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AUTHORS	Chao Chen Yanbo Jin		
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# A Robust Test on the Multifactor Pricing Model

Chao Chen<sup>1</sup>, Yanbo Jin<sup>2\*</sup>

#### **Abstract**

Since Chen, Roll and Ross' (1986) pioneering work on prespecified multifactor pricing model, the issue of inexact model specification as suggested by Shanken and Smith (1996) deserves further research. This study applies conditional mean encompassing test for model specification, which begins with the largest model with respect to our information set and performs the specification tests for restrictions on the explanatory variables consistent with financial theory to see if the smaller models encompass the larger models. This technique is parsimonious and robust against heteroscedasticity. To consider possible seasonality, we also include a seasonal dummy of the January effect for the specification tests. The empirical results demonstrate that the conditional excess rates of return are explained by four macroeconomic variables and the January effect.

**Key words**: Multifactor pricing model, model specification, encompassing test, January effect.

### 1. Introduction

Since the pioneering work on the Capital Asset-Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965a, 1965b), and the Arbitrage Pricing Theory (APT) of Ross (1976), asset pricing models have been an active point at issue in financial economics. Shanken and Smith (1996) point out that one of the unsolved problems of empirical studies in asset pricing models is inexact model specification<sup>3</sup>. White (1990) also argues that model selection based on the specification tests is crucial<sup>4</sup>. This study applies a different and interesting model selection technique -- the parsimonious encompassing test to provide further insight into Chen, Roll and Ross' (1986) prespecified multifactor pricing model.

Chen, Roll and Ross (1986) represent one approach in testing APT. Specifically, using simple and intuitive financial theory, they identify several macroeconomic variables and test whether these variables can systematically explain stock market returns. The potential risk factors include the growth rate of industrial production, expected inflation, unexpected inflation, default risk premium and term structure spread. They find that the default risk premium, term structure spread and industrial production are priced risk factors.

However, in their model, factor premiums are assumed to be constant. Mei (1993) uses a semi-autoregressive (SAR) approach to estimate factors of APT, and confirms that these macroeconomic factors are indeed priced by the market. In addition, he finds that the factor premiums move over time.

Along the same line of research, but following a different approach, many studies test APT by examining the relationship between overall economic conditions and time-varying risk premiums for a small group of portfolios. These include Campbell (1987), Chen (1991), Fama and French (1988, 1989), Ferson and Harvey (1991), among others.

<sup>&</sup>lt;sup>1</sup> Ph.D. Finance, Department of Finance, Real Estate and Insurance, College of Business and Economics, California State University, Northridge, USA.

<sup>&</sup>lt;sup>2</sup> Ph.D. Finance, Assistant Professor, Department of Finance, Real Estate and Insurance, College of Business and Economics, California State University, Northridge, USA.

<sup>&</sup>lt;sup>3</sup> Gibbons and Ferson (1985) also implicitly emphasize the importance of model specification. They argue that if a researcher overlooks some relevant variables, the cross-equation restrictions still hold, but the test will be less powerful.

<sup>&</sup>lt;sup>4</sup> White (1990) states that "In much of the empirical economics research the goal is to test hypotheses about parameters to which one wishes to attribute economic meaning. It is our view that this is inappropriate and unjustified without first establishing that the model within which the hypotheses are being tested is congruent with the data to at least some extent. Otherwise, one may only have confidence that one is testing hypotheses about parameters with an information theoretic interpretation; the economic interpretation desired is untenable."

In the last decade, researchers started to test APT using a device called "factor mimicking portfolios". This methodology was made popular by the famous study of Fama and French (1993). In their paper, two "mimicking" portfolios were constructed for firm size and book-to-market ratio besides the market portfolio to test a three-factor model. The benefit of this approach is that it allows for direct test of the multifactor model using time series regressions where both dependent and independent variables are portfolio returns. This methodology is also used in Daniel and Titman (1997), Ferson and Harvey (1991), among others. In addition, several studies also construct mimicking portfolio from pre-specified macroeconomic variables (Chan, Karceski and Lakonishok (1998 and 1999)). However, the "factor mimicking portfolio" methodology is being questioned by Asgharian and Hansson (2000).

More recently, researchers begin to model the stock market returns jointly with its conditional volatility, acknowledging the fact that the impact of macroeconomic variables on equity returns are neither time-invariant nor linear. For example, Flannery and Protopapadakis (2002) found that inflation affects the level of equity returns while Balance of Trade, Employment and Housing Starts affects the conditional volatility. In addition, they found that money supply affects both the return and the volatility.

Researchers have also started to link the relation between macroeconomic factors and stock and bond return correlations. For example, Li (2002) shows that correlation of stock and bond return can be explained by its common exposure to macroeconomic factors. In particular, he found that the major trend in stock-bond correlation is mainly determined by uncertainty about expected inflation.

This paper extends on the line of research by Chen, Roll and Ross (1986) and later by Mei (1993). We start with the macroeconomic factors identified in their study and implement the encompassing test proposed by Wooldridge (1990b).

The parsimonious encompassing test begins with the largest model with respect to our information set and performs tests for restrictions on some of the explanatory variables to see if the smaller models encompass the large models. The technique stops at the point where further reduction or simplification is no longer valid. In this paper, Wooldridge's (1990b) conditional mean encompassing test is applied to perform robust model selection on the pre-specified state variables of Chen, Roll and Ross (1986) and Mei (1993). Furthermore, this paper follows the approach of Ferson (1990) by assuming linearity for the risk premium. To consider possible seasonality, we also include a seasonal dummy of the January effect for the specification tests. In addition, the technique is robust against heteroscedasticity<sup>1</sup>.

The rest of this paper is organized as follows. In Section II, we discuss multifactor pricing models with an incomplete information set. A new methodology – Wooldridge's robust conditional mean encompassing test for model selection – is applied to test the multifactor pricing model. The details of the methodology are discussed in Section III. Section IV describes the data set. The empirical evidence is presented in Section V, followed by concluding remarks.

# 2. The Model

The statistical framework for testing multifactor pricing models can be shown as follows:

$$x_{t} = E[x_{t}|I_{t-1}] + f_{t}, (1)$$

$$r_{t} = E[r_{t}|I_{t-1}] + Bf_{t} + \varepsilon_{t}, \tag{2}$$

where  $r_t$  is an n-by-1 vector of excess rates of return,  $I_{t-1}$  is the information set up to time t-1,  $x_t$  is a k-by-1 vector of pre-specified variables,  $f_t$  stands for the innovations of  $x_t$ , B is an n-by-

<sup>&</sup>lt;sup>1</sup> Econometrically, the specification tests based on multifactor pricing models need to satisfy the following prerequisites: 1) the test should be robust against heteroscedasticity (of unknown form) even though the conditional mean is believed to be correctly specified; 2) it should also consider alternative models to search for the best specifications with pre-specified proxies since any set of reference variables can be applied; and 3) if the number of factors is finite, the model should be parsimonious. This study shows that the approximation of conditional excess rates of return and factors with prespecified variables and proxies for factors can be tested by a robust encompassing test which satisfies the above prerequisites.

k matrix of factor loadings with column rank k, and  $\varepsilon_t$  is the error term for the regression of  $r_t$  on the pre-specified variables. Also let  $E[x_t | I_{t-1}] = \text{Å } X_{t-1}$ , where Å is k-by-k matrix of parameters,  $\sigma(X_{t-1}) \subseteq I_{t-1}, X_{t-1} = (X'_{t-1}, X'_{t-2}, ..., X'_0)$  is a set of pre-specified variables. If linearity is assumed for the conditional expectation  $E[r_t | I_{t-1}]$ , it can be written as  $E[r_t | I_{t-1}] = vx_{t-1}$ , where v is an *n*-by-*k* matrix of nonstochastic coefficients.

The choice of state variables as appropriate explanatory variables for risk premiums is still an open issue for empirical studies. Conventional tests of pre-specified multifactor pricing models include all the variables of interest in one model and investigate whether their t-statistics (or F-statistics) are significant or not. In this paper, we consider a model selection procedure. More specifically, alternative (nested or non-nested) hypotheses for the multifactor pricing model are assessed in the specification test. Furthermore, since the numbers of factors are unknown a priori, a parsimonious approach is suggested to avoid overfitted models. In the following section, we introduce the encompassing principle of Mizon and Richard (1986), Mizon (1984), Hendry and Richard (1982, 1983), and Hendry (1989, 1995) to perform parsimonious model selection on the possibly incomplete information sets in specifying the excess rates of return. In particular, the conditional mean encompassing test of Wooldridge (1990a) is applied to the studies.

# 3. Methodology

Mizon and Richard (1986), and Mizon (1984) propose the encompassing principle for specification tests. Intuitively, the principle states that a correctly specified model (conditional on an information set) should be able to explain or predict the statistics or characteristics of alternative models. These encompassing principles do not require a complete information set of relevant variables to search for the model specification.

Following Hendry (1989, 1995), we can explain the encompassing principle with the following example. Let the model of interest be  $E[y_t|X_t]$ , where  $y_t$  is the dependent variable such as excess rates of return  $r_{it}$ ,  $X_t$  is a set of explanatory variables including forecasting variables, proxies for factors, and the entire past history of  $y_t$ . Suppose there are two competing models such that  $M_1: y_t = E[y_t | X_t] + \varepsilon_{1t} = X_{1t}' \gamma_0 + \varepsilon_{1t}$ ,

$$M_1: y_t = E[y_t | X_t] + \varepsilon_{1t} = X_{1t}' \gamma_0 + \varepsilon_{1t}, \tag{3}$$

$$M_2: y_t = E[y_t|X_t] + \varepsilon_{2t} = X_{2t}'\beta_0 + \varepsilon_{2t}, \tag{4}$$

where  $X_{1t}$  and  $X_{2t}$  may be non-nested,  $X_{1t}$  is a *U*-by-1 vector, and  $X_{2t}$  is a *V*-by-1 vector. Therefore, the encompassing test for model  $M_1$  to encompass  $M_2$  is to perform the specification test for  $\tau = \underline{0}$  on the auxiliary regression model  $M^*$ ,

$$M^*: y_t = X_{1t}' y^* + W_{2t}' \tau + \varepsilon_t^*, \tag{5}$$

where  $W_{2t}$  is a q-by-1 vector which contains the variables included in  $X_{2t}$  while not included in  $X_{1t}$ . Since parsimonious encompassing is transitive. That is, let model  $M_1$  encompass model  $M_2$ ,  $M_1$  is nested in  $M_2$ , where  $M_1 \delta M_2$ . Also let  $M_2$  encompass model  $M_3$ ,  $M_2$  is nested in  $M_3$ , where  $M_2 \delta M_3$ . Then, model  $M_1$  will parsimoniously encompass model  $M_3$ . This also satisfies the theory of reduction where simplification is desired.

The parsimonious encompassing test should stop at the point where further reduction or simplification is no longer valid. Notice that even if  $M_1$  and  $M_2$  are non-nested, there is a minimal nesting model  $M_m$  within which  $M_1$  and  $M_2$  are nested. If  $M_1$  parsimoniously encompasses  $M_m$ , then  $M_1$  also encompasses  $M_2$ .

This methodology provides us with a simple approach for the specification of risk premiums and latent variables. Thus, we may begin with the largest model with respect to our informa-

<sup>&</sup>lt;sup>1</sup> Since any ad hoc a priori information set applied by the modellers may not be complete, such model selection procedures should not require the information set (for each model) to be complete for the specification of risk premium.

tion set and see if smaller models encompass the large model. Since the parsimonious encompassing is transitive, the order of selecting variables in tests is not relevant. In particular, the model selection which started from the most general model yields certain optimal properties with regard to power according to Harvey (1990). Based on this rationale, our hypothesis is:

 $\mathbf{H_0}$ : If we have correctly specified the conditional excess rates of return, then, the innovations generated by the economic variables may be a candidate set for the approximate factors.

In other words, following the approach of Ferson, if  $\Psi^*(f_{j\,t}) = \chi^t W_{t-1}$  where  $\chi^t *0$ , we are interested in whether the  $f_t = (f_{1\,t}, ..., f_{k\,t})^t$  are appropriate for the proxies of factors. More specifically, with the parsimonious encompassing, we begin with the largest model consisting of  $W_{t-1}$  and  $f_t$ , and sequentially reduce the model to see if the smaller models explain the same (statistical) characteristics of the larger models.

In the following, in order to avoid dynamic misspecification, we focus on the information set  $\sigma(W_{t-1})$  that contains the entire past history of  $r_{i,t}$ 's and other economic (forecasting) variables  $Y_t$ 's. We also let  $I_t$  in equation (1) be the information set formed by the lags of economic variables  $Y_t$  plus a constant term. This is to ensure the correct specification for conditional mean  $E[Y_t | I_t]$  in the vector autoregression of  $Y_t$ ?

This study performs the encompassing test based on the extension of Wooldridge (1990b). Wooldridge (1990b) proposes a robust conditional mean encompassing test for (nonnested) nonlinear regression allowing conditional or unconditional heteroscedasticity of unknown form. Although various specification tests such as the likelihood ratio test of Vuong (1989) and Vuong and Wang (1993), the Chi-square test of Andrews (1988a, 1988b), and the generalized method of moments of Smith (1992) are available, they are either non-robust for heteroscedastic observations or complicated in estimating the asymptotic variance-covariance matrix. In particular, if the asymptotic variance-covariance matrix (for parameters) is inconsistently estimated, the limiting distribution is no longer a central Chi-square distribution. Therefore, the tests may not have the correct size<sup>3</sup>. The robust testing procedures developed by Wooldridge (1990b) are described hereon. For simplicity, we apply the above example of models  $M_1$  and  $M_2$  to describe Wooldridge's test. The testing procedures are as follows:

- 1. Obtain the consistent estimates for  $(\gamma_o', \beta_o')'$ . Save the residuals  $e_t$  from the model  $M_1^4$ .
- 2. Run the multivariate regression of  $W_{2t}$  on  $X_{1t}$ . Save the residuals in vector  $\xi t$ .
- 3. Run the regression 1 on  $e_t\xi_t$ , and use  $TRu^2 = T$  SSR as asymptotically  $x_q^2$  under the hypothesis that  $M_1$  is the correct specification, where  $Ru^2$  is the un-centered R-square of the regression, and SSR is the sum of squared residuals in the regression of step (3). This procedure provides a Chi-square distribution under  $M_1$  in the presence of heteroscedasticity (conditional or unconditional) of unknown form. In generalizing this procedure with the encompassing principle to the multivariate case, we may set  $y_t$  as an n-by-1 vector of dependent variables,  $X_{1t}$  as the n-by-U matrix, and  $X_{2t}$  as the n-by-V matrix. The models  $M_1$  and  $M_2$  can be expressed as

$$M_1: y_t = \Gamma X_{1t} + \varepsilon_{1t}, \tag{6}$$

$$M_2: y_t = BX_{2t} + \varepsilon_{2t}. \tag{7}$$

In addition, the auxiliary regression is now specified as

$$M^*: y_t = \Gamma X_{1t} + \Pi W_{2t} + \varepsilon_t^*, \tag{8}$$

<sup>&</sup>lt;sup>1</sup> See Harvey (1990, pp. 185-187).

<sup>&</sup>lt;sup>2</sup> Notice that the information set  $\sigma(W_{t-1})$  is not a complete information set for all alternative models.

<sup>&</sup>lt;sup>3</sup> See White (1993) for the details.

<sup>&</sup>lt;sup>4</sup> In fact, the ordinary least square estimates for these parametric models can be shown as the consistent estimates according to White (1984). In our study, the ordinary least square estimates are applied.

where  $W_{2t}$  is an n-by-q matrix of omitted variables not included in model  $M_1$ . In fact, since we assume that the returns share a similar specification of conditional risk premium with the pre-specified variables and latent variables, we set  $W_{2t} = [u_1', \dots, u_t']'$ , where  $u_t$  is a q-by-1 vector of omitted variables for each equation in model  $M_1$  under  $H_0$ . Therefore,  $e_t$  is the residual vector of n equations in model  $M_1$ . Replace the above step (2) with the matrix regression, where  $\xi_t$  now is an n-by-q matrix of misspecification indicator, step (3) of running the regression 1 on  $e_t'$   $\xi_t$  will still generate a Chi-square statistic with degree of freedom q under  $H_0$  according to Wooldridge (1990a).

Furthermore, we follow the approach of Ferson (1990) by assuming linearity for the risk premium. To consider possible seasonality, we also include a constant term and a seasonal dummy of the January effect for the specification tests. In order to generate the latent variables, we perform the vector autoregression (VAR) on the economic state variables. The residuals from the VAR system are treated as the latent variables for proxies of factors. Applying the Schwarz information criterion (SIC), we find that VAR(1) with one lag in each variable is sufficient for the dynamic specification.

With parsimonious encompassing, we begin the model search from the largest model which includes the lags of excess rates of returns on the selected portfolios, the lagged economic variables and the latent variables in our information set, then sequentially reduce the model until further reduction is invalid. The first task we perform is to see if further reduction only on the economic variables is possible or not. Then, we perform the tests to investigate further reduction in the latent variables. Notice that the encompassing tests are also performed with respect to lagged excess rates of returns. Therefore, the residuals are not serially dependent in the time series regression with the correctly specified conditional mean  $E[r_{i,l}|s(W_{t-1})]$ .

#### 4. Data Sources

The macroeconomic factors used in this study are similar to those variables identified by Chen, Roll and Ross (1986), Chan, Chen and Hsieh (1985), Chang and Pinegar (1990) and Chen (1991). The sample period is selected for compatibility. Monthly returns for 20 portfolios classified by firm sizes are compiled over the period from 1953 to 1984. Since literature on finance suggests different behavior between the stock prices of small firms and large firms, this study forms 20 portfolios in any given year ranked by the value of equity of previous year. Specifically, this study ranks firm size initially from the end of 1952 and rebalances the portfolios annually through the end of 1984.

The data for stock returns and firm sizes are obtained from the Center for Research in Securities Prices (CRSP) monthly tapes. Six economic variables (including their lags) and the lagged excess rates of return are applied to approximate the conditional risk premium. These economic variables are specified as follows:

- the term structure of interest rates, UTS, which is the difference between the return of a portfolio or long-term Treasury bonds and the return on a portfolio of short-term Treasury bills,
- the change in expected inflation, DEI, which is the first difference of expected inflation, where the expected inflation is the difference between monthly Treasury bill rates and the expected real rates. The expected real rate is estimated as the arithmetic average of real rates of return on T-bills over the 12-month period prior to the month of interests<sup>2</sup>,
- contemporaneous unexpected inflation, UI, which is measured by the difference between actual and expected inflation,
- the monthly growth rate in industrial production, MP, which is the natural log of the level of industrial production in month *t* to the level of industrial production in month *t*-1,

A similar method is also applied by Mei (1993).
 The approach is similar to Fama and Gibbon (1984).

- the risk premium for default, UPR, which is the return of a portfolio of low quality long-term corporate bonds which are rated no better than Baa in excess of the return on a portfolio of long-term government bonds, and
- the return on the value-weighted NYSE index, denoted as RM.

Monthly data for macroeconomic variables are collected from the following sources:

- industrial production is obtained from the Survey of Current Business,
- holding-period returns on a portfolio of long-term Treasury bonds are obtained from the Bank and Quotation Record,
- holding-period returns on a portfolio of long-term industrial bonds with rating Baa and under are obtained from the Bank and Quotation Record,
- holding-period returns on a portfolio of one month Treasury bills are collected from Ibbotson and Sinquefield (1987),
- inflation is the natural log of the level of the Consumer Price Index in month *t* divided by the level of the Consumer Price Index in month *t*-1, and
- returns on the value-weighted NYSE stock index are obtained from the CRSP monthly tape.

# 5. Empirical Results

Based on equations (1) and (2), we apply the above macroeconomic variables and their latent variables to verify the predictability of excess rates of returns and their systematic risk. Table 1 shows the Chi-square statistics of the robust conditional mean encompassing tests. Each Chi-square statistic is performed by sequentially deleting a certain variable of interest. In our applications, we start the order of deletion from the fourth lag of the excess rates of return, and continue with smaller lags of excess rates of return, the constant term, seasonal dummy, other economic variables, and end with the latent variables. If deleting the variable shows a significant Chi-square statistic, it indicates that the deletion of the variable isn't appropriate for model specification. In other words, the smaller model which deletes this particular variable does not parsimoniously encompass the larger model. Therefore, the other variables following in the list are further considered as possible candidates for the reduction of the model.

Due to the transitivity of parsimonious encompassing, this procedure will provide the smallest possible model that parsimoniously encompasses all larger models. After performing the encompassing tests, we also provide tests to verify the dynamic specifications by examining the serial correlations of error terms in the final model. The result of the robust conditional mean encompassing test indicates the predictive power of the lagged macroeconomic variables for the portfolio returns. In fact, the lags of four economic variables, market index (RM), change in expected inflation (DEI), unexpected change in risk premium for default (UPR), the unexpected change in the term structure of interest rates (UTS), and a seasonal dummy which identifies the January effect explain the conditional excess rates of returns. The evidence indicates that seasonality does exist for these portfolio returns when we include some macroeconomic variables in the conditional mean. This finding is inconsistent with the results of Chen, Roll, and Ross (1986). In brief, seasonality describes part of the time-varying risk premium which can not be explained by the (linear form of) included macroeconomic variables.

The table below reports the robust conditional mean encompassing tests. The list of variables indicates the order in performing the parsimonious encompassing tests. The initial step begins with the largest model which includes all the identified variables. The encompassing test is to examine whether the smaller model that deletes a particular variable can explain the larger model statistically. If the statistic is insignificant, this particular variable can be deleted to reduce the dimensionality of the model. On the other hand, the significant Chi-square statistic shows that the smaller model that excluded this particular variable does not encompass the large model. Consequently, the next step is to select a subsequent variable in the list for encompassing test as a possible reduction of the model while keeping the significant variables from earlier encompassing tests

and the other as-yet-untested prespecified variables in the model. The set {f1,..., f6} represents the set of latent variables generated from the VAR of economic variables. The asterisk sign indicates that the Chi-square statistic is significant at 5% level with 1 degree of freedom.

Table 1 Parsimonious Encompassing Tests for Conditional Risk Premiums and Latent Variables

Economic Variables and latent variables	Chi-square tests
4-lagged excess return	0.080
3-lagged excess return	0.131
2-lagged excess return	0.030
1-lagged excess return	0.435
constant term	51.484*
seasonal dummy for January	17.610*
RM(-1)	15.784*
MP(-1)	0.367
DEI(-1)	5.905*
UI(-1)	0.160
UPR(-1)	5.406*
UTS(-1)	4.242*
f <sub>1</sub>	99.288*
f <sub>2</sub>	6.486*
f <sub>3</sub>	0.779
f <sub>4</sub>	0.000

RM = the return on the value-weighted NYSE index,

MP = the monthly growth rat ein industrial production,

DEI = the change in expected inflation,

UI = contemporaneous unexpected inflation,

UPR = the risk premium for default,

UTS = the term structure of interest rates.

Four error terms of the economic variables based on the VAR(1) model constitute the set of proxy variables for the factors. In other words, the latent variables generated from the following four variables, market index (RM), lagged industrial production growth rate (MP), unexpected risk premium (UPR), and unexpected change of term structure of interest rates (UTS), constitute the proxies for factors. This result shows that the excess rates of return for assets and portfolios can be specified by the lagged economic variables. Furthermore, the results of Table 1 are not affected when the order of the variables is changed in conducting the robust conditional mean encompassing tests.

It is worth noting that the significance of the Chi-square of statistic in Table 1 only indicates that the model which excludes that particular variable does not encompass the large model with that particular variable included. In other words, the model selection tests implemented here are trying to solve the problem of choice of regressors. The comparison of Chi-square statistics for all the statistically significant variables does not necessarily imply which variable has more explanatory power than the other variables.

Table 2 shows all the conditional mean tests of Wooldridge (1991) for dynamic misspecification in each portfolio's excess rates of return after specifying the linear regression with economic variables and latent variables identified by the encompassing tests. Only one portfolio has a slightly significant serial correlation at the fourth lag in the residuals. However, its effect is only one in eighty statistics of twenty portfolios in the table. Therefore, it may be considered as statistically negligible. That is, no significant dynamic misspecification is discovered.

Notice that these results are based on the current information set of our pre-specified variables. If the information set is extended, the parsimonious procedures of model selection should be applied again for the extended information set. However, the model selection for specification of conditional excess rates of return and proxies for factors should be implemented before any meaningful interpretation (for the empirical results) is obtained. That is to say, for any empirical finding of conditional risk premium, the model specification and diagnostic tests should be cautiously examined before the interpretation on the parameters of fitted models. In this study, choices over the numbers of proxies for factors are performed by the model selection specification tests. In addition, the specification on the risk premium and the numbers of latent variables are analyzed by a simple encompassing test for model selections simultaneously.

Table 2 reports the diagnostic tests for serial correlations of residuals  $\epsilon_{it}$  in the following regression:

 $r_{it} = b_i D_t + r_{it} R M_{t-1} + r_{i2} DEI_{t-1} + r_{i3} UPR_{t-1} + r_{i4} UTS_{t-1} + \beta_{i1} f_{1t} + \beta_{i2} f_{2t} + \beta_{i5} f_{5t} + \beta_{i6} f_{6t} + \varepsilon_{it}$ , where i = 1, 2, 3, ..., 20 for 20 grouped portfolios,  $D_t$  represents the seasonal dummy for January effect. Details of the test are shown in Wooldidge (1991). The portfolios are grouped in the ascending order based on the size. The asterisk sign indicates that the Chi-square statistics is significant at 5% level with degrees of freedom equal to the total numbers of lags in checking the serial dependence of  $\varepsilon_{it}$ . The Chi-square statistics of 5% level of significance from degrees of freedom 1 to 4 are,  $x^2_1 = 3.841$ ,  $x^2_2 = 5.991$ ,  $x^2_3 = 7.815$ ,  $x^2_4 = 9.488$ , respectively.

Table 2
Diagnostic Tests for Dynamic Misspecifications

Order of portfolios	1 lag	2 lags	3 lags	4 lags
Portfolio 1	0.053	1.472	2.024	7.105
Portfolio 2	0.064	0.256	1.923	5.539
Portfolio 3	1.061	2.692	2.915	10.327*
Portfolio 4	0.125	0.854	0.981	8.331
Portfolio 5	0.020	0.063	1.675	7.862
Portfolio 6	0.021	0.166	2.125	6.930
Portfolio 7	0.662	3.306	3.547	8.096
Portfolio 8	0.029	0.041	1.116	6.330
Portfolio 9	0.410	1.538	5.699	7.900
Portfolio 10	0.079	0.667	3.061	4.354
Portfolio 11	0.077	0.617	4.112	5.422
Portfolio 12	0.840	0.849	0.898	1.342
Portfolio 13	0.001	0.202	5.574	6.427
Portfolio 14	0.082	0.092	2.097	4.008
Portfolio 15	0.115	1.231	1.268	1.295
Portfolio 16	0.729	0.941	1.509	1.760
Portfolio 17	1.448	1.504	1.536	1.760
Portfolio 18	2.418	2.606	3.987	4.014
Portfolio 19	3.830	5.392	5.478	5.757
Portfolio 20	2.852	2.885	3.099	4.908

#### 6. Conclusion

Traditional tests for the CAPM or multifactor pricing models, such as Chen, Roll and Ross (1986) and Chang and Pinegar (1990), apply univariate two-stage regressions to examine pricing factors. This paper uses a multivariate testing procedure similar to Mei (1993) for the system of equations to detect pricing factors. In particular, this paper investigates the model selection test to determine the best model given some information sets of pre-specified variables.

Due to the pricing errors and the difficulty of selecting the appropriate economic variables, applications of model selection for specification of multifactor pricing models is required. Such a difficulty is resolved by applying Wooldridge's (1990b) robust conditional mean encompassing test and parsimonious encompassing principle on the specification of conditional excess rates of return with Ferson's latent-variable approach. Following Chang and Pinegar (1990), this study takes the seasonality factor into account.

Our empirical result indicates that the conditional excess rates of returns for twenty size portfolios are explained by lagged expected inflation, lagged unexpected premium for default, lagged unexpected change in term structure, a seasonal dummy, and lagged market returns. Four error terms of vector autoregression in economic variables constitute the latent variables for factors.

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