dies, the allocation share of it is evenly distributed among all surviving funds in the next period and vice versa. We have therefore a time-series of an artificial FoF to estimate against the market index. In practice, FoF managers may of course use their own actual and current portfolio weights for the 100 day backward time-series generation. Investors to FoFs may use the actual time series of the FoF – thereby keeping in mind that it is an approximation due to allocation changes within the time period – or may go on with the equal weighted approach as some FoFs may be approximated as equal-weight schemes of their fund universe.

With the bivariate approach, we obtain the dependence structure by fitting the asymmetric *t* copula to the two return series in any window, and then generate simulations of the FoF and the index using the stable distribution for the univariate randomness. One benefit of the bivariate approach is that we do not need to estimate a large number of parameters, a pre-requisite for a dynamic approach with only limited data input, as we have here with only 100 trading days. An estimation of the parameters for each fund in the respective time period would make the analysis far more complex and would demand more data and/or calculation steps. The return series entered the estimation process unfiltered, that is, no time-varying effects, volatility clustering or similar features have been modelled upfront as the aim is to show directly the dependence structure of the variables. The framework may as well be combined and used on pre-filtered data, for example on the innovations of a multivariate AR(I)MA/GARCH model between the FoF and the Index or on results of time-series analysis with a decaying time influence, but the input data set needs to be larger then[1].

Our choice for the size of simulations was 1,000 simulations for each variable. This keeps the computational burden on a practical level that allows for daily application of the approach. In addition, for appropriate backtesting of the model over a considerable history the size of the simulations should be kept in a sensible range. Therefore, we are generating a 1,000 by 2 matrix of simulations for each estimation window, with the simulations, on the one hand, being based on the true dependence between the FoF and the broad index as being estimated by the copula, and on the other hand, mirroring the single return distributions adequately.

The resulting simulations may be used in a large variety of ways, for example for portfolio optimization or the calculation of risk measures. Moreover, the obtained results may be used by sector FoF managers or investors of sector FoFs to hedge their broad market exposure incurred by the sector investment when no industry-specific tools may be available. We track whether the model did adequately capture both the dependence structure and the structure of the single variables by comparing the simulations' properties with the actual properties of the FoF and the index. In addition, we compare the results obtained with other methods that were commonly used in financial markets and that were discussed above.

We have chosen the technology (tech) sector as an example in this study. The tech sector has undergone tremendous up-and-down phases in the late 1990s and the beginning of the new century, and the returns of tech stocks show high concentration in the tails that makes the need for application of sophisticated methods obvious. As a FoF analysis was done for measuring the dependence on broad market movements, the approach is interesting in light of diversification arguments too, as the benefit of diversification is an oft-heard argument by FoF proponents. In addition, the approach is straightforward, as an estimation of the dependence of each single fund on the index is not needed when considering a FoF that one is managing, neither is it possible to do so when one is invested in the FoF and is seeking to estimate the dependence of it on the market.

Selection of the funds and streaming of the total return series was done using Bloomberg based[2] on the following criteria. All funds included are mutual funds that (1) are listed in the Unites States, (2) have their investment focus on tech stocks of the domestic market, (3) are denominated in U.S. dollar, and (4) report daily net asset values. Fortunately, the resulting fund spectrum includes both dead and alive funds such that even the last return of any fund before going out of business enters the analysis. Daily data were used for the 10 years ending April 2009. The resulting return matrix consists of 2,527 daily returns for each of the 255 funds included. Measuring the broad stock market was done using the S&P 500 for the respective time-period. The S&P 500 was selected because it is the index that is typically used for benchmarking by institutional investors and an indicative check of FoFs that satisfy our selection criteria strengthened this notion. Because we use an equal weighted FoF construction, we have a 2,526 by 2 matrix of returns as our sample for the whole period, and 2,426 matrices of size 100 by 2 for the dynamic intertemporal estimations.

Concerning the measurement of risk for the index and the synthetic FoFs, we use the expected tail loss (ETL) which is the conditional value at risk (CVaR) for continuous distributions[3],

$$ETL_{1-\alpha}(r_a) = E\left(\max\left(-r_a, 0\right) \middle| -r_a > VaR_{1-\alpha}\left(r_a\right)\right)$$

with $ETL_{1-\alpha}(r_p)$ being the expected tail loss with tail probability α for asset returns r_a and *VaR* denoting the value at risk. In accordance with common confidence levels for other risk measures such as VaR are 1% or 5% for α, corresponding to confidence levels of 99% and 95%, respectively. For any confidence level, ETL is higher than VaR as it measures the expected losses in the case of a tail event rather than measuring the loss not to be exceeded with the respective confidence[4]. Concerning the measurement of risk the choice of an appropriate measure is another way to omit erroneous estimations, as, for example, the VaR at 95% confidence of a normal distribution may be the same as the corresponding measure for a stable distribution or a *t* distribution, but the ETLs or CVaRs (AVaRs) at 95% may be largely differing. For the sake of comparability, we report the classical measure as well.

[1] See Sun et al. (2009) for a multivariate approach to estimating tail risks using the ARMA-GARCH methodology and the Student's *t* copula.

[2] Datasource: Bloomberg Finance L.P.

[3] See Rachev et al. (2007) for discussions on risk, uncertainty and performance measures. The conditional value at risk (CVaR) corresponds to the average value at risk (AVaR), see Pflug and Romisch (2007) for example.

[4] See Sortino and Sachell (2001) and Rockafellar (2002) among others concerning VaR and CVaR / ETL.

3. Empirical results

Prior to appling the rolling window approach for successive 100 trading day periods, we check the data's full sample characteristics. Looking at the return scatter plot of the index and the synthetic FoF as shown in Figure 1, the elliptical shape indicates significant dependence, showing the immediate need for detailed modeling of the dependence structure of the two series.

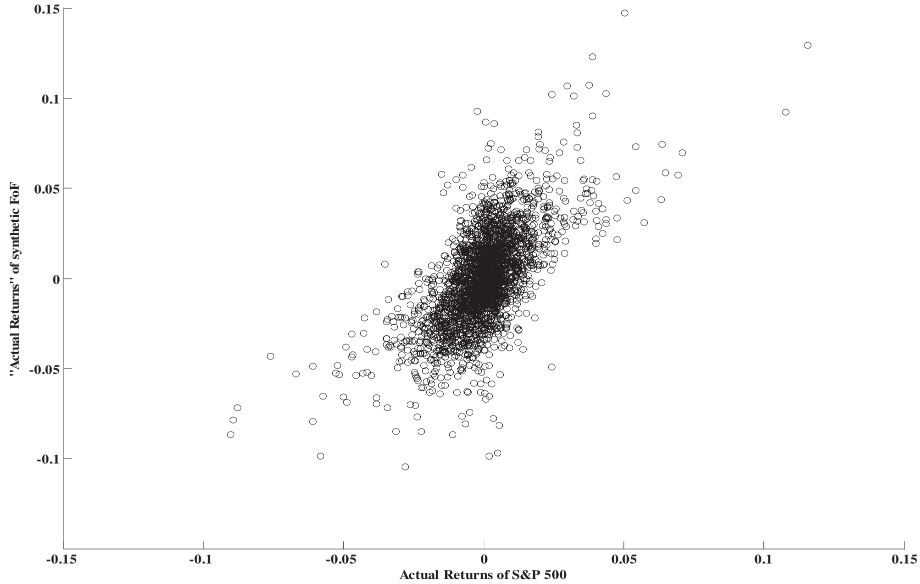


Fig. 1. The return scatter plot of the synthetic tech FoF and the index for the whole sample period

In general, to check whether the pair of tools we favor adequately models both the dependence structure and the univariate randomness, we estimate the asymmetric *t* copula and generate simulations using the stable distributions from the entire sample of observations. The result is a 2500 by 2 matrix with simulations for the FoF and the index. For comparison purposes, we used a number of simulations being approximately equal to the actual observation series. For comparability to other commonly used approaches, we have included the results of simulations using a normal copula and normal marginal distributions approach as well as the results of a directly applied multivariate *t* distribution (the distribution being applied on the returns rather than on the cumulative density function of the variables).

From Figure 2 it can be seen that the normal approach suffers from the fact that the normal copula cannot capture tail dependence and the marginal distribution does not account for univariate tail risks. The multivariate *t* distribution approach suffers from the fact that the dependence structure and the marginal distributions are not modeled separately, leading to a loss of information and a less detailed modeling. Therefore, a too radial and poor fitting shape is obtained. Increasing the number of simulations made this problem even more obvious when checking the approaches' behavior. In contrast, the simulations obtained from our approach with the asymmetric *t* copula and stable marginals appear to be a good tracking of the dependence structure of the FoF and the index.

Using the approach with rolling 100-day periods, we continued by modelling the bivariate set over time. When it comes to modelling the dependence structure over time, we need to check the ability of the approach to fit the data well even in the presence of a heavily reduced data set because only 100 days were selected as the time window in the example. Since we originally had 2,526 return observations, we have 2,426 windows for which we generated the simulations, Figure 3 shows the last period as an example. We checked the short sample properties of the other methods as well, and the deviations from the true data sets are even more severe than in the whole data sample, again strengthening the notion that the appropriate tools were chosen for the analysis.

As the simulations are of size 1000 and the returns were 100 each, the scatter diagram of the simulations is of course more crowded than the one of the observations. In addition, the realizations on the tail sides seem to be more pronounced in the simulations. To see whether this is due to overestimation of the tails or to a small sample bias, the quantile-quantile (q-q) plots were checked for both the index and the synthetic FoF. From the q-q plots we can see that the simulations fit the data very well and that for both variables only two simulated realizations are somewhat deviating. Indicative checks of other periods did not give rise to doubts concerning the estimation and fitting performance for the problem at hand.

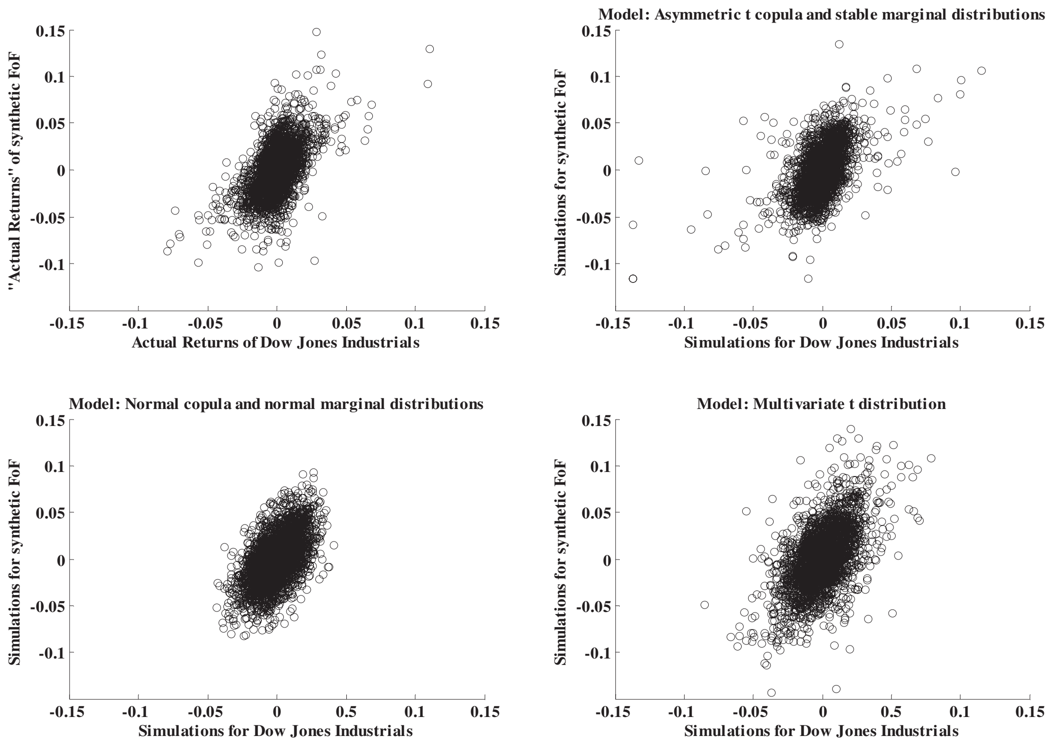


Fig. 2. The simulations of the synthetic tech FoF and the index for several approaches for the whole sample period

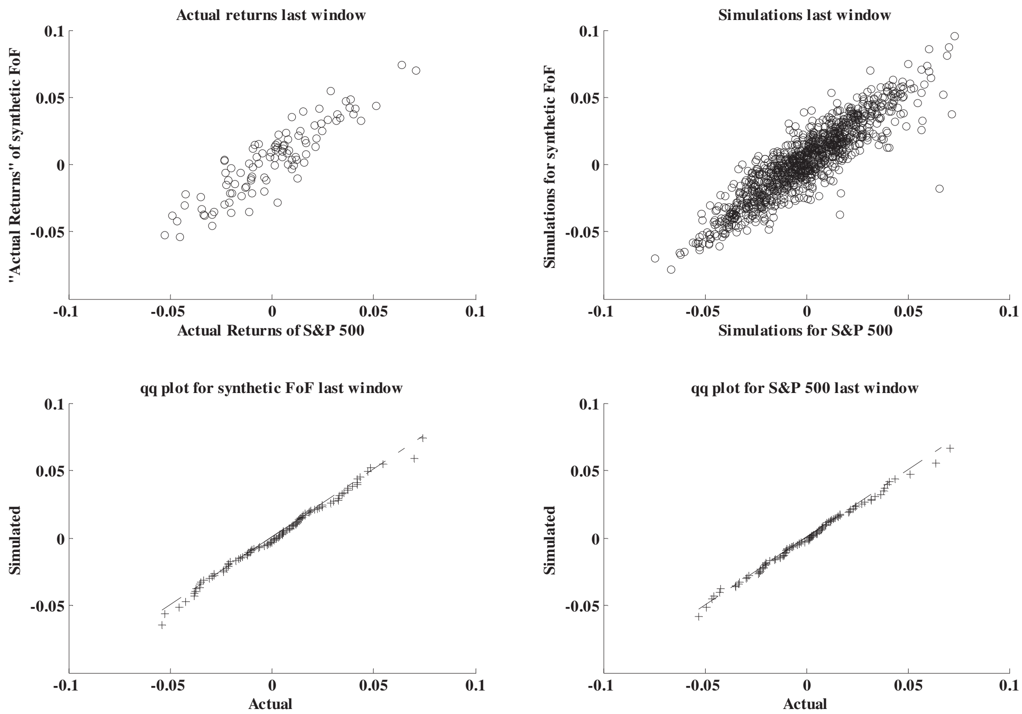
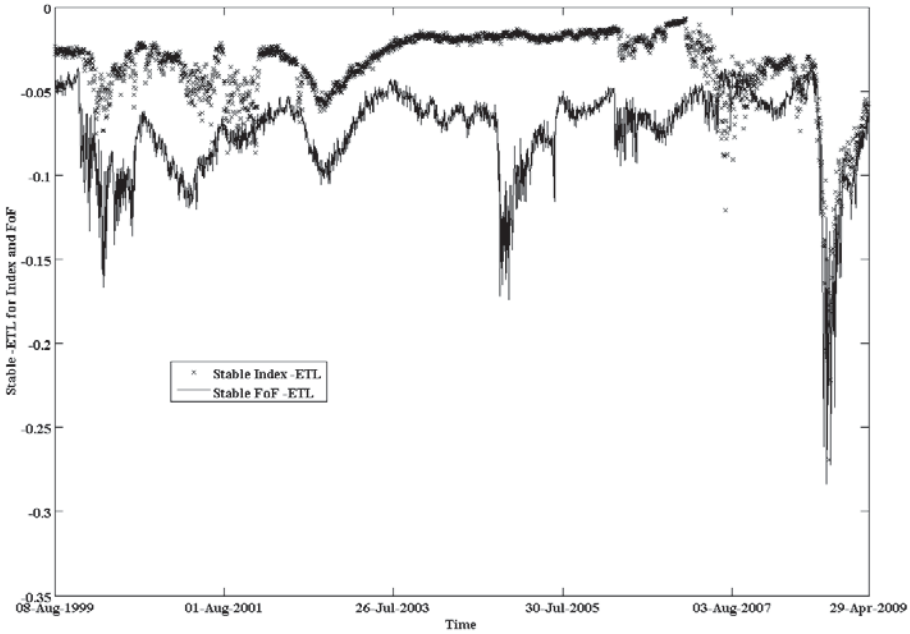


Fig. 3. Example of last period estimations

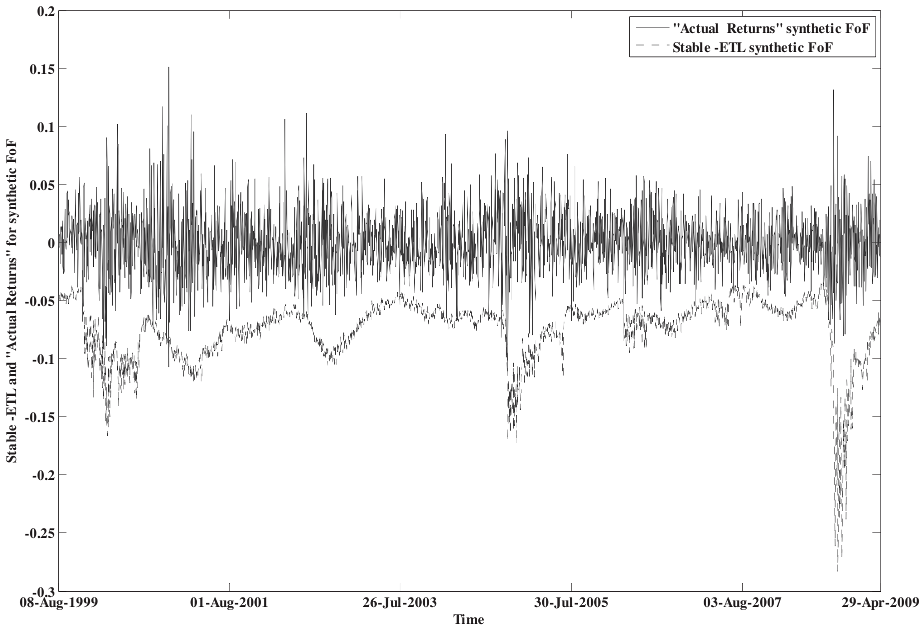
We can see from the calculations of the expected tail loss that it is good practice to model the broad market risk for the FoF in dynamic nature, as both the magnitude of the risk measure as well as the joint changes therein are heavily time-dependent, as can be seen in Figure 4.

With respect to the expected tail losses of the two variables, the fact that a large increase in the magnitude of this risk measure for the index leads to an increase of it for the FoF too, shows that the influence of the broad market risk on the FoF is substantial and modelled adequately. In addition, the tech sector had its own characteristic increases in the tail risk during drawdowns (besides more severe tail events throughout the sample) which did not appear in the broad market and did not affect the estimation results of the index expected tail loss. The latter fact is very favorable concerning the judgment of the measurement of dependence, showing that with the asymmetric *t* copula, increases in broad market risk lead to increases in sector FoF risk, but not the other way round and therefore no spurious causality seems to be generated during the asymmetric *t* copula fitting and simulation generating using the stable distributions.



Note: The negative of the ETL (99% level) is plotted, according to industry usage of the negative loss as risk measure.

Fig. 4. Stable negative expected tail losses over time



Note: The negative of the ETL (99% level) is plotted, according to industry usage of the negative loss as risk measure.

Fig. 5. Actual returns of synthetic FoF and stable negative expected tail losses over time

To check whether the methods used were good at estimating the expected tail losses for each time span, we tested how many times the expected tail loss on the 99% level was exceeded in the following trading day. With 10 exceedances (0.41% of observations) for the index and 9 exceedances (0.37% of observations) for the synthetic FoF, a considerable small number is obtained, see Figure 5 for a graphical representation of the FoF returns and estimated tail risks. Naturally, the number of exceedances was higher for the corresponding 99% VaR, but with 34 (1.40%) and 32 (1.32%) for the index and FoF respectively, the number is still very small, strengthening our notion of a sensible approach.

Conclusions

The asymmetric *t* copula approach for the estimation of the dependence of a sector FoF on broad market risk captured the independence structure very well. Combined with the stable distribution we obtained well-fitting simulations for the synthetic FoF and the index for each estimation window. Being applied to a very short window of data of 100 trading days, the approach suits estimation needs concerning short-term tracking of risks and risk dependencies and may be applied to problems with limited and small data sets in general. This is because the problem of measuring the interdependence is of the bivariate type, the estimation efficiency using the asymmetric *t* copula and the subsequent generation of simulations using the numeric solutions to the previous fitting.

As the procedure appears to generate well-fitting simulations, these may serve as input to a large variety of applications, from risk management and measurement, portfolio optimizations and scenario analyses to investment selection and hedging purposes as examples. It is critical to have an approach that identifies the joint risks of a sector FoF and the broad markets because for many industries or (sub) sectors no viable derivative market exists. The results obtained by using our approach may serve both FoF investors and FoF managers when it comes to not only measuring risks, but also isolating the sector portfolios from general market movements. Possible extensions or adjustments would be to take into account time-series effects such as volatility clustering and to combine the procedure with those, although this would demand more data points for each estimation, reducing the great benefit of a parsimonious approach as proposed in this paper.

References

1. Bradley, B.O., M.S. Taqqu. Financial Risk and Heavy Tails Management – Handbook of Heavy-Tailed Distributions in Finance, ed. Rachev, S.T. – Amsterdam: North Holland Handbooks of Finance, 2003 – pp. 95-103.
2. Cherubini, U., E. Luciano, W. Vecchiato. Copula Methods in Finance. – Chichester: Wiley, 2004.
3. Embrechts, P., F. Lindskog, A. McNeil. Modelling Dependence with Copulas and Applications to Risk Management – Handbook of Heavy-Tailed Distributions in Finance, ed. Rachev, S.T. – Amsterdam: North Holland Handbooks of Finance, 2003 – pp. 329-384.
4. Mandelbrot, B. The Variation of Certain Speculative Prices // *Journal of Business*, 1963. – № 36. – pp. 394-419.
5. Meucci, A. Risk and Asset Allocation. – New York: Springer, 2006.
6. Mittnik S., S.T. Rachev. Modelling Asset Returns with Alternative Stable Distribution // *Economic Review*, 1993. – №12. – pp. 261-330.
7. Nelsen, R. An Introduction to Copulas. – New_York: Springer, 2006.
8. Ortobelli, S., I. Huber, E. Schwartz. Portfolio Selection with Stable Distributed Returns // *Mathematical Methods of Operations Research*, 2002. – №55 – pp. 265-300.
9. Ortobelli, S. I. Huber, S.T. Rachev, E. Schwartz. Portfolio Choice Theory with Non-Gaussian Distributed Returns – Handbook of Heavy-Tailed Distributions in Finance, ed. Rachev, S.T. – Amsterdam: North Holland Handbooks of Finance, 2003.
10. Pflug, G.C., W. Römisch. Modeling, Measuring and Managing Risk. – Singapore: World Scientific Publishing, 2007.
11. Rachev, S.T., S. Han. Portfolio Management with Stable Distributions // *Mathematical Methods of Operations Research*, 2000. – №51. – pp. 341-353.
12. Rachev, S.T., S. Mittnik. Stable Paretian Models in Finance. – New York: Wiley, 2000.
13. Rachev, S.T., S.V. Stoyanov, F.J. Fabozzi. Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization: The Ideal Risk, Uncertainty, and Performance Measures. – New York: Wiley, 2007.
14. Rachev, S.T., W. Sun, M. Stein. Copula Concepts in Financial Markets. Technical Report, University of Karlsruhe. http://www.statistik.uni-karlsruhe.de/download/Copula_Concepts_in_Financial_Markets.pdf, 2009.
15. Rockafellar, R. T., S. Uryasev. Conditional Value-at-Risk for General Loss Distributions // *Journal of Banking and Finance*, 2002. – №26. – pp. 1443-1471.
16. Salmon, F. Recipe for Disaster: The Formula That Killed Wall Street. Wired, 23 February 2009.
17. Samorodnitsky G., M.S. Taqqu. Stable Non-Gaussian Random Variables – New York: Chapman and Hall, 1994.
18. Sklar, A. Fonctions de Repartition a N Dimensions et Leurs Marges // Publ. Inst. Statist. Univ. Paris, 1959. – №8. – pp. 229-231.
19. Sklar, A. Random Variables, Joint Distribution Functions and Copulas // *Kybernetika*, 1973. – №9. – pp. 449-460.
20. Sun, W., S.T. Rachev, F.J. Fabozzi, P.S. Kalev. A new approach to modeling co-movement of international equity markets: evidence of unconditional copula-based simulation of tail dependence // *Empirical Economics*, 2009. – №36. – pp. 201-229.
21. Whitehouse, M. How a Formula Ignited Market That Burned Some Big Investors. *Wall Street Journal*, 12 September 2005.
22. Sortino, F.A., S. Satchell, S. Managing Downside Risk in Financial Markets: Theory, Practice and Implementation. Oxford: Butterworth Heinemann, 2001.